Journal of Algebraic Systems
Vol. 7, No. 1, (2019), pp 25-32

COTORSION DIMENSIONS OVER GROUP RINGS

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Abstract. Let Γ be a group, Γ' a subgroup of Γ with finite index and M be a Γ-module. We show that M is cotorsion if and only if it is cotorsion as a Γ'-module. Using this result, we prove that the global cotorsion dimensions of rings $\mathbb{Z}\Gamma$ and $\mathbb{Z}\Gamma'$ are equal.

1. Introduction

Harrison [11], Nunke [13] and Fuchs [9], independently, introduced the notion of cotorsion abelian groups. An abelian group is said to be cotorsion, if every extension of it by a torsion-free group splits. This notion was extended to modules over integral domains by Matlis [12] and Warfield [15] in two different ways. Finally in [8], Enochs has defined cotorsion modules over arbitrary associative rings as the modules C for which $\mathrm{Ext}^1_R(F, C) = 0$ for all flat modules F. Actually, Enochs's definition generalizes the definitions of Harrison and Warfield and agrees with that of Fuchs but differs from that of Matlis.

In [6], Ding and Mao defined a homological dimension, the cotorsion dimension, $\mathrm{cd}_R M$, for any R-module M. It is defined as the least non-negative integer n satisfying $\mathrm{Ext}^{n+1}_R(F, M) = 0$ for all flat R-modules F. They also defined the global cotorsion dimension of a ring R, denoted by $\mathrm{Cot.D}(R)$, as the supremum of the cotorsion dimensions of all R-modules.

Recall that a ring R is perfect, if every R-module has a projective cover. Bass [2] has proved that perfect rings are precisely those whose

MSC(2010): Primary: 20J05, 16S34; Secondary: 16L30, 18G20.
Keywords: Cotorsion dimension, global cotorsion dimension, perfect ring.
Received: 17 June 2018, Accepted: 2 October 2018.
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every flat module is projective. The global cotorsion dimension of rings measures how far away a ring is from being perfect. This exactly means that, for a ring R and a positive integer n, $\mathrm{Cot.D}(R) \leq n$ if and only if every flat R-module F has projective dimension less than or equal to n; see [6, Theorem 7.2.5]. In particular, R is perfect if and only if $\mathrm{Cot.D}(R) = 0$. Ding and Mao have proved that the global cotorsion dimension gives an upper bound on the global dimension of rings. More precisely, they showed that for any ring R, we have the inequality

$$\mathrm{D}(R) \leq \mathrm{w.D}(R) + \mathrm{Cot.D}(R).$$

The purpose of this note is to study the cotorsion modules over group rings. The main result of this paper asserts that a given $\mathbb{Z}\Gamma$-module M is cotorsion if and only if it is cotorsion over $\mathbb{Z}\Gamma'$, where Γ' is a finite index subgroup of Γ. This result enables us to deduce that if M is a $\mathbb{Z}\Gamma$-module, then $\mathrm{cd}_\Gamma M = \mathrm{cd}_{\Gamma'} M$, where $\mathrm{cd}_\Gamma M$ (resp., $\mathrm{cd}_{\Gamma'} M$) denotes the cotorsion dimension of M as a $\mathbb{Z}\Gamma$-module (resp., $\mathbb{Z}\Gamma'$-module).

Let Γ be an abelian multiplicative group, and R be a ring with identity. It is shown by Woods in [16] that, the group ring $R\Gamma$ is perfect if and only if R is perfect and Γ is a finite group. Several decades later, Bennis and Mahdou [3] extended the result of Woods and proved:

$$\mathrm{Cot.D}(R) \leq \mathrm{Cot.D}(R\Gamma) \leq \mathrm{Cot.D}(R) + \mathrm{pd}_{R\Gamma}(R).$$

Furthermore, if $\mathrm{pd}_{R\Gamma} R$ is finite and Γ is a finite group, then the equality $\mathrm{Cot.D}(R) = \mathrm{Cot.D}(R\Gamma)$ holds.

In this paper, we consider the global cotorsion dimension of the integral group ring of a group Γ, $\mathbb{Z}\Gamma$, and denote it by $\mathrm{Cot.D}(\Gamma)$. We prove that $\mathrm{Cot.D}(\Gamma) = \mathrm{Cot.D}(\Gamma')$, where Γ' is a finite index subgroup of Γ. Also, it is shown that there is a tight connection between the global cotorsion dimension of $\mathbb{Z}\Gamma$, the supremum of flat length of injective $\mathbb{Z}\Gamma$-modules, $\mathrm{sfli}\Gamma$, and the supremum of injective length of flat $\mathbb{Z}\Gamma$-modules, $\mathrm{silf}\Gamma$.

