Journal of Algebraic Systems
Vol. 5, No. 2, (2017), pp 139-148

ON p-NILPOTENCY OF FINITE GROUPS WITH SS-NORMAL SUBGROUPS

G. R. REZAEEZADEH* AND Z. AGHAJARI

Abstract. A subgroup H of a group G is said to be SS-embedded in G if there exists a normal subgroup T of G such that HT is subnormal in G and $H \cap T \leq H_{sG}$, where H_{sG} is the maximal s-permutable subgroup of G contained in H. We say that a subgroup H is an SS-normal subgroup in G if there exists a normal subgroup T of G such that $G = HT$ and $H \cap T \leq H_{SS}$, where H_{SS} is an SS-embedded subgroup of G contained in H. In this paper, we study the influence of some SS-normal subgroups on the structure of a finite group G.

1. Introduction

All groups considered in this paper are finite.
Recall that for a group G, n-maximal subgroup is defined recursively: if U is a maximal subgroup of G, U is said to be 1-maximal in G; for $n > 1$, a subgroup U is said to be n-maximal in G if U is $(n-1)$-maximal in a maximal subgroup M of G (see [2]). Let $\mathcal{F}$ be a class of groups. We call $\mathcal{F}$ a formation, provided that (i) if $G \in \mathcal{F}$ and $N \trianglelefteq G$, then $G/N \in \mathcal{F}$, and (ii) if $N_1, N_2 \trianglelefteq G$ such that $G/N_1, G/N_2 \in \mathcal{F}$, then $G/N_1 \cap N_2 \in \mathcal{F}$. A formation $\mathcal{F}$ is said to be saturated if $G/\Phi(G) \in \mathcal{F}$ implies that $G \in \mathcal{F}$ (see [7]).

Recently, the relationship between the subgroups of a finite group G and the structure of the group G has been extensively studied in the literature. For instance, Wang [11] introduced the concept of c-normal

---

MSC(2010): Primary: 20D10; Secondary: 20D20, 20D35.
Keywords: SS-normal subgroup, SS-embedded subgroup, p-nilpotent group.
Received: 09 January 2017, Accepted: 06 October 2017.
∗Corresponding author.

subgroup and used the c-normality of maximal subgroups to determine the structure of some groups. A subgroup H of G is called c-normal in G if there is a normal subgroup T of G such that $G = HT$ and $H \cap T \leq H_G$, where H_G is the normal core of H in G.

Following Kegel [8], a subgroup H of a group G is said to be S-permutable in G if H permutes with every Sylow subgroup P of G. Guo et al. [4] introduced the concept of S-embedded subgroup. A subgroup H of a group G is said to be S-embedded in G if there exists a normal subgroup N such that HN is S-permutable in G and $H \cap N \leq H_{sG}$, where H_{sG} is the largest S-permutable subgroup of G contained in H.

Also, there exist other fruitful related concepts which have been introduced by many scholars and a lot of meaningful results have been obtained by them, such as S-permutably embedded subgroup [1], nearly S-normal [6], weakly S-permutable subgroup [10], $\cdots$.

More recently, Zhao [12] introduced the concept of SS-embedded subgroup, which covers S-permutability, c-normality and S-embedded subgroups. Recall that a subgroup H of a group G is said to be SS-embedded in G if there exists a normal subgroup T of G such that HT is subnormal in G and $H \cap T \leq HsG$. Zhao obtained many interesting results, by assuming that some subgroups of G satisfy the SS-embedded property. We now introduce the following concept:

Definition 1.1. Let H be a subgroup of a group G. H is called SS-normal in G if there exists a normal subgroup T of G such that $G = HT$ and $H \cap T \leq H_{SS}$, where H_{SS} is an SS-embedded subgroup of G contained in H.

In this paper, we study the influence of some SS-normal subgroups on the structure of a finite group G and we achieve some new results.

2. Preliminaries

Here, we collect some basic results which are useful in the sequel.

