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TOTAL DOMINATION POLYNOMIAL OF GRAPHS FROM PRIMARY SUBGRAPHS

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Abstract. Let $G = (V, E)$ be a simple graph of order n. The total dominating set is a subset D of V that every vertex of V is adjacent to some vertices of D. The total domination number of G is equal to minimum cardinality of total dominating set in G and is denoted by $\gamma_t(G)$. The total domination polynomial of G is the polynomial $D_t(G, x) = \sum d_t(G, i)x^i$, where $d_t(G, i)$ is the number of total dominating sets of G having size i. A root of $D_t(G, x)$ is called a total domination root of G. Let G be a connected graph constructed from pairwise disjoint connected graphs $G_1, \ldots, G_k$ by selecting a vertex of G_1, a vertex of G_2, and identify these two vertices. Then continue in this manner inductively. We say that G is obtained by point-attaching from $G_1, \ldots, G_k$ and that G_i's are the primary subgraphs of G. In this paper, we consider some particular cases of these graphs that most of them are of importance in chemistry and study their total domination polynomials.

1. Introduction

Let $G = (V, E)$ be a simple graph of order n. For any vertex $v \in V$, the open neighborhood of v is the set $N(v) = \{u \in V | uv \in E\}$ and the closed neighborhood is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subset V$, the open neighborhood of S is the set $N(S) = \bigcup\limits_{v \in S} N(v)$ and the closed neighborhood of S is the set $N[S] = N(S) \cup S$. A set $D \subset V$ is a total dominating set if every vertex of V is adjacent to some vertices

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of D, or equivalently, $N(D) = V$. The total dominating number $\gamma_t(G)$ is the minimum cardinality of a total dominating set in G. A total dominating set with cardinality $\gamma_t(G)$ is called a γ_t-set. An i-subset of V is a subset of V of cardinality i. Let $D_t(G, i)$ be the family of total dominating sets of G which are i-subsets and let $d_t(G, i) = |D_t(G, i)|$. The polynomial $D_t(G; x) = \sum_{i=1}^{n} d_t(G, i)x^i$ is defined as total domination polynomial of G. Let G be a connected graph constructed from pairwise

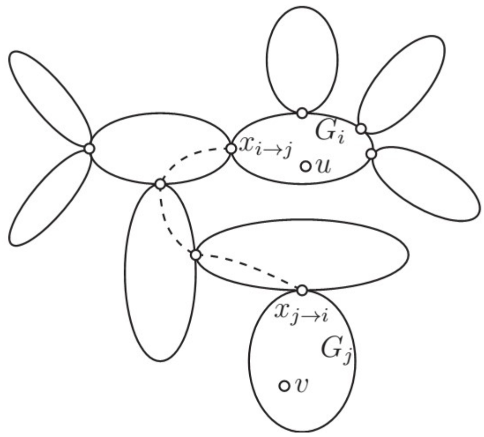

FIGURE 1. Graph G obtained by point-attaching from $G_1, \dots, G_k$.

disjoint connected graphs $G_1, \dots, G_k$ as follows. Select a vertex of G_1, a vertex of G_2, and identify these two vertices. Then continue in this manner inductively. Note that the graph G constructed in this way has a tree-like structure, the G_i's being its building stones (see Figure 1). Usually say that G is obtained by point-attaching from $G_1, \dots, G_k$ and that G_i's are the primary subgraphs of G. A particular case of this construction is the decomposition of a connected graph into blocks (see [7]).

