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ON α-SEMI-SHORT MODULES

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ABSTRACT. We introduce and study the concept of α-semi short modules. Using this concept we extend some of the basic results of α-short modules to α-semi short modules. We observe that if M is an α-semi short module then the dual perfect dimension of M is α or $\alpha+1$.

1. INTRODUCTION

Lemonnier [26] has introduced the concept of deviation (resp., codeviation) of an arbitrary poset, which in particular, when applied to the lattice of all submodules of a module M_R give the concept of Krull dimension, see [17], [16] and [28] (resp., the concept of dual Krull dimension of M. The dual Krull dimension in [14], [13], [15], [19], [20], [21], [22], [8], [11],[9], [10], and [24] is called Noetherian dimension and in [7] is called N-dimension. This dimension is called Krull dimension in [29]. The name of dual Krull dimension is also used by some authors, see [2], [4] and [1]). The Noetherian dimension of an R-module M is denoted by $n\text{-dim}\, M$ and by $k\text{-dim}\, M$ we denote the Krull dimension of M. We recall that if an R-module M has Noetherian dimension and α is an ordinal number, then M is called α-atomic if $n\text{-dim}\, M = \alpha$ and $n\text{-dim}\, N < \alpha$, for all proper submodule N of M. An R-module M is called atomic if it is α-atomic for some ordinal α (note, atomic modules are also called conotable, dual critical and N-critical in some other articles; see for example [27], [2], and [7]). The author introduced

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and extensively investigated perfect dimension and dual perfect dimension of an R-module M, see [13]. The dual perfect dimension (resp., perfect dimension), which is denoted $dp\text{-}\dim M$ (resp., $p\text{-}\dim M$) is defined to be the codeviation (resp., deviation) of the poset of the finitely generated submodules of M. It is convenient, when we are dealing with the latter dimensions, to begin our list of ordinals with -1. We recall that an R-module M is called α-perfect atomic, where α is an ordinal, if $dp\text{-}\dim M = \alpha$ and $dp\text{-}\dim N < \alpha$ for any proper finitely generated submodule N of M. M is said to be perfect-atomic if it is α-perfect atomic for some α. Bilhan and Smith have introduced and extensively investigated short modules and almost Noetherian modules, see [6]. Later Davoudian, Karamzadeh and Shirali undertook a systematic study of the concepts of α-short modules and α-almost Noetherian modules, see [14]. We recall that an R-module M is called an α-short module, if for each submodule N of M, either $n\text{-}\dim N \leq \alpha$ or $n\text{-}\dim \frac{M}{N} \leq \alpha$ and α is the least ordinal number with this property. We shall call an R-module M to be α-semi short, if for each finitely generated submodule N of M, either $dp\text{-}\dim N \leq \alpha$ or $dp\text{-}\dim \frac{M}{N} \leq \alpha$ and α is the least ordinal number with this property. Using this concept, we show that each α-semi short module M has dual perfect dimension and $\alpha \leq dp\text{-}\dim M \leq \alpha + 1$. We observe that an Artinian serial module M is α-short if and only if it is β-semi short, where α and β are ordinal numbers and $\beta \leq \alpha \leq \beta + 1$. We also recall that an R-module M is called α-almost Noetherian, if for each proper submodule N of M, $n\text{-}\dim N < \alpha$ and α is the least ordinal number with this property, see [14]. We shall call an R-module M to be α-semi Noetherian if for each proper finitely generated submodule N of M, $dp\text{-}\dim N < \alpha$ and α is the least ordinal number with this property. In section 2 of this paper we investigate some basic properties of α-semi Noetherian and α-semi short modules. We show that if M is an α-semi short module (resp., α-semi Noetherian module), then $dp\text{-}\dim M = \alpha$ or $dp\text{-}\dim M = \alpha + 1$ (resp., $dp\text{-}\dim M \leq \alpha$). In the last section we also investigate some properties of α-semi Noetherian and α-semi short modules. Finally, we should emphasize here that the results in sections 2 and 3 are new and are similar to the corresponding results in [14].

2. α-semi short modules and α-almost semi Noethrian modules

We recall that an R-module M is called α-almost Noetherian, if for each proper submodule N of M, $n\text{-}\dim N < \alpha$ and α is the least ordinal number with this property. In the following definition we consider a related concept.