Lemma 2.1. ([8]) *Suppose that H is an S-permutable subgroup of a group G and $N \trianglelefteq G$. Then the following statements hold:*

(1) *If $H \leq K \leq G$, then H is S-permutable in K.*
(2) *HN and $H \cap N$ are S-permutable in G.*
(3) *HN/N is S-permutable in G/N.*
(4) *H is subnormal in G.*

Lemma 2.2. ([12], *Lemma 2.2*) *Suppose that H is an SS-embedded subgroup of a group G and $N \trianglelefteq G$. Then the following statements hold:*

(1) *If $H \leq K \leq G$, then H is SS-embedded in K.*
(2) *If $N \leq H$, then H/N is SS-embedded in G/N.*
(3) *Let H be a π-subgroup and N be a normal π'-subgroup of G. Then HN/N is SS-embedded in G/N.*

Lemma 2.3. *Suppose that H is an SS-normal subgroup of a group G and $N \trianglelefteq G$. Then the following statements hold:*

(1) *If $H \leq K \leq G$, then H is SS-normal in K.*
(2) *If $N \leq H_{SS}$, then H/N is SS-normal in G/N.*
(3) *If $N \leq H$ such that H_{SS} is a π-subgroup and N is a π'-subgroup, then H/N is SS-normal in G/N.*
(4) *Let H be a π-subgroup and N be a normal π'-subgroup of G. Then HN/N is SS-normal in G/N.*

Proof. By hypothesis, there exists a normal subgroup T of G such that $G = HT$ and $H \cap T \leq H_{SS}$, where H_{SS} is an SS-embedded subgroup of G contained in H.

(1) It is clear that $K \cap T$ is a normal subgroup of K. We have $H(K \cap T) = K \cap G = K$ and $H \cap (K \cap T) = H \cap T \leq H_{SS}$. It is easy to see that H_{SS} is SS-embedded in K. Hence H is SS-normal in K.
(2) Clearly, TN/N is a normal subgroup of G/N. Since $N \leq H_{SS}$, it follows that $(H/N)(TN/N) = G/N$ and
$$(H/N) \cap (TN/N) = (H \cap TN)/N = (H \cap T)N/N \leq H_{SS}/N.$$
By Lemma (2.2), H_{SS}/N is SS-embedded in G/N. Therefore H/N is SS-normal in G/N, as required.
(3) We know that TN/N is a normal subgroup of G/N. Since $N \leq H$, it follows that $(H/N)(TN/N) = G/N$ and
$$(H/N) \cap (TN/N) = (H \cap TN)/N = (H \cap T)N/N \leq H_{SS}N/N.$$
Now, if H_{SS} be a π-subgroup and N be a π'-subgroup, then $H_{SS}N/N$ is SS-embedded in G/N by Lemma (2.2). Therefore HN/N is SS-normal in G/N.
(4) We know that $TN/N \trianglelefteq G/N$ and we have
$$(HN/N)(TN/N) = HTN/N = G/N.$$
Since $(|H|, |N|) = 1$, it follows that
$$|H \cap TN| = \frac{|H||TN|_\pi}{|HTN|_\pi} = \frac{|H||T|_\pi}{|HT|_\pi} = |H \cap T|.$$
Hence $H \cap TN = H \cap T$, so
$$(HN/N) \cap (TN/N) = (HN \cap TN)/N =$$

$$(H \cap TN)N/N = (H \cap T)N/N \leq H_{SS}N/N.$$

By Lemma (2.2), $H_{SS}N/N$ is SS-embedded in G/N. Therefore HN/N is SS-normal in G/N.

□

Lemma 2.4. ([7], *IV, Theorem 5.4*) *Suppose that G is a group which is not p-nilpotent but whose all proper subgroups are p-nilpotent. Then the following statements hold:*

1. *Every proper subgroup of G is nilpotent.*
2. *$|G| = p^a q^b$, where $p \neq q$.*
3. *G has a normal Sylow p-subgroup P for some prime p and $G/P \cong Q$, where Q is a non-normal cyclic q-subgroup for some prime $q \neq p$.*
4. *$P/\Phi(P)$ is a minimal normal subgroup of $G/\Phi(P)$.*

Theorem 2.5. ([7], *IV, Theorem 2.8*) *Let p be the smallest prime divisor of the order of $|G|$. If G has a cyclic Sylow p-subgroup P, then there is a normal subgroup N of G such that $G/N \cong P$. (In particular, the Sylow 2-subgroup of a simple non-abelian group can never be cyclic.)*

Lemma 2.6. ([5], *lemma 2.5*) *Let G be a group and p a prime such that $p^{n+1} \nmid |G|$ for some integer $n \geqslant 1$. If $(|G|, (p-1)(p^2-1)...(p^n-1)) = 1$, then G is p-nilpotent.*

Theorem 2.7. ([9], *Theorem 10.1.9*) *Let p be the smallest prime dividing the order of the finite group G and assume that G is not p-nilpotent. Then the Sylow p-subgroups of G are not cyclic. Moreover $|G|$ is divisible by p^3 or by 12.*

Let π is a set of primes. We shall say that G is π-separable if every composition factor of G is either a π'-group or a π-group; and we shall say that G is π-solvable if every composition factor of G is either a π'-group or a p-group for some prime p in π. For a single prime p, the notions of p-separable and p-solvable are obviously equivalent (see [3]).