As an example, the n-barbell graph Bar_n with $2n$ vertices, is formed by joining two copies of a complete graph K_n by a single edge (Figure 2). Actually, this graph is a specific kind of point-attaching of two complete graphs K_n and the graph P_2. Observe that the total domination polynomial of n-barbell graph is

$$D_t(Bar_n, x) = \sum_{i=2}^{n} \binom{2n-2}{i-2} x^i + \sum_{i=n+1}^{2n} \binom{2n}{i} x^i.$$

This formula obtain easily from counting the total dominating sets of Bar_n. Calculating the total domination polynomial of a graph G is

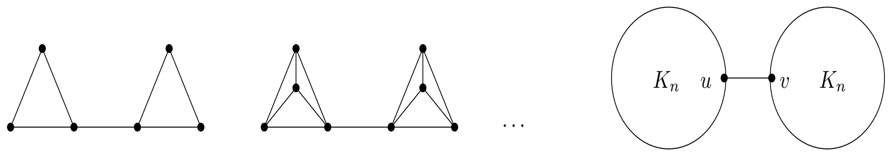

FIGURE 2. Barbell graphs Bar_3, Bar_4 and Bar_n, respectively.

difficult in general, as the smallest power of a non-zero term is the total domination number $\gamma_t(G)$ of the graph, and determining whether $\gamma_t(G) \leq k$ is known to be NP-complete. So presenting a closed formula for the total domination polynomial of any kind of point-attaching graphs is difficult, but for certain classes of graphs, we can find a closed form expression for the total domination polynomial.

In this paper, we consider some particular cases of point attaching graphs and study their total domination polynomials. In Section 2, we consider graphs which obtain by a special point-attaching of a graph H and $|V(H)|$ copies of graph P_3. We prove that some graphs whose total domination polynomial have just two roots $\{-2, 0\}$ are in this form. Also we study the total domination polynomial of some kind of generalized friendship graphs in this section. In Section 3, we investigate the total domination polynomial of cactus chains.

2. Total domination polynomial of graphs from primary subgraphs

In this section, we consider graphs constructed from primary subgraphs and study their total domination polynomial. Some kind of these graphs have interesting properties. In the Subsection 2.1, we prove that a special kind of graphs from primary subgraphs have exactly two total domination roots. In Subsection 2.2 we study the total domination polynomial of the generalized friendship graph.

2.1. **Graphs with exactly two total domination roots $\{-2, 0\}$.**

Graphs whose certain polynomials have few roots can sometimes give interesting information about the structure of the graph. The characterization of graphs with few distinct roots of characteristic polynomials (i.e., graphs with few distinct eigenvalues) have been the subject of many researchers [3, 4, 5, 6]. Also the first authors has studied graphs with few domination roots in [1]. Let H be an arbitrary graph of order n and consider n copies of graph P_3. Let $H(3)$ be a graph is obtained by identifying each vertex of H with an end vertex of a P_3. See Figure

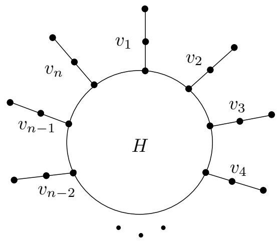

FIGURE 3. The graph $H(3)$.

3. To obtain the total domination polynomial of $H(3)$, we need the following result.

Theorem 2.1. [3] *Let G be a connected graph of order $n \geq 3$. Then $\gamma_t(G) = \frac{2n}{3}$ if and only if G is C_3, C_6 or $H(3)$ for some connected graph H.*

Theorem 2.2. *For any graph H of order n, $D_t(H(3), x) = x^{2n}(x+2)^n$.*

Proof. Let D be a total dominating set of $H(3)$ of size $k \geq n$ in Figure 3. Obviously $\{v_1, v_2, \ldots, v_n\} \subset D$. To choose $n + i$ $(0 \leq i \leq n)$ other vertices of $V(H(3)) \setminus \{v_1, v_2, \ldots, v_n\}$, we have $\binom{n}{i}2^{n-i}$ possibilities. So we have the result. $\square$

Now, we state and prove the following result.