Definition 2.1. An R-module M is called α-semi Noetherian if for each proper finitely generated submodule N of M, $dp\text{-}\dim N < \alpha$ and α is the least ordinal number with this property.

It is manifest that if M is an α-semi Noetherian module, then each submodule and each factor module of M is β-semi Noetherian for some $\beta \leq \alpha$ (note, see [13, Lemmas 2.5, 2.10]).

In view of [13, Proposition 2.7], we have the next three trivial, but useful facts.

Lemma 2.2. *If M is an α-semi Noetherian module, then M has dual perfect dimension and $dp\text{-}\dim M \leq \alpha$. In particular, $dp\text{-}\dim M = \alpha$ if and only if M is α-perfect atomic.*

Lemma 2.3. *If M is a module with $dp\text{-}\dim M = \alpha$, then either M is α-perfect atomic, in which case it is α-semi Noetherian, or it is $\alpha+1$-semi Noetherian.*

Lemma 2.4. *If M is an α-semi Noetherian module, then either M is α-perfect atomic or $\alpha = dp\text{-}\dim M + 1$. In particular, if M is α-semi Noetherian module, where α is a limit ordinal, then M is α-perfect atomic.*

Proposition 2.5. *An R-module M has dual perfect dimension if and only if M is α-semi Noetherian for some ordinal α.*

Next, we give our definition of α-semi short modules.

Definition 2.6. An R-module M is called α-semi short module, if for each finitely generated submodule N of M, either $dp\text{-}\dim N \leq \alpha$ or $dp\text{-}\dim \frac{M}{N} \leq \alpha$ and α is the least ordinal number with this property.

In view of [13, Corollary 2.13], we have the following results.

Remark 2.7. If M is an R-module with $dp\text{-}\dim M = \alpha$, then M is β-semi short for some $\beta \leq \alpha$.

Remark 2.8. If M is an α-semi short module, then each submodule and each factor module of M is β-semi short for some $\beta \leq \alpha$.

We cite the following result from [13, Proposition 2.9].

Lemma 2.9. *If M is an R-module and for each finitely generated submodule N of M, either N or $\frac{M}{N}$ has dual perfect dimension, then so does M.*

The previous lemma and Remark 2.7, immediately yield the next result.

Corollary 2.10. *Let M be an α-semi short module. Then M has dual perfect dimension and $\alpha \leq dp\text{-}\dim M$.*

The following is now immediate.

Proposition 2.11. *An R-module M has dual perfect dimension if and only if M is α-semi short for some ordinal α.*

Proposition 2.12. *If M is an α-semi short R-module, then either $dp\text{-}\dim M = \alpha$ or $dp\text{-}\dim M = \alpha + 1$.*

Proof. Clearly in view of Corollary 2.10, we have $dp\text{-}\dim M \geq \alpha$. If $dp\text{-}\dim M \neq \alpha$, then $dp\text{-}\dim M \geq \alpha+1$. Now let $M_1 \subseteq M_2 \subseteq ...$ be any ascending chain of finitely generated submodules of M. If there exists some k such that $dp\text{-}\dim \frac{M}{M_k} \leq \alpha$, then $dp\text{-}\dim \frac{M_{i+1}}{M_i} \leq dp\text{-}\dim \frac{M}{M_i} = dp\text{-}\dim \frac{M/M_k}{M_i/M_k} \leq dp\text{-}\dim \frac{M}{M_k} \leq \alpha$ for each $i \geq k$, see [13, Corollary 2.13]. Otherwise $dp\text{-}\dim M_i \leq \alpha$ (M is α-semi short) for each i, hence $dp\text{-}\dim \frac{M_{i+1}}{M_i} \leq dp\text{-}\dim M_{i+1} \leq \alpha$ for each i. Thus in any case there exists an integer k such that for each $i \geq k$, $dp\text{-}\dim \frac{M_{i+1}}{M_i} \leq \alpha$. This shows that $dp\text{-}\dim M \leq \alpha + 1$, i.e., $dp\text{-}\dim M = \alpha + 1$. □

Remark 2.13. An R-module M is -1-semi short if and only if it is simple.