Theorem 2.8. ([3], *VI, Theorem 3.2*) *If G is π-separable and $\overline{G} = G/O_{\pi'}(G)$, then*

$$C_{\overline{G}}(O_\pi(\overline{G})) \subseteq O_\pi(\overline{G})$$

In particular, if $O_{\pi'}(G) = 1$, then $C_G(O_\pi(G)) \subseteq O_\pi(G)$.

If π is a set of primes, a subgroup H of a group G will be called an S_π-subgroup of G provided H is a π-group and $|G : H|$ is divisible by no primes in π. Such a subgroup is also called a Hall subgroup of G.

Theorem 2.9. ([3], *VI, Theorem 3.5*) *If G is π-separable group and p, q are primes in π, π', respectively, then G possesses an S_σ-subgroup for $\sigma = \pi$, $\sigma = \{\pi, q\}$ and $\sigma = \{p, q\}$.*

3. Main results

We start our main results with the followig theorem.

Theorem 3.1. *Let P be a Sylow p-subgroup of a solvable group G, where p is a prime divisor of $|G|$. If the following conditions hold, then G is p-nilpotent:*

(1) $(|G|, (p-1)(p^2-1)...(p^n-1)) = 1$, *where $n \in \mathbb{Z}$,*
(2) *every n-maximal subgroup of P (if exists), which does not have a p-nilpotent supplement in G, is SS-normal in G, and*
(3) *every SS-embedded subgroup of G contained in P contains $O_p(G)$.*

Proof. Assume that the result is false and let G be a counterexample of minimal order. We break the proof into several steps:

Step(1) $|P| \geq p^{n+1}$ and every n-maximal subgroup of P is SS-normal in G.

By Lemma (2.6), we have $|P| \geq p^{n+1}$.

Assume that there exists an n-maximal subgroup P_1 of P which has a p-nilpotent supplement T in G. We claim that G is p-nilpotent. Otherwise we would find a non-p-nilpotent subgroup H of G which contains P and all its proper subgroups are p-nilpotent. Then by Theorem (2.4), H is a minimal non-nilpotent group. We have $G = P_1T$, so

$$H = H \cap P_1T = P_1(H \cap T) \ (1).$$

Since $H \cap T \leq T$ is p-nilpotent and H is not p-nilpotent, it follows that $L = H \cap T$ is a proper subgroup of H. Hence L is nilpotent and so $L = L_pL_q$. We have $P = P_1L_p$, so L_p is not contained in $\Phi = \Phi(P)$. Now, we consider the factor group H/Φ. The fact $L_q \leq N_H(L_p)$ implies that

$$L_q\Phi/\Phi \leq N_{H/\Phi}(L_p\Phi/\Phi) \ (2).$$

On the other hand, since P/Φ is an elementary abelian group, we have

$$L_p\Phi/\Phi \trianglelefteq P/\Phi \ (3).$$

Obviously, L_q is also a Sylow q-subgroup of H. Thus $L_p\Phi/\Phi \trianglelefteq H/\Phi$ by (2) and (3). Moreover $L_p\Phi/\Phi \neq 1$. By Theorem (2.4), P/Φ is a chief factor of H, whence $L_p\Phi/\Phi = P/\Phi$. Hence $L_p = P$, so $L = H$. This contradiction completes the proof of Step 1.

Step(2) $O_{p'}(G) = 1$ and $O_p(G) \neq 1$.
If $O_{p'}(G) \neq 1$, then $\overline{P} = PO_{p'}(G)/O_{p'}(G)$ is a Sylow p-subgroup of $\overline{G} = G/O_{p'}(G)$. We have

$$(|\overline{G}|, (p-1)(p^2-1)...(p^n-1)) = 1.$$

By Step 1, $|\overline{P}| \geqslant p^{n+1}$. Let $\overline{P_1} = P_1O_{p'}(G)/O_{p'}(G)$ be an n-maximal subgroup of $\overline{P}$. Then P_1 is an n-maximal subgroup of P. By Step 1, P_1 is SS-normal in G hence $\overline{P_1}$ is SS-normal in $\overline{G}$ by Lemma (2.3)(3). Therefore $\overline{G}$ is p-nilpotent by induction. It follows that G is p-nilpotent. By this contradiction $O_{p'}(G) = 1$. Since G is soluble, we have $O_p(G) \neq 1$.