Theorem 2.3. *$D_t(G, x) = x^{2n}(x + 2)^n$ if and only if $G = H(3)$ for some graph H of order n.*

Proof. ($\Leftarrow$) It follows from Theorem 2.2.
($\Rightarrow$) Let G be a graph with $D_t(G, x) = x^{2n}(x + 2)^n$. Thus $|V(G)| = 3n$ and G has no isolated vertex. Since $\gamma_t(G) = 2n$, by Theorem 2.1, every component of G is a cycle C_3, C_6 or $H(3)$ for some connected graph H. Since $D_t(C_3, x) = x^2(x + 3)$ and $D_t(C_6, x) = x^4(x + 3)^2$ does not divide $x^{2n}(x + 1)^n$, we conclude that there exists a graph H such that $G = H(3)$ and the proof is complete. $\square$

Remark 2.4. The characterization of graphs whose certain polynomials have few roots have been an interesting problem and studied well in the literature ([1, 9]). Also there is a conjecture in [2] which states that every integer total domination roots is in the set $\{-3, -2, -1, 0\}$. So finding the graphs whose total domination polynomial have these few roots can be a good start for solving this conjecture. Theorem 2.3 introduces some graphs whose total domination polynomial have just two distinct roots -2 and 0.

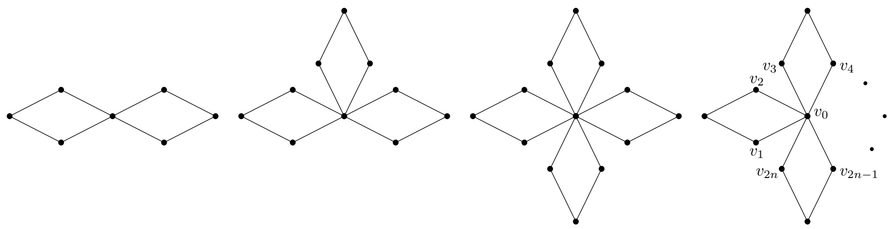

FIGURE 4. Friendship graphs $F_{2,4}, F_{3,4}, F_{4,4}$ and $F_{n,4}$, respectively.

2.2. **Total domination polynomial of the generalized friendship graph.** Here we consider another kind of point-attaching graphs and study their total domination polynomials. The generalized friendship graph $F_{n,q}$ is a collection of n cycles (all of order q), meeting at a common vertex (see Figure 4). The generalized friendship graph may also be referred to as a flower [10]. For $q = 3$ the graph $F_{n,q}$ is denoted simply by F_n and is friendship graph. The total domination polynomial of F_n and its roots studied in [2]. Here, first we compute the total domination number of $F_{n,4}$.

Theorem 2.5. *For any $n \geq 1$, we have $\gamma_t(F_{n,4}) = n + 1$.*

Proof. Let $\{v_1, \ldots, v_{2n}\}$ be vertex set of $F_{n,4}$ that adjacent by v_0 (common vertex in $F_{n,4}$). Then $\{v_0, v_1, v_3, \ldots, v_{2n-1}\}$ is a total dominating set for $F_{n,4}$ (see Figure 4) and the set $D \subseteq V(F_{n,4})$ of size less than or equal n is not total dominating set for $F_{n,4}$, therefore $\gamma_t(F_{n,4}) = n + 1$. □

The following theorem is useful for finding the recurrence relations of the total domination polynomials of graphs. The vertex contraction G/u of a graph G by a vertex u is the operation under which all vertices in $N(u)$ are joined to each other and then u is deleted (see [11]).

Theorem 2.6. [8]

(i) *For any vertex u in the graph G we have $D_t(G, x)$*

$$= D_t(G \setminus u, x) + xD_t(G/u, x) + x^2 \sum_{v \in N(u)} D_t(G \setminus N[\{u, v\}], x)$$
$$- (1 + x)p_u(G),$$

where $p_u(G, x)$ is the polynomial counting the total dominating sets of $G \setminus u$ which do not contain any vertex of $N(u)$ in G.