Throughout the paper, Γ is a group and $\mathbb{Z}\Gamma$ is its integral group ring. By a Γ-module, we mean a $\mathbb{Z}\Gamma$-module. We follow this abbreviation in all of our notations. For example, for a Γ-module M, projective dimension of M over $\mathbb{Z}\Gamma$ is denoted by $\mathrm{pd}_\Gamma M$. The tensor product and Hom functor over $\mathbb{Z}\Gamma$ denoted by $-\otimes_\Gamma-$ and $\mathrm{Hom}_\Gamma(-,-)$, respectively. We also denote the tensor product and Hom functor over $\mathbb{Z}$ by $-\otimes-$ and $\mathrm{Hom}(-,-)$, respectively.

2. Results and proofs

Let Γ be a group. Following [8], a Γ-module C is called cotorsion if $\mathrm{Ext}^1_\Gamma(F,C)=0$ for any flat Γ-module F. The class of cotorsion modules contains all pure-injective (and hence all injective) modules, and is closed under finite direct sums and direct summands.

Lemma 2.1. *Let Γ' be a subgroup of Γ and let M be a cotorsion Γ-module. Then M is a cotorsion Γ'-module.*

Proof. Suppose that F is a flat Γ'-module. Then $\mathbb{Z}\Gamma\otimes_{\Gamma'}F$ is a flat Γ-module. So $\mathrm{Ext}^1_\Gamma(\mathbb{Z}\Gamma\otimes_{\Gamma'}F,M)=0$. But

$$\mathrm{Ext}^1_\Gamma(\mathbb{Z}\Gamma\otimes_{\Gamma'}F,M)\cong\mathrm{Ext}^1_{\Gamma'}(F,\mathrm{Hom}_\Gamma(\mathbb{Z}\Gamma,M))\cong\mathrm{Ext}^1_{\Gamma'}(F,M),$$

and then $\mathrm{Ext}^1_{\Gamma'}(F,M)=0$. Hence M is a cotorsion Γ'-module. □

Theorem 2.2. *Let Γ' be a finite index subgroup of Γ and let M be a Γ-module. Then M is cotorsion if and only if it is cotorsion as a Γ'-module.*

Proof. According to the Lemma 2.1, we only need to show the 'if' part. So, assume that M is a Γ-module which is cotorsion over Γ'. We must show that M is cotorsion as a Γ-module. To this end, consider an arbitrary flat Γ-module F. Due to Lazard's Theorem, there is a direct system $\{P_i\}_{i\in I}$ of finitely generated projective Γ-modules such that $F\cong\varinjlim_i P_i$. Take for any i, a projective Γ'-module P_i' such that $P_i\cong\mathbb{Z}\Gamma\otimes_{\Gamma'}P_i'$ as Γ-modules. Letting $F'\cong\varinjlim_i P_i'$, one infers that F' is a flat Γ'-module and $F\cong\mathbb{Z}\Gamma\otimes_{\Gamma'}F'$. Hence, we may have the following isomorphisms:

$$\begin{aligned}\mathrm{Ext}^1_\Gamma(F,M)&\cong\mathrm{Ext}^1_\Gamma(\mathbb{Z}\Gamma\otimes_{\Gamma'}F',M)\\&\cong\mathrm{Ext}^1_{\Gamma'}(F',\mathrm{Hom}_\Gamma(\mathbb{Z}\Gamma,M))\\&\cong\mathrm{Ext}^1_{\Gamma'}(F',M),\end{aligned}$$

in which, the second isomorphism obtains by adjointness of Hom and $\otimes$. Since, by the assumption M is a cotorsion Γ'-module, $\mathrm{Ext}^1_{\Gamma'}(F',M)=0$, implying that $\mathrm{Ext}^1_\Gamma(F,M)=0$, as desired. □

Definition 2.3. Let M be a nonzero Γ-module. The cotorsion dimension of M, denoted by $\mathrm{cd}_\Gamma M$, is defined to be the least non-negative integer n such that $\mathrm{Ext}^{n+1}_\Gamma(F,M)=0$, for every flat Γ-module F. If no such n exists, set $\mathrm{cd}_\Gamma M=\infty$.

Remark 2.4. Suppose that Γ is a group. It is easy to show that, for any Γ-module M and integer $n\geq 0$, $\mathrm{cd}_\Gamma M\leq n$ if and only if there exists an exact sequence

$$0\longrightarrow M\longrightarrow C^0\longrightarrow C^1\longrightarrow\cdots\longrightarrow C^n\longrightarrow 0,$$

where each C^i is a cotorsion Γ-module, $i = 1, 2, \dots, n$; see [6, Corollary 19.2.1].