Step(3) $O_p(G)$ is unique minimal normal subgroup of G, $\Phi(G) = 1$ and $G/O_p(G)$ is p-nilpotent.
Let N be a minimal normal subgroup of G. Since G is solvable and Step 2, it follows that N is an elementary abelian p-group and $N \leq O_p(G)$. Now, we consider P/N so the following two cases arise:
Case i) If $|P/N| \leq p^n$, then G/N is p-nilpotent by Lemma (2.6).
Case ii) If $|P/N| \geq p^{n+1}$, then G/N is p-nilpotent by Lemma (2.3)(2), hypothesis of the theorem and the minimality of G.
Since the class of all p-nilpotent groups forms a saturated formation, it follows that N is an unique minimal normal subgroup of G and $\Phi(G) = 1$. Thus there is a maximal subgroup M of G such that $G = NM$ and $N \cap M = 1$. We have

$$O_p(G) \leq F(G) \leq C_G(N)$$

and

$$C_G(N) \cap M \trianglelefteq G.$$

The uniqueness of N yields that $N = O_p(G) = F(G) = C_G(N)$.

Step(4) $|O_p(G)| \geq p^{n+1}$.
We know $G/O_p(G)$ is p-nilpotent. Let $K/O_p(G)$ be the normal p-complement of $G/O_p(G)$. If $|O_p(G)| \leq p^n$, then $|K|_p \leq p^n$. Lemma (2.6) implies that K is p-nilpotent. The normal p-complement of K is also a normal p-complement of G, that is, G is p-nilpotent, this contradiction shows that $|O_p(G)| \geq p^{n+1}$.

Step(5) The final contradiction.
Since $\Phi(G) = 1$, there exists a maximal subgroup M of G such that $G = O_p(G)M$ and $O_p(G) \cap M = 1$. Let $P = O_p(G)M_p$ be a Sylow p-subgroup of G, where M_p is a Sylow p-subgroup of M. Since $|O_p(G)| \geq p^{n+1}$, we can pick an n-maximal subgroup

P_1 of P containing M_p. Since $O_p(G) \leq (P_1)_{SS} \leq P_1$, it follows that $P = P_1$. This is the final contradiction.

□

Corollary 3.2. *Let P be a Sylow p-subgroup of a solvable group G, where $p = min(\pi(G))$. If the following conditions hold, then G is p-nilpotent:*

(1) *every maximal subgroup of P, which does not have a p-nilpotent supplement in G, is SS-normal in G, and*
(2) *every SS-embedded subgroup in G contained in P contains $O_p(G)$.*

Theorem 3.3. *Let p be a prime divisor of $|G|$ and P be a Sylow p-subgroup of a solvable group G. If the following conditions hold, then G is p-nilpotent:*

(1) *$N_G(P)$ is p-nilpotent,*
(2) *every maximal subgroup of P, which does not have a p-nilpotent supplement in G, is SS-normal in G, and*
(3) *every SS-embedded subgroup in G is contained in P contains $O_p(G)$.*

Proof. If $p = min\pi(G)$, then G is p-nilpotent by Corollary (3.2). Hence we only need to consider the case which p is not the minimal prime divisor of $|G|$ (so it is an odd prime). Assume that the result is false and let G be a counterexample of minimal order. Then we break the proof into a several steps:

Step(1) Every maximal subgroup of P is SS-normal in G.
See the proof of Step 1 in Theorem (3.1).

Step(2) $O_{p'}(G) = 1$ and $O_p(G) = 1$.
Suppose that $O_{p'}(G) \neq 1$. Clearly, $\overline{P} = PO_{p'}(G)/O_{p'}(G)$ is a Sylow p-subgroup of $\overline{G} = G/O_{p'}(G)$ and

$$N_{\overline{G}}(\overline{P}) = N_G(P)O_{p'}(G)/O_{p'}(G)$$

is p-nilpotent. Let $\overline{M} = M/O_{p'}(G)$ be a maximal subgroup of $\overline{P}$. Then $M = P_1O_{p'}(G)$ for some maximal subgroup P_1 of P. We have $\overline{M}$ is SS-normal in $\overline{G}$ by Step 1 and Lemma (2.3)(3). This shows that $\overline{G}$ satisfies the hypothesis of the theorem. Thus $G/O_{p'}(G)$ is p-nilpotent by induction, so G is p-nilpotent. This contradiction shows that $O_{p'}(G) = 1$ and $O_p(G) = 1$.