(ii) *Let $u, v \in V(G)$ be two non-adjacent vertices of G with $N(v) \subseteq N(u)$. Then $D_t(G, x)$*

$$= D_t(G \setminus u, x) + xD_t(G/u, x) + x^2 \sum_{w \in N(u)\cap N(v)} D_t(G \setminus N[\{u, w\}], x).$$

(iii) *Let $u, v \in V(G)$ be two vertices of G with $N[v] \subseteq N[u]$. Then*

$$D_t(G, x) = D_t(G \setminus u, x) + xD_t(G/u, x) + x^2 \sum_{w \in N(u)} D_t(G \setminus N[\{u, w\}], x).$$

(iv) *Let $e = \{u, v\} \in E(G)$ and $N[v] = N[u]$. Then*

$$D_t(G, x) = D_t(G \setminus e, x) + x^2 D_t(G \setminus N[u], x).$$

Now we state and prove a recurrence relation for the total domination polynomial of $F_{n,4}$.

Theorem 2.7. *For any $n \geq 1$, we have*

$$D_t(F_{n,4}, x) = x(x+2)[(x+1)D_t(F_{n-1,4}, x) - (x^3 + 2x)^{n-1}],$$

with initial value $D_t(F_{1,4}, x) = x^4 + 4x^3 + 4x^2$.

Proof. Consider graph $F_{n,4}$ and u, v as shown in Figure 5. By Theorem 2.6, we have

$$\begin{aligned}
D_t(F_{n,4}, x) &\overset{part(ii)}{=} D_t(F_{n,4} \setminus u, x) + xD_t(F_{n,4}/u, x) + x^2 D_t(P_3, x)^{n-1} \\
&\overset{part(iii)}{=} \overbrace{D_t((F_{n,4} \setminus u) \setminus v, x)}^{0} + xD_t((F_{n,4} \setminus u)/v, x) \\
&+ x^2 D_t(P_3, x)^{n-1} + x[D_t((F_{n,4}/u) \setminus v, x) \\
&+ xD_t((F_{n,4}/u)/v, x) + x^2 D_t(P_3, x)^{n-1}] + x^2 D_t(P_3, x)^{n-1} \\
&= (x^2 + 2x)D_t((F_{n,4} \setminus u)/v, x) + (x^3 + 2x^2)D_t(P_3, x)^{n-1} \\
&\overset{part(i)}{=} (x^2 + 2x)[D_t(F_{n-1,4}, x) + xD_t(F_{n-1,4}, x) \\
&- (x+1)p_w((F_{n,4} \setminus u)/v, x)] + (x^3 + 2x^2)D_t(P_3, x)^{n-1} \\
&= x(x+1)(x+2)[D_t(F_{n-1,4}, x) - p_w((F_{n,4} \setminus u)/v, x)] \\
&+ (x^3 + 2x^2)D_t(P_3, x)^{n-1}.
\end{aligned}$$

Since $p_w((F_{n,4} \setminus u)/v, x) = D_t(P_3, x)^{n-1}$, so we have the result. □

3. Total domination polynomial of cactus chains

In this section, we consider another kind of point-attaching graphs and study their total domination polynomials. These kind of graphs are important in Chemistry. A cactus graph is a connected graph in which no edge lies in more than one cycle. Consequently, each block