Proposition 2.14. *Let M be an R-module, with $dp\text{-}\dim M = \alpha$, where α is a limit ordinal. Then M is α-semi short.*

Proof. We know that M is β-semi short for some $\beta \leq \alpha$. If $\beta < \alpha$, then by Proposition 2.12, $dp\text{-}\dim M \leq \beta + 1 < \alpha$. Which is a contradiction. Thus M is α-semi short. □

Proposition 2.15. *Let M be an R-module and $dp\text{-}\dim M = \alpha = \beta+1$. Then M is either α-semi short or it is β-semi short.*

Proof. We know that M is γ-semi short for some $\gamma \leq \alpha$. If $\gamma < \beta$, then by Proposition 2.12, we have $dp\text{-}\dim M \leq \gamma + 1 < \beta + 1$, which is impossible. Hence we are done. □

Proposition 2.16. *Let M be an α-perfect atomic R-module, where $\alpha = \beta + 1$, then M is a β-semi short module.*

Proof. Let N be a finitely generated submodule of M. Hence, we have $dp\text{-}\dim N < \alpha$. This shows that for some $\beta' \leq \beta$, M is β'-semi short. If $\beta' < \beta$, then $\beta' + 1 \leq \beta < \alpha$. But $dp\text{-}\dim M \leq \beta' + 1 \leq \beta < \alpha$, by Proposition 2.12, which is a contradiction. Thus $\beta' = \beta$ and we are done. □

The following remark, which is a trivial consequence of the previous fact, shows that the converse of Proposition 2.14, is not true in general.

Remark 2.17. Let M be an $\alpha+1$-perfect atomic R-module, where α is a limit ordinal. Then M is an α-semi short module but $dp\text{-}\dim M \neq \alpha$.

Proposition 2.18. *Let M be an R-module such that $dp\text{-}\dim M = \alpha + 1$. Then M is either α-semi short R-module or there exists a finitely generated submodule N of M such that $dp\text{-}\dim N = dp\text{-}\dim \frac{M}{N} = \alpha + 1$.*

Proof. We know that M is α-semi short or an $\alpha+1$-semi short R-module, by Proposition 2.15. Let us assume that M is not α-semi short R-module, hence there exists a finitely generated submodule N of M such that $dp\text{-}\dim N \geq \alpha+1$ and $dp\text{-}\dim \frac{M}{N} \geq \alpha+1$. This shows that $dp\text{-}\dim N = \alpha+1$ and $dp\text{-}\dim \frac{M}{N} = \alpha+1$ and we are through. □

Proposition 2.19. *Let M be a non-zero α-semi short R-module. Then either M is β-semi Noetherian for some ordinal $\beta \leq \alpha+1$ or there exists a finitely generated submodule N of M with $dp\text{-}\dim \frac{M}{N} \leq \alpha$.*

Proof. Suppose that M is not β-semi Noetherian for any $\beta \leq \alpha+1$. This means that there must exist a finitely generated submodule N of M such that $dp\text{-}\dim N \nleq \alpha$. Inasmuch as M is α-semi short, we infer that $dp\text{-}\dim \frac{M}{N} \leq \alpha$ and we are done. □

Finally we conclude this section by providing some examples of α-semi Noetherian (resp., α-semi short) modules, where α is any ordinal. Recall that a left R-module M, (note, R is not necessarily commutative) is called uniserial if its submodules are linearly orderded by inclusion. A serial module is a module that is a direct sum of uniserial modules. First, we recall that given any ordinal α there exists an Artinian serial module M such that $n\text{-}\dim M = \alpha$, see [22, Example 1] and [15, Lemma 2.4]. Thus $dp\text{-}\dim M = \alpha$, see [13, Corollary 4.4]. Consequently, we may take M to be an Artinian serial module with $dp\text{-}\dim M = \alpha$. Hence $dp\text{-}\dim M = \alpha$ and for any ordinal $\beta \leq \alpha$, we take N to be its β-perfect atomic submodule, see [13, Corollary 3.10], then by Lemma 2.3, N is β-semi Noetherian. We recall that the only α-semi Noetherian modules, where α is a limit ordinal, are α-perfect atomic modules, see Lemma 2.4. Therefore to see an example of α-semi Noetherian module which is not α-perfect atomic, the ordinal α must be a non-limit ordinal. Thus we may take M to be a non-perfect atomic module with $dp\text{-}\dim M = \beta$, where $\alpha = \beta+1$, hence its follows trivially that M is an α-semi Noetherian. As for examples of α-semi short modules, one can similarly use the facts that there are Artinian serial modules M with Noetherian dimension equal to α, see [22, 15].