Corollary 2.5. *Let Γ' be a subgroup of Γ with finite index and let M be a Γ-module. Then $\mathrm{cd}_{\Gamma} M = \mathrm{cd}_{\Gamma'} M$.*

Proof. The inequality $\mathrm{cd}_{\Gamma'} M \leq \mathrm{cd}_{\Gamma} M$ follows immediately from Theorem 2.2. For the reverse inequality, we may assume that $\mathrm{cd}_{\Gamma'} M = n < \infty$. If F is an arbitrary flat Γ-module, then by a similar argument as that in the proof of Theorem 2.2, one may obtain the isomorphism $\mathrm{Ext}^{n+1}_{\Gamma}(F, M) \cong \mathrm{Ext}^{n+1}_{\Gamma'}(F', M)$, in which F' is a flat Γ'-module. By assumption $\mathrm{Ext}^{n+1}_{\Gamma'}(F', M) = 0$. Therefore, $\mathrm{Ext}^{n+1}_{\Gamma}(F, M) = 0$. This implies the inequality, and the proof is complete. □

Definition 2.6. Assume that R is an associative ring with identity. The left (resp., right) global cotorsion dimension of R, denoted by $l.\mathrm{Cot.D}(R)$ (resp., $r.\mathrm{Cot.D}(R)$) is defined as the supremum of the cotorsion dimensions of left (resp., right) R-modules. If $R = \mathbb{Z}\Gamma$, where Γ is a group, then R is isomorphic with the opposite ring R^{op} and so the distinction between left and right module is redundant. In this case, we drop the superfluous letters l and r and we write $\mathrm{Cot.D}(\Gamma)$ instead of $\mathrm{Cot.D}(\mathbb{Z}\Gamma)$.

Lemma 2.7. *Let Γ' be an arbitrary subgroup of Γ and C be a cotorsion Γ'-module. Then $\mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, C)$ is a cotorsion Γ-module.*

Proof. Suppose that F is an arbitrary flat Γ-module. Using the adjointness of Hom and $\otimes$, we have the following isomorphisms:

$$\mathrm{Ext}^1_{\Gamma}(F, \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, C)) \cong \mathrm{Ext}^1_{\Gamma'}(\mathbb{Z}\Gamma \otimes_{\Gamma} F, M) \cong \mathrm{Ext}^1_{\Gamma'}(F, C).$$

Since F is flat over Γ', hence $\mathrm{Ext}^1_{\Gamma'}(F, C) = 0$ and then

$$\mathrm{Ext}^1_{\Gamma}(F, \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, C)) = 0.$$

The proof is now finished. □

Lemma 2.8. *Let Γ' be a subgroup of Γ with finite index. Then for any Γ-module M,*

$$\mathrm{cd}_{\Gamma'} M = \mathrm{cd}_{\Gamma} M = \mathrm{cd}_{\Gamma} \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M) = \mathrm{cd}_{\Gamma'} \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M).$$

Proof. The first and third equalities follows from Theorem 2.2. So, it is enough to show the second equality. To this end, first we show that $\mathrm{cd}_{\Gamma} \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M) \leq \mathrm{cd}_{\Gamma} M$. If $\mathrm{cd}_{\Gamma} M = \infty$, then there is no thing to prove. So assume that $\mathrm{cd}_{\Gamma} M = n < \infty$. By Remark 2.4, there exists an exact sequence of Γ-modules;

$$0 \longrightarrow M \longrightarrow C^0 \longrightarrow C^1 \longrightarrow \cdots \longrightarrow C^n \longrightarrow 0,$$

where each C^i is a cotorsion, and so cotorsion as a Γ'-module. Since $\mathbb{Z}\Gamma$ is a free $\mathbb{Z}\Gamma'$-module, applying the functor $\mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, -)$ to this sequence, gives rise to the following exact sequence of Γ-modules

$$0 \to \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M) \to \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, C^0) \to \cdots \to \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, C^n) \to 0.$$

By Lemma 2.7, $\mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, C^i)$'s are cotorsion Γ-modules. This means that $\mathrm{cd}_\Gamma \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M) \leq n$.
For the converse inequality, consider the exact sequence of Γ-modules

$$0 \longrightarrow M \longrightarrow \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M) \longrightarrow K \longrightarrow 0,$$

which splits over Γ'. So, M is isomorphic to a direct summand of $\mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M)$ over Γ'. Hence $\mathrm{cd}_{\Gamma'} M \leq \mathrm{cd}_{\Gamma'} \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M)$. In particular, by Theorem 2.2, $\mathrm{cd}_\Gamma M \leq \mathrm{cd}_\Gamma \mathrm{Hom}_{\Gamma'}(\mathbb{Z}\Gamma, M)$. This implies that the second equality. The proof is now finished. □