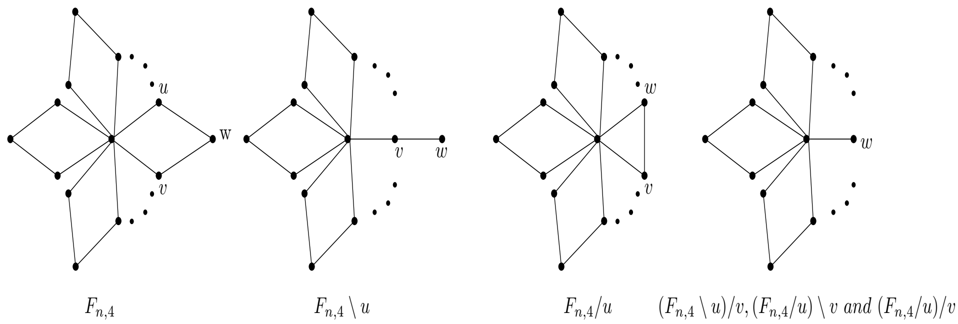

FIGURE 5. Graphs which considered in Theorem 2.7

of a cactus graph is either an edge or a cycle. If all blocks of a cactus G are cycles of the same size i , the cactus is i -uniform. A triangular cactus is a graph whose blocks are triangles, i.e., a 3 -uniform cactus. We call the number of triangles in G the length of the chain. Obviously, all chain triangular cactus of the same length are isomorphic. Hence, we denote the chain triangular cactus of length n by T_n (see Figure 6). By replacing triangles in this definition by cycles of length 4 we obtain cacti whose every block is C_4. In this section we shall study the total domination polynomial of some cactus chains.

3.1. **Total domination polynomial of the chain triangular cactus.** In this subsection we shall study the total domination polynomial of chain triangular cactus. To do this, we consider graph G_n as shown in Figure 6, which is also a kind of point attaching graphs. Note that G_n is a point attaching of T_n and P_2. First we state and prove the following theorem:

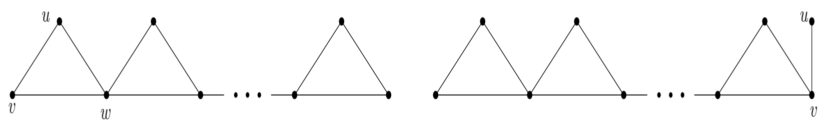

FIGURE 6. The graphs T_n and G_n, respectively.

Theorem 3.1. *For every* $n \geq 2$,

$$D_t(G_n, x) = (x+1)[D_t(T_n, x) - D_t(G_{n-1}, x)] + x^2 D_t(G_{n-2}, x),$$

where $D_t(G_0, x) = x^2$, $D_t(G_1, x) = x^4 + 3x^3 + 3x^2$ *and* $D_t(T_2, x) = x^5 + 5x^4 + 6x^3 + 4x^2$.

Proof. Consider the graph G_n as shown in Figure 6. Since $G_n \setminus u$ is isomorphic to G_n/u, by Theorem 2.6(i), we have

$$\begin{aligned}D_t(G_n,x) &= (x+1)D_t(G_n/u,x)\\ &\quad + x^2 D_t(G_n \setminus N[\{u,v\}],x) - (x+1)p_u(G_n)\\ &= (x+1)D_t(T_n,x) + x^2 D_t(G_{n-2},x) - (x+1)D_t(G_{n-1},x).\end{aligned}$$

□

Theorem 3.2. *For every $n \geq 3$,*

$$D_t(T_n,x) = (x+1)D_t(G_{n-1},x) + x^2[D_t(G_{n-2},x) + D_t(G_{n-3},x)].$$

Proof. Consider the graph T_n and its vertex u as shown in the Figure 6. By Theorem 2.6(iii), we have

$$\begin{aligned}D_t(T_n,x) &= (x+1)D_t(T_n/u,x) + x^2[D_t(T_n \setminus N[\{u,v\}],x)\\ &\quad + D_t(T_n \setminus N[\{u,w\}],x)]\\ &= (x+1)D_t(G_{n-1},x) + x^2[D_t(G_{n-2},x) + D_t(G_{n-3},x)].\end{aligned}$$

□

3.2. **Total domination polynomial of para-chain square cactus graphs.** In this subsection we consider a para-chain of length n , Q_n , as shown in Figure 7 and obtain a recurrence relation for the total domination polynomial of Q_n.

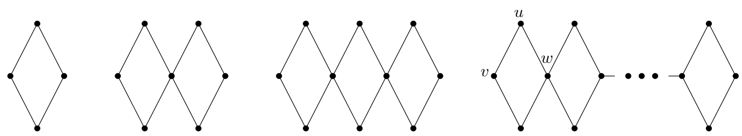

FIGURE 7. Para-chain square cactus graph Q_1, Q_2, Q_3 and Q_n, respectively.

