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ON THE $S_\lambda(X)$ AND λ-ZERO DIMENSIONAL SPACES

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Dedicated to professor O.A.S. Karamzadeh to appreciate in honor of his profound contributions to the popularization of mathematics for nearly half a century in the worldwide

Abstract. Let $S_\lambda(X) = \{f \in C(X) : |X \setminus Z(f)| < \lambda\}$, such that λ is a regular cardinal number with $\lambda \leq |X|$. It is generalization of $C_F(X) = S_{\aleph_0}(X)$ and $SC_F(X) = S_{\aleph_1}(X)$. Using this concept we extend some of the basic results concerning the socle to $S_\lambda(X)$. It is shown that if X is a λ-pseudo discrete space, then $C_{K,\lambda}(X) \subseteq S_\lambda(X)$. S_λ-completely regular spaces are investigated. Consequently, X is a $S_{\aleph_1}$-completely regular space if and only if X is $\aleph_1$-zero dimensional space. $S_\lambda P$-spaces are introduced and studied.

1. Introduction

Undoubtedly, one of the most beautiful connections of algebra and topology appears in the structure of the ring of real valued continuous functions. In [7, 8, 9, 17, 23, 24], the subrings of $C(X)$ that containing $C^*(X)$ are investigated. It can be said that in order to achieve the goal of establishing a relation between algebra and topology and describing the topology on X, the subalgebras of $C(X)$ have played an important role. There has always been an attempt to generalize the results of research to $C^*(X)$. We remind the reader that $C^*(X)$ is in fact $C(Y)$,

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where $Y=\beta X$, that is to say $C(X)$ and $C^*(X)$ are of the same type. In [13], Karamzadeh et al. introduced and studied the subalgebra $C_c(X)$ of $C(X)$ consists of functions with countable image. Their results show that $C_c(X)$, although not isomorphic to any $C(Y)$ in general, enjoys most of the important properties of $C(X)$, see [13, 6, 14, 18, 5]. In this paper, unless otherwise mentioned all topological spaces are infinite completely regular Hausdorff and we will employ the definitions and notations used in [11, 16]. For an element f of $C(X)$, the zero-set (resp., cozero-set) of f is denoted by $Z(f)$ (resp., $Coz(f)$) which is the set $\{x\in X : f(x)=0\}$ (resp., $X\setminus Z(f)$). We use $Z(X)$ (resp., $Coz(X)$) to denote the collection of all the zero-sets (resp., cozero-sets) of elements of $C(X)$. An ideal I in $C(X)$ is called a z-ideal if whenever $f\in I$, $g\in C(X)$ and $Z(f)\subseteq Z(g)$, then $g\in I$. For every ideal I in $C(X)$, if $\bigcap Z[I]=\bigcap_{f\in I} Z(f)$ is empty, I is said to be free, else fixed. We remind the reader that the intersection of every family of open sets with cardinality less than λ is called a G_λ-set and especially every $G_{\aleph_1}$-set is a G_δ-set, see [22]. A topological space X is a P_λ-space, whenever every G_λ-set in it is open, in particular X is a P-space whenever it is a $P_{\aleph_1}$-space, see [16, 22]. $C(X)$ is a lot in common with noncommutative rings. Those ideals of $C(X)$ that are not usually studied in commutative rings, namely minimal ideals, socle, essential ideal are investigated in [3, 4, 12, 19]. The socle of $C(X)$ (i.e., $C_F(X)$) which is in fact a direct sum of minimal ideals of $C(X)$ is characterized topologically in [19, Proposition 3.3], and it turns out that