In view of [13, Corollary 4.4], we infer that $dp\text{-}\dim M = \alpha$. By [13, Corollary 3.10], for each $\beta \leq \alpha$ there are β-perfect atomic submodules of M and then apply Propositions 2.14, 2.15, 2.16, to give various examples of α-semi short modules (for example, by Proposition 2.16, every $\alpha + 1$-perfect atomic module is α-semi short).

3. Properties of α-semi short modules and α-semi Noetherian modules

In this section some properties of α-semi short modules over an arbitrary ring R are investigated.

In the following two propositions we investigate the connection between α-short modules and α-semi short modules, where M is an Artinian serial module.

Proposition 3.1. *Let M be an Artinian serial R-module. If M is a β-semi short module, then M is α-short for some $\alpha \leq \beta + 1$.*

Proof. In view of Proposition 2.12, we get $dp\text{-}\dim M \leq \beta + 1$. Thus by [13, Corollary 4.4], we have $n\text{-}\dim M \leq \beta + 1$. This shows that M is an α-short module for some $\alpha \leq \beta + 1$, see [14, Remark 1.2]. □

Proposition 3.2. *If M is an α-short R-module, then it is β-semi short for some $\beta \leq \alpha$.*

Proof. Let N be a finitely generated submodule of M, then $n\text{-}\dim N \leq \alpha$ or $n\text{-}\dim \frac{M}{N} \leq \alpha$ (note, M is α-short). In view of [13, Lemma 2.3], we infer that $dp\text{-}\dim N \leq \alpha$ or $dp\text{-}\dim \frac{M}{N} \leq \alpha$. This implies that M is β-semi shore for some $\beta \leq \alpha$. □

In view of Propositions 3.1 and 3.2, we have the following corollary.

Corollary 3.3. *Let M be an Artinian serial R-module and α and β are ordinal numbers. Then M is β-semi short if and only if it is α-short, where $\beta \leq \alpha \leq \beta + 1$.*

The next example shows that in the previous corollary all the cases for α can occur.

Example 3.4. Let $\mathbb{Z}$ be the ring of integers. Then the $\mathbb{Z}$-module $\mathbb{Z}_{p^\infty}$ is both 0-short and 0-semi short. And the $\mathbb{Z}$-module $\mathbb{Z}_{p^\infty} \oplus \mathbb{Z}_{p^\infty}$ is 1-short but it is 0-semi short.

In view of Corollary 3.3, we have the following result.

Corollary 3.5. *If M is an α-short module, where α is a limit ordinal number, then M is α-semi short.*

Proposition 3.6. *Let R be a ring and M be a nonzero α-semi short module, which is not a perfect atomic module, then M contains a finitely generated submodule L such that dp-dim $\frac{M}{L} \leq \alpha$.*

Proof. Since M is not perfect atomic, we infer that there exists a finitely generated submodule $L \subsetneq M$, such that $dp\text{-}\dim L = dp\text{-}\dim M$. We know that $dp\text{-}\dim M = \alpha$ or $dp\text{-}\dim M = \alpha + 1$, by Proposition 2.12. If $dp\text{-}\dim M = \alpha$ it is clear that $dp\text{-}\dim \frac{M}{L} \leq \alpha$. Hence we may suppose that $dp\text{-}\dim L = dp\text{-}\dim M = \alpha + 1$. Consequently, $dp\text{-}\dim \frac{M}{L} \leq \alpha$ and we are done. □

Theorem 3.7. *Let α be an ordinal number and M be an R-module. If every proper finitely generated submodule of M is γ-semi short for some ordinal number $\gamma \leq \alpha$. Then dp-dim $M \leq \alpha + 2$, in particular, M is μ-semi short for some ordinal $\mu \leq \alpha + 1$.*

Proof. Let $N \subsetneq M$ be any finitely generated submodule of M. Since N is γ-semi short for some ordinal number $\gamma \leq \alpha$, we infer that $dp\text{-}\dim N \leq \gamma + 1 \leq \alpha + 1$, by Proposition 2.12. This immediately implies that $dp\text{-}\dim M \leq \alpha + 2$, see [13, Proposition 2.7]. The final part is now evident. □

The next result is the dual of Theorem 3.7.