Remark 2.9. Related to the problem of extending the Farrell-Tate cohomology, two homological invariants were assigned to a group Γ by Gedrich and Gruenberg, $\mathrm{spli}\Gamma$, the supremum of the projective length of the injective Γ-modules, and $\mathrm{silp}\Gamma$, the supremum of the injective lengths of the projective Γ-modules [10]. They studied these invariants and showed that for any group Γ, $\mathrm{silp}\Gamma \leq \mathrm{spli}\Gamma$ and if $\mathrm{spli}\Gamma$ is finite, then $\mathrm{silp}\Gamma = \mathrm{spli}\Gamma$. These invariants then have been considered by several authors; see [4, 5]. For a long time, it was not known if the finiteness of $\mathrm{silp}\Gamma$ implies the finiteness of $\mathrm{spli}\Gamma$. In 2010, it is proved that by Emmanouil [7] that, for any group Γ, $\mathrm{silp}\Gamma = \mathrm{spli}\Gamma$. While proving his interesting result, Emmanouil applied two new invariants $\mathrm{silf}\Gamma$, the supremum of the injective length of the flat Γ-modules, and $\mathrm{sfli}\Gamma$, the supremum of the flat length of the injective Γ-modules. For any group Γ, let $\mathrm{sclf}\Gamma$ denote the supremum of the cotorsion length of the flat Γ-modules. Note that since injective modules are cotorsion, one has the inequality $\mathrm{sclf}\Gamma \leq \mathrm{silf}\Gamma$.

The following proposition obtains immediately from [6, Theorem 7.2.5], but here we provide a short proof for it.

Proposition 2.10. *Let Γ be a group. Then,*

(i) $\mathrm{Cot.D}(\Gamma) = \mathrm{sclf}\Gamma$.
(ii) $\mathrm{Cot.D}(\Gamma) \leq \mathrm{silp}\Gamma$.

Proof. (i). It is clear that $\mathrm{sclf}\Gamma \leq \mathrm{Cot.D}(\Gamma)$. To prove the inverse inequality, we may assume that $\mathrm{sclf}\Gamma$ is finite, say n. Let M be an arbitrary Γ-module. Clearly we are done, if we can show that

$\mathrm{Ext}^{n+1}_{\Gamma}(F,M) = 0$ for any flat Γ-module F. To do this, consider a short exact sequence of Γ-modules;

$$0 \longrightarrow K \longrightarrow G \longrightarrow M \longrightarrow 0,$$

in which $G \to M$ is a flat cover of the Γ-module M. So by [17, Lemma 2.1.1], K is cotorsion. Assume that F is any flat Γ-module. Apply the functor $\mathrm{Hom}_{\Gamma}(F,-)$ to this sequence, to get the exact sequence

$$\mathrm{Ext}^{n+1}_{\Gamma}(F,G) \longrightarrow \mathrm{Ext}^{n+1}_{\Gamma}(F,M) \longrightarrow \mathrm{Ext}^{n+2}_{\Gamma}(F,K).$$

The first term vanishes because $\mathrm{cd}_{\Gamma}G \leq n$, and the last term vanishes because K is cotorsion; see [1, 2.2]. Hence, $\mathrm{Ext}^{n+1}_{\Gamma}(F,M) = 0$, as needed.
(ii). In view of part (i), $\mathrm{Cot.D}(\Gamma) \leq \mathrm{silf}\Gamma$. On the other hand, by [1, Theorem 3.3], $\mathrm{silf}\Gamma = \mathrm{silp}\Gamma$, implying that $\mathrm{Cot.D}(\Gamma) \leq \mathrm{silp}\Gamma$. The proof is complete. □

Theorem 2.11. *Let Γ' be a subgroup of Γ with finite index. Then* $\mathrm{Cot.D}(\Gamma) = \mathrm{Cot.D}(\Gamma')$.