$$C_F(X)=\{f\in C(X) : |X\setminus Z(f)|<\aleph_0\}$$

is a useful object in the context of $C(X)$. It is folklore that one of the main objectives of working in the context of $C(X)$ is to characterize topological properties of a given space X in terms of suitable algebraic properties of $C(X)$ and $C_F(X)$ is an important object in this way, see [19, 3, 12, 4, 25, 10]. Recall that $C_F(X)$ is a z-ideal and X is discrete if and only if $C_F(X)$ is a free ideal. In [20], the concept of $SC_F(X)=\{f\in C(X) : |X\setminus Z(f)|<\aleph_1\}$ has been generalized to λ-super socle of $C(X)$. The set of elements of $C(X)$ that cardinality of their cozerosets are less than λ where λ is a regular cardinal number such that $\lambda\leq|X|$, is called the λ-super socle of $C(X)$ and denoted by $S_\lambda(X)$. It is manifest that $C_F(X)=S_{\aleph_0}(X)$ and $SC_F(X)=S_{\aleph_1}(X)$, see also [15, 21, 13]. As it is remarked in the introduction of [21], the subject of $C_F(X)$ and $S_\lambda(X)$ seems to be receiving increasing attention in the literature. Moreover, we also believe that the present article together with [13, 15, 20, 21, 25] can perhaps provide some basic and necessary results for the study of the subject, in the future. The support of f is the closure of $X \setminus Z(f)$ and $C_K(X)$ denotes the family of all functions in $C(X)$ having compact support, see in [16, 4D]. If X is compact, then since the support of any function is a closed set and hence compact we infer that $C(X) = C_K(X)$. If X is not compact then $C_K(X)$ is an ideal in $C(X)$. $C_K(X)$ is a free ideal if and only if X is locally compact but not compact. We remind the reader that $C_K(X)$ is in every free ideal in $C(X)$ and $C_K(X)$ is actually the intersection of these ideals. We define pseudo countable discrete space and show that whenever X is a countable pseudo discrete space with $|X| > \aleph_1$, then $C_K(X) \subseteq S_{\aleph_1}(X)$. Recall that $C_\infty(X)$ denotes the family of all functions f in $C(X)$ for which the set $\{x \in X : |f(x)| \geq \frac{1}{n}\}$ is compact for every $n \in \mathbb{N}$ such functions are said to vanish at infinity, see [16]. We extend the latter definitions and results. λ-zero dimensional spaces are introduced and it is shown that X is an $\aleph_1$-zero dimensional space if and only if its topology coincides with the weak topology induced by $S_{\aleph_1}(X)$. We prove that X is a $S_{\aleph_1}$-completely regular space if and only if X is $\aleph_1$-zero dimensional space. A space X is called a $S_\lambda P$-space if $Z(f)$ is open for each $f \in S_\lambda(X)$. It is shown that every $\aleph_1$-zero dimensional $S_{\aleph_1}P$-space is a P-space.

2. $S_{\aleph_1}(X)$ VERSUS $C_K(X)$ AND $C_\infty(X)$

Recall that a topological space X is called λ-compact if each open cover of X can be reduced to an open cover whose cardinality is less than λ, where λ is the least infinite cardinal number with this property, see [22]. We extend the definition of $C_K(X), C_\infty(X)$.

Definition 2.1. Let X be a topological space,

$$C_{K,\lambda}(X) = \{f \in C(X) :\ cl(X \setminus Z(f)) \text{ is } \lambda\text{-compact}\},$$

$$C_{\infty,\lambda}(X) = \{f \in C(X) : \{x \in X : |f(x)| \geq \frac{1}{n}\} \text{ is } \lambda\text{-compact for all } n\}.$$

In particular, $C_{K,\aleph_0}(X) = C_K(X)$ and $C_{\infty,\aleph_0}(X) = C_\infty(X)$.

If X is λ-compact, then $C_{K,\lambda}(X) = C_{\infty,\lambda}(X) = C(X)$. Let us recall that every closed subset of a λ-compact space is a μ-compact space for some $\mu \leq \lambda$, see [22]. It is evident that $C_{K,\lambda}(X) \subseteq C_{\infty,\mu}(X)$ for some $\mu \leq \lambda$. It would be interesting to characterize topological spaces X such that the containment is actually an equality.

Definition 2.2. A Hausdorff space X is called almost locally λ-compact if every nonempty open set of X contains a nonempty set with λ-compact closure.

We remind the reader that almost locally $\aleph_0$-compact space is called almost locally compact space, see [2]. A Hausdorff space X is said to be locally λ-compact if every point in X has a λ-compact neighborhood, see [22]. If $\mu \leq \lambda$, then every μ-compact subspace of a P_λ-space is closed, see [22].