Step(3) If L is a proper subgroup of G containing P, then L is p-nilpotent.
We know $N_L(P) \leq N_G(P)$ is p-nilpotent. Also, L satisfies the hypothesis of the theorem by Step 1 and Lemma (2.3)(1). The minimality of G implies that L is p-nilpotent.

Step(4) $G = PQ$, where Q is a Sylow q-subgroup of G with $p \neq q$.
By Theorem (2.9), there exists a Sylow q-subgroup Q of G such that $PQ \leq G$, where q is a prime divisor of G and $p \neq q$. If $PQ < G$, then PQ is p-nilpotent by Step 3. This implies that $Q \leq C_G(O_p(G)) \leq O_p(G)$ by Theorem (2.8). This contradiction shows that $G = PQ$.

Step(5) G has an unique minimal normal subgroup N such that $G = NM$ and $N \cap M = 1$, where M is a maximal subgroup of G. Moreover, $N = O_p(G) = F(G) = C_G(N)$.
Let N be a minimal normal subgroup of G. Then N is an elementary abelian p-group and $N \leq O_p(G)$. Clearly, G/N satisfies the hypothesis of the theorem. The minimality of G implies that G/N is p-nilpotent.
Since the class of all p-nilpotent groups is a saturated formation, N is an unique minimal normal subgroup of G and $N \not\leq \Phi(G)$. Thus G holds in Step 5.

Step(6) $|N| = p$.
It is easy to see that, $P = NM_p$, where M_p is a Sylow p-subgroup of M. Let P_1 be a maximal subgroup of P containing M_p.
If $P_1 \neq 1$, then there exists $T \trianglelefteq G$ such that $G = P_1T$ and $P_1 \cap T \leq (P_1)_{SS}$. Since $(P_1)_{SS}$ is an SS-embedded subgroup of G, there exists $N' \trianglelefteq G$ such that $(P_1)_{SS}N' \trianglelefteq\trianglelefteq G$ and $(P_1)_{SS} \cap N' \leq ((P_1)_{SS})_{sG}$. Now, we should consider two following cases:
Case i) If $N' = 1$, then $(P_1)_{SS} \trianglelefteq\trianglelefteq G$. Since $(P_1)_{SS} \leq O_p(G)$, it follows that $P = P_1$, a contradiction.
Case ii) If $N' \neq 1$, then $O_p(G) \leq N'$. Since

$$O_p(G) = O_p(G) \cap N' \leq ((P_1)_{SS})_{sG} \leq O_p(G),$$

it follows that $O_p(G) = ((P_1)_{SS})_{sG}$. Hence $P_1 \cap O_p(G) = O_p(G)$ so $O_p(G)$ is subgroup of P_1. We have

$$P = O_p(G)M_p \leq P_1M_p = P_1.$$

It is a contradiction.
Now, if $P_1 = 1$, then $|N| = |P| = p$.

Step(7) The finial contradiction.
We have $M \cong G/N = N_G(N)/C_G(N)$ is isomorphic to a subgroup of $Aut(N)$. We know $Aut(N)$ is a cyclic group of order $p - 1$. Hence M and Q are cyclic groups. It follows from Theorem (2.5) that G is a q-nilpotent. Thus $P \trianglelefteq G$ so by the hypothesis of the theorem $G = N_G(P)$ is p-nilpotent. This final contradiction completes the proof of the theorem.

□

Let G be a group and $|G| = p_1^{r_1} p_2^{r_2} ... p_s^{r_s}$, where $p_1, p_2, ..., p_s$ are different primes. Recall that G is said to be a Sylow tower group if there exists a normal series $1 = G_0 \leq G_1 \leq ... \leq G_s = G$ of G such that $|G_i : G_{i-1}| = p_i^{r_i}$ for $1 \leq i \leq s$. In addition, if $p_1 > p_2 > ... > p_s$, then G is called a Sylow tower group of supersoluble type.