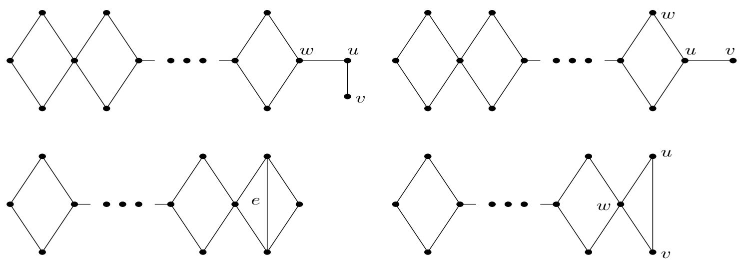

FIGURE 8. graphs $Q_n(2), Q_n(1), Q_n + e$ and Q_n^{Δ}, respectively.

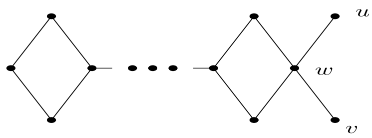

FIGURE 9. graph Q_n'.

Lemma 3.3. *For graphs in Figures 8 and 9:*

(i) $D_t(Q_n(1),x) = xD_t(Q_n,x) + x^2[D_t(Q_{n-1},x) + 3D_t(Q_{n-1}',x)]$, *where* $D_t(Q_0(1),x) = x^2$.

(ii) $D_t(Q_n(2),x) = x^2[D_t(Q_n,x) + (x+1)D_t(Q_{n-1},x) + (3x+1)D_t(Q_{n-1}',x)]$, *where* $D_t(Q_0(2),x) = x^3 + 2x^2$.

(iii) $D_t(Q_n',x) = x(x+1)D_t(Q_n,x) + x^2(x+2)D_t(Q_{n-1},x) + 3x^2(x+1)D_t(Q_{n-1}',x)$, *where* $D_t(Q_0',x) = x^3 + 2x^2$.

(iv) $D_t(Q_n^{\Delta},x) = x(x+1)D_t(Q_n,x) + x^2(x+2)D_t(Q_{n-1},x) + x^2(3x+4)D_t(Q_{n-1}',x)$, *where* $D_t(Q_0^{\Delta},x) = x^3 + 3x^2$.

Proof. (i) Consider graph $Q_n(1)$ and its vertex u in Figure 8. By Theorem 2.6(iii),

$$\begin{aligned} D_t(Q_n(1),x) &= \overbrace{D_t(Q_n(1)\setminus u,x)}^{0} + xD_t(Q_n(1)/u,x) \\ &\quad + x^2[D_t(Q_n(1)\setminus N[\{u,v\}],x) + 2D_t(Q_n(1)\setminus N[\{u,w\}],x)] \\ &= xD_t(Q_n+e,x) + x^2[D_t(Q_{n-1}x) + 2D_t(Q_{n-1}',x)]. \end{aligned}$$

By applying Theorem 2.6(iv) on Q_n+e, we have

$$D_t(Q_n+e,x) = D_t(Q_n,x) + x^2D_t(Q_{n-1}',x).$$

So we have result.

(ii) Consider the vertex u as shown in Figure 8. By Theorem 2.6(iii), we have

$$\begin{aligned} D_t(Q_n(2),x) &= \overbrace{D_t(Q_n(2)\setminus u,x)}^{0} + xD_t(Q_n(2)/u,x) \\ &\quad + x^2[D_t(Q_n(2)\setminus N[\{u,v\}],x) + D_t(Q_n(2)\setminus N[\{u,w\}],x)] \\ &= xD_t(Q_{n-1}(1),x) + x^2[D_t(Q_{n-1}',x) + D_t(Q_{n-1},x)]. \end{aligned}$$

By using Part (i) in the above equation, we have result.