Proposition 2.3. *Every locally λ-compact P_λ-space is an almost locally μ-compact space, for some $\mu \leq \lambda$.*

Proof. Suppose that X is a locally λ-compact space, then for every $x \in X$ there exists a λ-compact neighborhood U_x. If A is a nonempty open set of X and $x \in A$, there exists an open neighborhood V_x that containing x where $x \in V_x \subseteq A$. Put $G = U_x \cap V_x$ so $x \in G$ and G is an open subset of A. Since U_x is a λ-compact set of a P_λ-space X we infer that U_x is closed. $cl(G) \subseteq cl(U_x) = U_x$ and U_x is λ-compact, so $cl(G)$ is μ-compact for some $\mu \leq \lambda$ and the proof is complete. □

Proposition 2.4. *Every open subspace of an almost locally λ-compact space is an almost locally μ-compact space for some $\mu \leq \lambda$.*

Proof. Let X be almost locally λ-compact and Y be an open subspace of X. If A be a nonempty open subset of Y, then A is open in X and consequently there exists a nonempty set G where $cl_X(G)$ is λ-compact and $G \subseteq A$. $cl_Y(G) \subseteq cl_X(G)$, so $cl_Y(G)$ is μ-compact for some $\mu \leq \lambda$ and we are done. □

Definition 2.5. A topological space X is said to be a λ-pseudo discrete space if every λ-compact subset of X has interior of cardinality less than λ, where λ is the least infinite cardinal number with this property.

$\aleph_0$-pseudo discrete space is called pseudo discrete space, see [1]. Clearly, if every λ-compact subset of X has cardinality less than λ then X is a λ-pseudo discrete space and in particular, every P-space is a pseudo discrete space. The next two results are the counterparts of facts in [2].

Proposition 2.6. *Every open subspace of a λ-pseudo discrete space is λ-pseudo discrete space.*

Proof. Suppose that X is a λ-pseudo discrete space and Y is an open subspace of X. If A is a λ-compact subset of Y, then A is a λ-compact subset of X and $|int_X(A)| < \lambda$. We know

$$int_X(A) = int_Y(A) \cap int_X(Y) = int_Y(A)$$

hence the proof is complete. □

We recall that a point $x \in X$ is called λ-isolated if it has a neighborhood with cardinality less than λ, and $I_\lambda(X)$ is denoted the set of λ-isolated points of X. A topological space X is said to be λ-discrete if every point of X is λ-isolated i.e., $I_\lambda(X) = X$, see [20].

Proposition 2.7. *Every locally λ-compact and λ-pseudo discrete space is a λ-discrete space.*

Proof. Since X is locally λ-compact we infer that $x \in X$ has a neighborhood V where $cl(V)$ is λ-compact. The definition of λ-pseudo discrete implies that the interior of $cl(V)$ has cardinality less than λ and consequently $|V| < \lambda$ i.e., x has a neighborhood with cardinality less than λ. Therefore X is λ-discrete. □

Proposition 2.8. *If X is a λ-discrete space, then $C_{K,\lambda}(X) \subseteq S_\lambda(X)$.*

Proof. Let X be λ-discrete, then every λ-compact space has cardinality less than λ. Hence $f \in C_{K,\lambda}(X)$ implies that $|cl[X \setminus Z(f)]| < \lambda$. So $|X \setminus Z(f)| < \lambda$ and we are done. □

Definition 2.9. A topological space X is said to be a λ-pseudo space if every λ-compact subset of X has cardinality less than λ.

We recall that $\aleph_0$-pseudo space is called pseudo finite space, see [19]. Clearly, every subspace of a λ-pseudo space is a λ-pseudo space.

Example 2.10. Every λ-pseudo space is λ-pseudo discrete but the converse is not true in general. For instance, we consider the free union of a discrete space D and the rational numbers set $\mathbb{Q}$, it is a pseudo discrete space which is not pseudo finite.