Theorem 3.8. *Let M be a nonzero R-module and α be an ordinal number. Let for every non-zero finitely generated submodule N of M, $\frac{M}{N}$ be γ-semi short for some ordinal number $\gamma \leq \alpha$. Then dp-dim $M \leq \alpha + 1$, in particular, M is μ-semi short for some ordinal $\mu \leq \alpha + 1$.*

Proof. Let N be any non-zero finitely generated submodule of M, then $\frac{M}{N}$ is γ-semi short for some ordinal number $\gamma \leq \alpha$. In view of Proposition 2.12, we infer that $dp\text{-}\dim \frac{M}{N} \leq \gamma + 1 \leq \alpha + 1$. Therefore $dp\text{-}\dim M \leq \sup\{dp\text{-}\dim \frac{M}{N} : 0 \neq N \subseteq M, N \textit{ is f.g.}\} \leq \alpha + 1$, see [13, Proposition 2.6]. The final part is now evident. □

The next immediate result is the counterparts of Theorems 3.7, 3.8, for α-semi Noetherian modules.

Proposition 3.9. *Let M be an R-module and α be an ordinal number. If each proper finitely generated submodule N of M (resp., for each non-zero finitely generated submodule N of M, $\frac{M}{N}$) is γ-semi Noetherian with $\gamma \leq \alpha$, then M is a μ-semi Noetherian module with $\mu \leq \alpha+1$ and dp-dim $M \leq \alpha + 1$ (resp., with $\mu \leq \alpha + 1$ and dp-dim $M \leq \alpha$).*

Proposition 3.10. *Let R be a semiprime right Goldie ring. Then the right R-module R is α-semi short if and only if dp-dim $R = \alpha$.*

Proof. Let R be α-semi short as an R-module. We are to show that $dp\text{-}\dim R = \alpha$. If for each essential right ideal E of R, $dp\text{-}\dim \frac{R}{E} \leq \alpha$ then $dp\text{-}\dim R = \sup\{dp\text{-}\dim \frac{R}{E} : E \subseteq_e R\} \leq \alpha$, see [13, Proposition 2.15]. Since R is α-semi short we have $dp\text{-}\dim R = \alpha$, by Proposition 2.12. Now suppose that there exists an essential right ideal E' of R such that $dp\text{-}\dim \frac{R}{E'} \not\leq \alpha$. But R is a right Goldie ring, hence there exists a regular element c in E', which implies that $dp\text{-}\dim \frac{R}{cR} \not\leq \alpha$, see [13, Lemma 2.10]. Thus $dp\text{-}\dim R = dp\text{-}\dim cR \leq \alpha$, see [13, Lemma 2.5]. Consequently, we must have $dp\text{-}\dim R = \alpha$, by Proposition 2.12. Conversely, by Remark 2.7, R is β-semi short for some $\beta \leq \alpha$. But, by the first part of the proof, we must have $dp\text{-}\dim R = \beta$, i.e., $\beta = \alpha$, and we are through. □

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ON α-SEMI SHORT MODULES

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دربارەی مدول‌های α -شبه کوتاه

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در این مقاله مفهوم مدول‌های α -شبه کوتاه معرفی شده است. با استفاده از این مفهوم، برخی از نتایج مدول‌های α -کوتاه به مدول‌های α -شبه کوتاه تعمیم داده شده است. نشان می‌دهیم اگر M یک مدول α -کوتاه باشد، آن‌گاه دوگان بعد تام دارد و دوگان بعد تام آن α یا $\alpha + ۱$ است.

کلمات کلیدی: مدول‌های α -کوتاه، مدول‌های α -تقریبا نوتری، مدول‌های α - شبه کوتاه، بعد نوتری، دوگان بعد تام، بعد کرول.