Proof. In view of Corollary 2.5, we only need to show that $\mathrm{Cot.D}(\Gamma') \leq \mathrm{Cot.D}(\Gamma)$. If $\mathrm{Cot.D}(\Gamma) = \infty$, there is nothing to prove. So assume that $\mathrm{Cot.D}(\Gamma) = n < \infty$. Take an arbitrary Γ'-module M. By the hypothesis, Γ-module $\mathbb{Z}\Gamma \otimes_{\Gamma'} M$ has cotorsion dimension at most n, and hence Corollary 2.5, yields that the inequality $\mathrm{cd}_{\Gamma'}(\mathbb{Z}\Gamma \otimes_{\Gamma'} M) \leq n$, implying that $\mathrm{cd}_{\Gamma'}M \leq n$, since M is a direct summand of $\mathbb{Z}\Gamma \otimes_{\Gamma'} M$ as a Γ'-module. Consequently, $\mathrm{Cot.D}(\Gamma') \leq n$, as desired. □

Corollary 2.12. *If Γ is a finite group, then* $\mathrm{Cot.D}(\mathbb{Z}) = \mathrm{Cot.D}(\Gamma)$.

Remark 2.13. Recall that a ring R is called (left) perfect if every (left) R-module has a projective cover. In [2], Bass proved that perfect rings are those rings such that every flat module is projective. This rings were characterized in term of the vanishing of $\mathrm{Cot.D}(R)$ by Ding and Mao as: R is a perfect ring if and only if $\mathrm{Cot.D}(R) = 0$; see [6, Corollary 19.2.9]. It is clear that $\mathbb{Z}$ is not perfect. Hence, if Γ is a finite group, then the previous corollary implies that $\mathbb{Z}\Gamma$ is not a perfect ring.

Corollary 2.14. *If Γ is a finite group, then* $\mathrm{Cot.D}(\Gamma) = 1$.

Proof. By Proposition 2.10, we have $\mathrm{Cot.D}(\Gamma) \leq \mathrm{silp}\Gamma$. Since Γ is finite, by [7, Theorem 4.6] $\mathrm{silp}\Gamma = 1$. So $\mathrm{Cot.D}(\Gamma) \leq 1$. On the other hand, by the above Remark, $\mathrm{Cot.D}(\Gamma) \neq 0$. Hence $\mathrm{Cot.D}(\Gamma) = 1$. □

Theorem 2.15. *For any group Γ,* $\mathrm{silf}\Gamma \leq \mathrm{Cot.D}(\Gamma) + \mathrm{sfli}\Gamma$.

Proof. Assume that $\mathrm{Cot.D}(\Gamma) = n$ and $\mathrm{sfli}\Gamma = m$ are both finite. By [1, Theorem 3.3], in conjunction with Remark 2.9, it is enough for us to show that $\mathrm{pd}_\Gamma(I) \leq n + m$, for all injective Γ-modules I. Since $\mathrm{sfli}\Gamma = m$, $\mathrm{fd}_\Gamma I \leq m$. Thus, there exists an exact sequence of Γ-modules;

$$0 \longrightarrow F_m \longrightarrow F_{m-1} \longrightarrow \cdots \longrightarrow F_1 \longrightarrow F_0 \longrightarrow I \longrightarrow 0,$$

in which F_i, for any i, is flat. Take short exact sequences

$$0 \longrightarrow L_i \longrightarrow F_i \longrightarrow L_{i-1} \longrightarrow 0,$$

where, $L_i = \ker(F_i \longrightarrow F_{i-1})$, $i = 0, 1, 2, \ldots, m-1$, $F_{-1} = I$ and $F_m = L_{m-1}$. By [6, Theorem 19.2.5] together with [14, Lemma 9.26], we have $\mathrm{pd}_\Gamma(L_{m-2}) \leq 1 + n$. So $\mathrm{pd}_\Gamma(I) \leq m + n$. This means that $\mathrm{spli}\Gamma \leq m + n$. Therefore, $\mathrm{silf}\Gamma \leq m + n$, as required. □

Remark 2.16. Let Γ be a finite group. Then by [1, Corollary 3.9] $\mathrm{silf}\Gamma = \mathrm{sfli}\Gamma = 1$. Also by Corollary 2.14, $\mathrm{Cot.D}(\Gamma) = 1$. So in this case, the inequality is strict.

Acknowledgments

The author would like to thank the referee for valuable suggestions and comments which improved the present article.

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Journal of Algebraic Systems

COTORSION DIMENSIONS OVER GROUP RINGS

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بُعدهای همتابی روی حلقه گروه‌ها

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فرض کنید Γ یک گروه و Γ' زیرگروهی از Γ با اندیس متناهی باشد. فرض کنید M یک Γ-مدول باشد. نشان می‌دهیم که M همتابی است اگر و تنها اگر به عنوان Γ'-مدول نیز همتابی باشد. با استفاده از این نتیجه، ثابت می‌کنیم که بُعدهای همتابی جامع حلقه‌های $Z\Gamma$ و $Z\Gamma'$ نیز با هم مساوی هستند.

کلمات کلیدی: بُعد همتابی، بُعد همتابی جامع، حلقه کامل.