We know that $C_K(X) = C_F(X)$ if and only if X is a pseudo-discrete space. So if X is a pseudo finite space, then $C_K(X)$ and $C_F(X)$ coincide. $C_\infty(X) = C_F(X)$ if and only if X is a pseudo discrete space with only a finite number of isolated points, i.e., if and only if $C_F(X) = C_K(X) = C_\infty(X)$. For instance Q is a pseudo discrete space with no isolated points, hence $C_F(Q) = C_K(Q) = C_\infty(Q) = 0$, see [3, 4, 19]. If every compact subset of X has countable interior and $|X| > \aleph_1$, then $C_K(X) \subseteq S_{\aleph_1}(X)$. Now we extend the latter facts.

Theorem 2.11. *If X is a λ-pseudo discrete space with $|X| > \lambda$, then $C_{K,\lambda}(X) \subseteq S_\lambda(X)$.*

Proof. Let X be a λ-pseudo discrete space and $f \in C_{K,\lambda}(X)$. Hence $cl[X \setminus Z(f)]$ is λ-compact and so $int(cl[X \setminus Z(f)])$ has cardinality less than λ. Since $X \setminus Z(f) \subseteq int(cl[X \setminus Z(f)])$ we infer that $f \in S_\lambda(X)$. □

We recall the reader that if the set of λ-isolated points of X is finite then $I_\lambda(X) = I(X)$. So $S_\lambda(X)$ and $C_F(X)$ coincide if and only if $|I_\lambda(X)| < \aleph_0$, see [20].

3. λ-zero dimensional spaces

We remind the reader that a subset of X that is closed and open is called clopen. If X is a zero-dimensional space, then each zero-set in $Z(X)$ contains a member of $Z_c(X) = \{Z(f) : f \in C_c(X)\}$.

Definition 3.1. A topological space X with $|X| > \lambda$ is called λ-zero dimensional if X has a base consisting of clopen sets with cardinality less than λ, where λ is the least infinite cardinal number with this property.

Example 3.2. The space of the countable ordinals is an $\aleph_1$-zero dimensional space. For each point of this space has a neighborhood basis of countable clopen sets.

Definition 3.3. A Hausdorff space X is called S_λ-completely regular if whenever $F \subseteq X$ is a closed set and $x \notin F$, then there exists $f \in S_\lambda(X)$ with $f(F) = 0$ and $f(x) = 1$.

The proof of the following proposition is evident.

Proposition 3.4. *Let X be a topological space and λ, μ be two regular cardinals such that $|X| \geq \lambda > \mu$. If X is S_μ-completely regular, then X is S_λ-completely regular.*

Proposition 3.5. *A topological space X is a $S_{\aleph_1}$-completely regular space but not $S_{\aleph_0}$-completely regular if and only if X is an $\aleph_1$-zero dimensional space.*

Proof. Let X be $\aleph_1$-zero dimensional and F be a closed subset of X such that $x \notin F$. There exists a countable clopen subset U where $x \in U \subseteq X \setminus F$. Now, we define $f : X \to \mathbb{R}$ such that $f(x) = 1$, if $x \in U$ and $f(x) = 0$, otherwise. Clearly, $f \in S_{\aleph_1}(X)$, $x \in U$ and $f(F) = 0$. Conversely, let $x \in X$ and U_x be a neighborhood of x. There exists $f \in S_{\aleph_1}(X)$ where $f(x) = 1$ and $f(X \setminus U_x) = 0$. Hence there exists $c \in [0, 1]$ such that $c \notin f(X)$. Therefore $f^{-1}(c, \infty)$ is clopen which contains x and $f^{-1}(c, \infty) \subseteq X \setminus Z(f)$, hence it is countable, i,e., X is λ-zero dimensional where $\lambda \leq \aleph_1$. If $\lambda = \aleph_0$, then X is $S_{\aleph_0}$-completely regular which is a contradiction. Therefore we are done. $\square$

Proposition 3.6. *A Hausdorff space X is S_λ-completely regular if and only if whenever $F \subseteq X$ is closed and $x \in X \setminus F$, then x and F have two disjoint zero-set neighborhoods in $Z[S_\lambda(X)]$.*