(iii) Consider graph Q_n' in Figure 9. By Theorem 2.6,

$$\begin{aligned} D_t(Q_n',x) &= D_t(Q_n'\setminus u,x) + xD_t(Q_n'/u,x) + x^2D_t(Q_n'\setminus N[\{u,w\}],x) \\ &= (x+1)D_t(Q_n(1),x) + x^2D_t(Q_{n-1},x). \end{aligned}$$

Using Part (i) in the above equation, we have result.

(iv) By Theorem 2.6(iii), we have

$$\begin{aligned}D_t(Q_n^{\Delta}, x) &= D_t(Q_n^{\Delta} \setminus u, x) + xD_t(Q_n^{\Delta}/u, x)\\ &\quad + x^2[D_t(Q_n^{\Delta} \setminus N[\{u, v\}], x) + D_t(Q_n^{\Delta} \setminus N[\{u, w\}], x)]\\ &= (x + 1)D_t(Q_n(1), x) + x^2[D_t(Q'_{n-1}, x) + D_t(Q_{n-1}, x)].\end{aligned}$$

So by using Part (i) in the above equation, we have result.

□

Theorem 3.4. *The total domination polynomial of para-chain Q_n is given by*

$$\begin{aligned}D_t(Q_n, x) &= x^2(x + 2)[D_t(Q_{n-1}, x) + (x + 1)D_t(Q_{n-2}, x)]\\ &\quad + x^2(3x^2 + 7x + 2)D_t(Q'_{n-2}, x),\end{aligned}$$

where $D_t(Q_1, x) = x^4 + 4x^3 + 4x^2$ and $D_t(Q_2, x) = D_t(F_{2,4}, x)$.

Proof. With regards to Figure 7 and Theorem 2.6, we have

$$\begin{aligned}D_t(Q_n, x) &= D_t(Q_n \setminus u, x) + xD_t(Q_n/u, x)\\ &\quad + x^2[D_t(Q_n \setminus N[\{u, v\}], x) + D_t(Q_n \setminus N[\{u, w\}], x)]\\ &= D_t(Q_{n-1}(2), x) + xD_t(Q_{n-1}^{\Delta}, x)\\ &\quad + x^2[D_t(Q'_{n-2}, x) + D_t(Q_{n-2}, x)].\end{aligned}$$

Now by Lemma 3.3 results is obtained. □

3.3. **Total domination polynomial of ortho-chain square cactus graphs.** In this subsection we consider an ortho-chain of length n, O_n, as shown in Figure 10. We shall obtain a recurrence relation for the total domination polynomial of O_n. Similar to Lemma 3.3 using

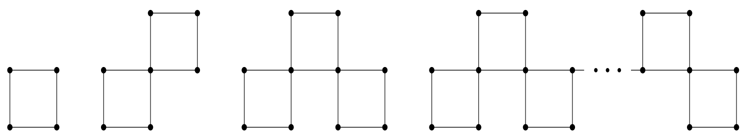

FIGURE 10. ortho-chain square cactus graphs O_1, O_2, O_3 and O_n, respectively.

Theorem 2.6 we can have the following result. Note that in the Part (i), $p_u(O_n) = D_t(O_{n-1}(2), x)$.

Lemma 3.5. *For graphs in Figure 11, we have*

(i) $D_t(O_n(1), x) = (x + 1)[D_t(O_n, x) - D_t(O_{n-1}(2), x)]$, *where $D_t(O_0(1), x) = x^2$.*

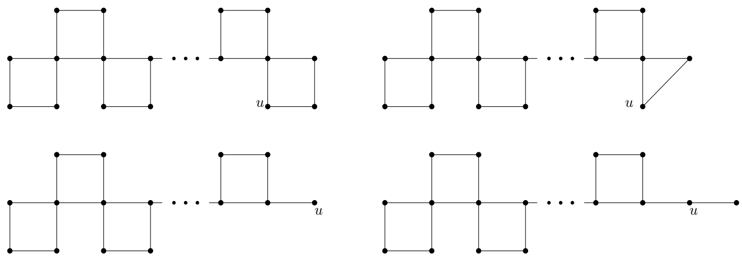

FIGURE 11. graphs O_n, O_n^{Δ}, $O_n(1)$ and $O_n(2)$, respectively.