Theorem 3.4. *Let G a solvable group. If every non-cyclic Sylow p-subgroup P of G satisfies the following conditions, then G is a Sylow tower group of supersoluble type:*

(1) *$N_G(P)$ is p-nilpotent,*
(2) *every maximal subgroup of P is SS-normal in G, and*
(3) *every SS-embedded subgroup of G is contained in P contains $O_p(G)$,*

Proof. Let p_1 be the minimal prime divisor of $|G|$ and $P_1 \in Syl_{p_1}(G)$. First, we prove that G is p_1-nilpotent. If P_1 is cyclic, then G is p_1-nilpotent by Theorem (2.7). If P_1 is not cyclic, then G is p_1-nilpotent by hypothesis of the theorem and Corollary (3.2).

Now, we let K be the normal p_1-complement of G. We have $N_K(Q) \leq N_G(Q)$ is q-nilpotent for every non-cyclic Sylow q-subgroup Q of K. Every maximal subgroup of Q is SS-normal in K. By induction, we can deduce that K is a Sylow tower group of supersoluable type. It follows that G is a Sylow tower group of supersoluable type. □

References

1. A. Ballester-Bolinches and M. C. Pedraza-Aguilera, Sufficient conditions for supersolvability of finite groups, *J. Pure Appl. Algebra*, **127** (1998), 113–118.
2. C. M. Campbell, E. F. Robertson and G. C. Smith, *Groups St Andrews 2001 in Oxford*, Cambridge University Press, 2003.
3. D. Gorenstein, *Finite Groups*, Chelsea Publishing Company, New York, 1968.
4. W. B. Guo, K. P. Shum and A. N. Skiba, On solubility and supersolubility of some classes of finite groups, *Sci. China (Ser. A)*, **52** (2009), 272–286.
5. W. B. Guo, K. P. Shum and F. Y. Xie, Finite groups with some weakly s-supplemented subgroups, *Glasg. Math. J.*, **53** (2011), 211–222.
6. W. B. Guo, Y. Wang and L. Shi, Nearly s-normal subgroups of a finite group, *J. Algebra Discrete Struct.*, **6** (2008), 95–106.
7. B. Huppert, *Endliche Gruppen*, Vol. I, Springer, New York, 1967.
8. O. H. Kegel, Sylow-Gruppen und abnormalteiler endlicher Gruppen, *Math. Z.*, **78** (1962), 205–221.
9. D. J. S. Robinson, *A Course in the Theory of Groups*, Springer-Verlag, New York-Berlin, 1993.
10. A. N. Skiba, On weakly s-permutable subgroups of finite groups, *J. Algebra*, **315** (2007), 192–209.

11. Y. M. Wang, C-normality of groups and its properties, *J. Algebra*, **180** (1996), 954–965.
12. T. Zhao, Finite groups whit some SS-embedded subgroups, *IJGT*, **3** (2013), 63–70.

Gholamreza Rezaeezadeh
Department of Mathematics, University of Shahrekord, P.O.Box 115, Shahrekord, Iran.
Email: `rezaeezadeh@sci.sku.ac.ir`

Zahra Aghajari
Department of Mathematics, University of Shahrekord, P.O.Box 115, Shahrekord, Iran.
Email: `Z.Aghajari@stu.sku.ac.ir`

Journal of Algebraic Systems

ON p-NILPOTENCY OF FINITE GROUPS WITH SS-NORMAL SUBGROUPS

G. R. REZAEEZADEH AND Z. AGHAJARI

p-پوچ‌توانی گروه‌های متناهی دارای زیرگروه‌های SS-نرمال

غلامرضا رضایی‌زاده و زهرا آقاجری
شهرکرد، دانشگاه شهرکرد، دانشکده علوم ریاضی

فرض کنیم G یک گروه باشد. زیرگروه H از G را SS-نشانده شده در G گویند، هرگاه زیرگروه نرمال T از G وجود داشته باشد به‌طوری که HT زیرنرمال در G و $H \cap T \leq H_{SG}$، جایی‌که H_{SG} بزرگترین زیرگروه S-جابه‌جاپذیر در G مشمول در H است. زیرگروه H از G را SS-نرمال در G گوییم، هرگاه زیرگروه نرمال T از G وجود داشته باشد به‌طوری که $G = HT$ و $H \cap T \leq H_{SS}$، جایی‌که H_{SS} بزرگترین زیرگروه SS-نشانده‌شده در G مشمول در H می‌باشد. در این مقاله، سعی می‌کنیم تأثیر برخی از زیرگروه‌های SS-نرمال یک گروه را بر ساختار آن مورد مطالعه قرار می‌دهیم.

کلمات کلیدی: زیرگروه SS-نرمال، زیرگروه SS-نشانده‌شده، گروه p-پوچ‌توان.