Proof. Let $Z_1, Z_2 \in Z[S_\lambda(X)]$ and $x \in intZ_1$, $F \subseteq int\ Z_2$ and $Z_1 \cap Z_2 = \emptyset$. Hence there exist $f, g \in S_\lambda(X)$ where $Z_1 = Z(f)$ and $Z_2 = Z(g)$. We put $h = \frac{|g|}{|f|+|g|}$, it is evident that $h \in S_\lambda(X)$, $h(x) = 1$ and $h(F) = 0$. Conversely, let X be S_λ-completely regular and $F \subseteq X$ be closed and $x \notin F$. There exists $f \in S_\lambda(X)$ such that $f(x) = 1$ and $f(F) = 0$. We put $Z'_1 = \{x \in X : f(x) \leq 1/3\}$ and $Z'_2 = \{x \in X : f(x) \geq 2/3\}$. It is evident that $Z'_1, Z'_2 \in Z[S_\lambda(X)]$, $Z'_1 \cap Z'_2 = \emptyset$ and we have $F \subseteq intZ'_1$, $x \in intZ'_2$. □

Corollary 3.7. *Every closed set in an λ-zero dimensional space X is an intersection of zero-set neighborhoods in $S_\lambda(X)$.*

Proof. Let F be a closed set in λ-zero dimensional space X. By the previous propositions, for each $x \notin F$ there exists a zero-set neighborhood $Z_x \in Z[S_\lambda(X)]$ such that $x \notin Z_x$ and $F \subseteq Z_x$. Therefore $F = \cap_{x \notin F} Z_x$ and we are done. □

Corollary 3.8. *Every neighborhood of each point in an λ-zero dimensional space X contains a zero-set neighborhood $Z(f)$, where $f \in S_\lambda(X)$.*

Let $Z[S_\lambda(X)]$ be a base of closed set and F be a closed subset of X where $x \notin F$. There exists $Z(f) \in Z[S_\lambda(X)]$ such that $F \subseteq Z(f)$ and $x \notin Z(f)$ i.e., $f(F) = 0$ and $f(x) \neq 0$, now we can get the following corollary.

Corollary 3.9. *A topological space X is a λ-zero dimensional space if and only if $F = \{Z(f) : f \in S_\lambda(X)\}$ is a base for the closed sets in X or equivalently if and only if $B = \{\mathrm{int(Z(f)) : f \in S_\lambda(X)}\}$ is a base for the open sets in X.*

If we apply the proof of [16, 3.11(a)] word-for-word, we obtain the following.

Proposition 3.10. *Let X be a $\aleph_1$-zero dimensional space and A, B be two disjoint closed sets in X such that A is compact, then there is $f \in S_{\aleph_1}(X)$ with $f(A) = 0$, $f(B) = 1$.*

Proof. Let A, B be two disjoint closed set in X and A be compact. For each $x \in A$ there exist $Z_x, Z'_x \in Z[S_{\aleph_1}(X)]$ such that Z_x be a neighborhood of x and $B \subseteq Z'(X)$. Therefore the cover $\{Z_x : x \in A\}$ has a finite subcover $\{Z_{x_1}, Z_{x_2}, \dots, Z_{x_n}\}$. Hence

$$A \subseteq Z = Z_{x_1} \cup \dots \cup Z_{x_n},\ B \subseteq Z = Z'_{x_1} \cap \dots \cap Z_{x_n}$$

and clearly Z, Z' are disjoint zero sets in $Z[S_{\aleph_1}(X)]$. □

Corollary 3.11. *X is a $\aleph_1$-zero dimensional space if and only if its topology coincides with the weak topology induced by $S_{\aleph_1}(X)$.*

4. $S_\lambda P$-space

In this section we introduce $S_\lambda P$-spaces. We recall that X is a P-space if and only if $C(X)$ is a regular ring if and only if $Z(f)$ is open for each $f \in C(X)$, see [16, 4J]. We observe trivially if $C(X)$ is regular, then so is $S_\lambda(X)$, but the converse is not true in general.

Example 4.1. Let $X = [-1,0] \dot{\bigcup} \mathbb{N}$ be the free sum of $[-1,0]$ and the natural numbers $\mathbb{N}$, then X is $S_{\aleph_1}P$-space which is not a P-space.

Motivated by this, we offer the following definition.

Definition 4.2. A space X is called a $S_\lambda P$-space if $Z(f)$ is open for each $f \in S_\lambda(X)$.