(ii) $D_t(O_n(2),x) = x[(x+1)D_t(O_n,x) - D_t(O_{n-1}(2),x)]$, *where* $D_t(O_0(2),x) = x^3 + 2x^2$.

(iii) $D_t(O_n^{\Delta},x) = (x+1)^2 D_t(O_n,x) - (2x+1)D_t(O_{n-1}(2),x)$, *where* $D_t(O_0^{\Delta},x) = x^3 + 3x^2$.

Theorem 3.6. *For graph O_n in Figure 10, we have*

$$D_t(O_n,x) = x(x+2)[(x+1)D_t(O_{n-1},x) - D_t(O_{n-2}(2),x)],$$

where $D_t(O_1,x) = x^4 + 4x^3 + 4x^2$ and $D_t(O_2,x) = D_t(F_{2,4},x)$.

Proof. Consider graph O_n and vertex u as shown in Figure 11. By Theorem 2.6 for vertex u, we have

$$D_t(O_n,x) = D_t(O_{n-1}(2),x) + xD_t(O_{n-1}^{\Delta},x) + x^2 D_t(O_{n-2}(2),x).$$

Now by using Lemma 3.5(ii) and (iii), we have result. □

4. CONCLUSION

Calculating the total domination polynomial of a graph G is difficult in general, as the smallest power of a non-zero term is the total domination number $\gamma_t(G)$ of the graph, and determining whether $\gamma_t(G) \leq k$ is known to be NP-complete. Graphs with specific construction (which call them point attaching graphs) considered and their total domination polynomials have studied. Graphs which obtain by a special point-attaching of a graph H and $|V(H)|$ copies of graph P_3 considered and proved that all graphs whose total domination polynomial have just two roots $\{-2,0\}$ are in this form. Also we studied the total domination polynomial of some kind of generalized friendship graphs and some cactus chains, which are special cases of point attaching graphs. We think that the study of the total domination polynomial of point-attaching graphs can be a good start for proving or disproving of a conjecture in [2] which states that every integer total domination roots is in the set $\{-3,-2,-1,0\}$.

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TOTAL DOMINATION POLYNOMIAL OF GRAPHS FROM PRIMARY SUBGRAPHS

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چندجمله‌ای غالب تام گراف‌ها بر اساس زیرگراف های ابتدایی

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فرض کنید $G = (V, E)$ یک گراف ساده است. یک زیرمجموعه D از V را مجموعه غالب تام می‌نامیم، هرگاه هر رأس از V با رأسی از D مجاور باشد. کوچکترین اندازه مجموعه‌های غالب تام گراف G را عدد غالب تام گراف G نامیده و با $\gamma_t(G)$ نشان می‌دهیم. چندجمله‌ای غالب تام گراف G به صورت $D_t(G, x) = \sum d_t(G, i)x^i$ تعریف می‌شود که در آن $d_t(G, i)$ تعداد مجموعه‌های غالب تام G از اندازه i است. فرض کنید G گراف همبندی است که از گراف‌های همبند دوبه‌دو مجزای $G_1, \ldots, G_k$ با چسباندن یک رأس دلخواه از G_1 به یک رأس دلخواه از G_2 و ادامه دادن این روند، به دست می‌آید. به گراف G گراف اتصال نقطه‌ای $G_1, \ldots, G_k$ گویند که هر یک از G_i ها زیرگراف ابتدایی G می‌باشند. در این مقاله حالت‌های خاصی از این نوع گراف‌های اتصال نقطه‌ای که بیشتر آنها در علم شیمی مهم هستند را در نظر گرفته و چندجمله‌ای غالب تام آن‌ها را مورد مطالعه قرار می‌دهیم.

کلمات کلیدی: عدد غالب تام، چندجمله‌ای غالب تام، مجموعه‌ی غالب تام.