Proposition 4.3. *X is a $S_\lambda P$-space if and only if $\bigcap_{i=1}^{\infty} Z(f_i)$ where $f_i \in S_\lambda(X)$ is an open zero set in $Z[S_\lambda(X)]$.*

Proof. Since λ is a regular cardinal number we infer that for a collection $\{f_n : n \in \mathbb{N}\} \subseteq S_\lambda$, if $f = \sum_{n\in\mathbb{N}} \frac{|f_n| \wedge 1}{2^n}$ then $Z(f) = \bigcap_{n\in\mathbb{N}} Z(f_n)$. The regularity of the infinite cardinal number λ implies that $|X \setminus Z(f)| < \lambda$. It follows that $f \in S_\lambda(X)$ and we are done. $\square$

Proposition 4.4. *If $\lambda \leq \mu$ then every $S_\lambda P$-space is a $S_\mu P$-space.*

Proof. Let X be a $S_\lambda P$-space. We must show that for each $f \in S_\mu(X)$, there exists $g \in S_\mu(X)$ with $f = f^2 g$. Since $S_\lambda(X)$ is regular, there is $h \in S_\lambda(X)$ with $f = f^2 h$. Consequently, $f = f^2 g$, where $g = h^2 f$. It is also evident that $Z(f) \subseteq Z(g)$ and $g(x) = \frac{1}{f(x)}$, whenever $x \notin Z(f)$. Since $X \setminus Z(g) \subseteq X \setminus Z(f)$ we infer that $g \in S_\mu(X)$ and we are done. $\square$

Corollary 4.5. *Let X be a $\aleph_1$-zero dimensional $S_{\aleph_1}P$-space. So each G_δ-set A containing a compact set S contains a zero-set in $Z[S_{\aleph_1}(X)]$ containing S. In particular, every $\aleph_1$-zero dimensional $S_{\aleph_1}P$-space is a P-space.*

Proof. Let $A = \bigcap_{n=1}^{\infty} U_n$ where U_n's are open subsets of X. There exists zero set $F_n \in Z[S_{\aleph_1}(X)]$ such that $K \subseteq F_n \subseteq U_n$. Therefore $K \subseteq \bigcap_{n=1}^{\infty} F_n$ and $\bigcap_{n=1}^{\infty} F_n$ as a countable intersection of zero sets in $Z[S_{\aleph_1}(X)]$ is a zero set in $Z[S_{\aleph_1}(X)]$, and we are done. $\square$

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References

1. F. Azarpanah, Algebraic properties of some compact spaces, *Real Anal. Exchange*, **21** (1999), 105–112 .
2. F. Azarpanah, Intersection of essential ideals in $C(X)$, *Proc. Amer. Math. Soc.*, **125**(7) (1997), 2149–2154 .
3. F. Azarpanah and O. A. S. Karamzadeh, Algebraic characterization of some disconnected spaces, *Italian. J. Pure Appl. Math.* **12** (2002), 155–168.
4. F. Azarpanah, O. A. S. Karamzadeh and S. Rahmati, $C(X)$ VS. $C(X)$ modulo its socle, *Coll. Math.* **3** (2008), 315–336.
5. F. Azarpanah, O. A. S. Karamzadeh, Z. Keshtkar and A. R. Olfati, On maximal ideals of $C_c(X)$ and the uniformity of its localizations, *Rocky Mt. J. Math.*, **48**(2) (2018), 1–9.
6. P. Bhattacharjee, M.L. Knox, W. McGovern, The classical ring of quotient of $C_c(X)$, *App. Gen.Topol.*, **15** (2014), 147–154.
7. L. Byun and S. Watson, Local invertibility in subrings of $C^*(X)$, *Bull. Aust. Math. Soc.*, **46** (1992), 449–458.
8. L. Byun and S. Watson, Prime and maximal ideals in subrings of $C(X)$, *Topology Appl.*, **40**(1) (1991), 45–62.
9. J. M. Dominguez, J. Gomez and M.A. Mulero, Intermediate algebras between $C^*(X)$ and $C(X)$ as rings of fractions of $C^*(X)$, *Topology Appl.*, **77** (1997), 115–130.
10. T. Dube, Contracting the Socle in Rings of Continuous Functions, *Rend. Semin. Mat. Univ. Padova*, **123** (2010), 37–53.
11. R. Engelking, General Topology, Berlin, Germany, Heldermann Verlag (1989).
12. A. A. Estaji and O. A. S. Karamzadeh, On $C(X)$ modulo its socle, *Comm. Algebra*, **31** (2003), 1561–1571.
13. M. Ghadermazi, O. A. S. Karamzadeh and M. Namdari, On the functionally countable subalgebra of $C(X)$, *Rend. Sem. Mat. Univ. Padova*, **129** (2013), 47–69.
14. M. Ghadermazi, O. A. S. Karamzadeh and M. Namdari, $C(X)$ versus its functionally countable subalgebra, *Bull. Iran. Math. Soc.*, **245** (2019), 173–187.
15. S. G. Ghasemzadeh, O. A. S. Karamzadeh and M. Namdari, The super socle of the ring of continuous functions, *Math. Slovaca*, **67** (2017), 1001–1010.
16. L. Gillman and M. Jerison, Rings of continuous functions, Springer-Verlag, 1976.
17. H. Hewitt, Rings of real-valued continuous functions I, *Trans. Amer. Math. Soc.*, **64** (1948), 54–99.
18. O. A. S. Karamzadeh, M. Namdari and S. Soltanpour, On the locally functionally countable subalgebra of $C(X)$, *App. Gen. Topology*, **16** (2015), 183–207.
19. O. A. S. Karamzadeh and M. Rostami, On the intrinsic topology and some related ideals of $C(X)$, *Proc. Amer. Math. Soc.*, **93** (1985), 179–184.
20. S. Mehran and M. Namdari, The λ-super socle of the ring of continuous functions, Categories Gen. *Algebraic Struct. Appl.*, **6** (2017), 37–50.
21. S. Mehran, M. Namdari and S. Soltanpour, On the essentiality and primeness of λ- super socle of $C(X)$, *Appl. Gen. Topol.*, **19** (2018), 261–268.
22. M. Namdari and M. A. Siavoshi, A generalization of compact spaces, *JP J. Geom. Topol.*, **11**(3) (2011), 259–270.

23. L. Redlin and S. Watson, Maximal ideals in subalgebras of $C(X)$, *Proc. Amer. Math. Soc.*, **100** (1987), 763–766.
24. L. Redlin and S. Watson, Structure spaces for rings of continuous functions with applications to realcompactifications, *Fundamenta Mathematicae*, **152** (1997), 151–163.
25. S. Soltanpour, On the locally socle of $C(X)$ whose local cozeroset is cocountable (cofinite), *Hacet. J. Math. Stat.*, **48**(5) (2019), 1430–1436.

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$S_\lambda(X)$ و فضاهای λ-صفربعدی

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گیریم $S_\lambda(X) = \{f \in C(X) : |X \setminus Z(f)| < \lambda\}$ به طوری که λ یک عدد کاردینال منظم و $|X| \leq \lambda$ است. $S_\lambda(X)$ تعمیمی از $C_F(X) = S_{\aleph_0}(X)$ و $SC_F(X) = S_{\aleph_1}(X)$ است. با استفاده از این مفهوم، تعدادی از نتایج پایه‌ای درمورد ساکل را برای $S_\lambda(X)$ توسیع می‌دهیم. نشان می‌دهیم که اگر X یک فضای λ-شبه گسسته باشد، آن‌گاه $C_{K,\lambda}(X) \subseteq S_\lambda(X)$. فضاهای S_λ-کاملاً منظم بررسی شده‌اند. ثابت می کنیم که X یک فضای $S_{\aleph_1}$-کاملاً منظم است اگر و تنها اگر X فضای $\aleph_1$-صفربعدی باشد. در انتها $S_\lambda P$-فضاها معرفی و مورد مطالعه قرار گرفته‌اند.

کلمات کلیدی: λ-سوپر ساکل $C(X)$، فضای λ-صفر بعدی، فضای S_λ-کاملاً منظم، $S_\lambda P$-فضا.