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THE COST NUMBER AND THE DETERMINING NUMBER OF A GRAPH

S. ALIKHANI* AND S. SOLTANI

Abstract. The distinguishing number $D(G)$ of a graph G is the least integer d such that G has an vertex labeling with d labels that is preserved only by a trivial automorphism. The minimum size of a label class in such a labeling of G with $D(G) = d$ is called the cost of d-distinguishing G and is denoted by $\rho_d(G)$. A set of vertices $S \subseteq V(G)$ is a determining set for G if every automorphism of G is uniquely determined by its action on S. The determining number of G, $\text{Det}(G)$, is the minimum cardinality of determining sets of G. In this paper we compute the cost and the determining number for the friendship graphs and corona product of two graphs.

1. Introduction

Let $G = (V, E)$ be a simple graph with n vertices. We use the standard graph notation [8]. The set of all automorphisms of G, with the operation of composition of permutations, is a permutation group on V and is denoted by $\text{Aut}(G)$. A labeling of G, $\phi : V \to \{1, 2, \ldots, r\}$, is r-*distinguishing*, if no non-trivial automorphism of G preserves all of the vertex labels. In other words, ϕ is r-distinguishing if for every non-trivial $\sigma \in \text{Aut}(G)$, there exists x in V such that $\phi(x) \neq \phi(\sigma(x))$. The *distinguishing number* of a graph G, $D(G)$ which has been defined in [1], is the minimum number r such that G has a labeling that it is r-distinguishing. To consider the cost number of a graph,

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we also need to know what it means for a subset of vertices to be d-distinguishable. For $W \subseteq V(G)$, a labeling $f : W \to \{1, \ldots, d\}$ is called *d-distinguishing* if whenever an automorphism fixes W setwise and preserves the label classes of W then it fixes W pointwise. Note that though such an automorphism fixes W pointwise, it is not necessarily trivial; it may permute vertices in the complement of W. A set W is called *d-distinguishable* if it has a d-distinguishing labeling. By definition, W is 1-distinguishable if every automorphism that preserves W fixes it pointwise. The introduction of the distinguishing number in [1] was a great success; by now about one hundred papers were written motivated by this seminal paper! The core of the research has been done on the invariant $D(G)$ itself, either on finite [6, 9, 11] or infinite graphs [7, 12, 13]; see also the references therein.

In 2007, Imrich posed the following question [5, 10]: "What is the minimum number of vertices in a label class of a 2-distinguishing labeling for the hypercube Q_n?" To aid in addressing this question, Boutin [5] called a label class in a 2-distinguishing labeling of G a *distinguishing class*. She called the minimum size of such a class in G the *cost of 2-distinguishing G* and denoted it by $\rho(G)$. More generally, for a graph G with the distinguishing number $D(G) = d$, the minimum size of a label class in any d-distinguishing labeling of G, is called the cost of d-distinguishing of G and denoted it by $\rho_d(G)$. The motivation for the cost of 2-distinguishing is that since virtually all graphs are 2-distinguishable, the cost can help differentiate between these and so the parameter $\rho_2(G)$ (or simply, $\rho(G)$) is more important.

Boutin showed that $\lceil \log_2 n \rceil - 1 \leq \rho(Q_n) \leq \lceil \log_2 n \rceil + 1$. She used the *determining set* [4], a set of vertices whose pointwise stabilizer is trivial. In other words, a subset S of the vertices of a graph G is called a determining set if whenever $g, h \in \mathrm{Aut}(G)$ agree on the vertices of S, they agree on all vertices of G. That is, S is a determining set if whenever g and h are automorphisms with the property that $g(s) = h(s)$ for all $s \in S$, then $g = h$. Albertson and Boutin proved the following theorem in [2].

Theorem 1.1. [2] *A graph is d-distinguishable if and only if it has a determining set that is $(d-1)$-distinguishable.*

In particular, the complement of such a determining set is a label class in a d-distinguishing labeling of G. Thus, a graph is 2-distinguishable if and only if it has a determining set for which any automorphism that fixes it setwise must also fix it pointwise. In such a case, the determining set and its complement provide the two necessary label classes for a 2-distinguishing labeling. Thus, in particular, the cost of 2-distinguishing a graph G is bounded below by the size of a smallest determining set, denoted $\mathrm{Det}(G)$. In other words:

Observation 1.2. For any graph G, $Det(G) \leq \rho(G)$.

This paper is organized as follows. Some properties of the cost of d-distinguishing labeling of G, is given in Section 2. Also by finding the cost number and the determining number of the friendship graph in Section 2, we show that for any positive integer m, there exists a graph G with $D(G) = d$ such that $|\mathrm{Det}(G) - \rho_d(G)| = m$. The cost of d-distinguishing corona product of two graphs are given in Section 3.

2. The cost of d-distinguishing graphs

It can be easily seen that the cost of n-distinguishing of complete graph K_n and complete bipartite graph $K_{n,m}$ ($m < n$) and $K_{n-1,n-1}$ is 1. The following results are immediate consequences of definition of d-distinguishing labeling.

Proposition 2.1. *Let G be a graph of order n and $D(G) = d$. Then*

(i) $\rho_d(G) \leq \frac{n}{d}$.
(ii) $d = 1$ *if and only if* $\rho_d(G) = n$.
(iii) *If* $d \geq 2$, *then* $\rho_d(G) \leq \frac{n}{2}$. *In particular, if* $\rho_d(G) = \frac{n}{2}$, *then* $d = 2$.
(iv) *If* ψ *is a* d*-distinguishing labeling of* G *with distinguishing classes of sizes* $t_1 \leq \cdots \leq t_d$ *such that* $t_1 = \rho_d(G)$, *then*

$$\mathrm{Det}(G) \leq \rho_d(G) + t_2 + \cdots + t_{d-1}.$$

(v) $\rho_d(G) \leq n - \mathrm{Det}(G)$.

Corollary 2.2. *Let G be a graph of order n with $D(G) = d$. If A is the determining set of G such that $|A| = \mathrm{Det}(G)$ with distinguishing number $d - 1$, then*

$$\rho_d(G) \leq \min\{n - \mathrm{Det}(G), \rho_{d-1}(G[A])\},$$

where $G[A]$ is the induced subgraph of G generated by vertices in A.

Proof. Set $\mathrm{Det}(G) = t$ and let $A = \{v_1, \ldots, v_t\}$ be a determining set of G with the distinguishing number $d - 1$. If we label the vertices of $G[A]$ with labels $1, \ldots, d - 1$ distinguishingly, and label all vertices $v_{t+1}, \ldots, v_n$ with new label d, then it can be seen that we have a distinguishing labeling of G with d labels. Since the minimum size of distinguishing classes of this labeling is $\min\{n - t, \rho_{d-1}(G[A])\}$, so we have the result. □

By Proposition 2.1 (v) and Observation 1.2, we can prove the following result.

Corollary 2.3. *Let G be a graph of order n and the distinguishing number $D(G) = d$.*

(i) *If* $\mathrm{Det}(G) \leq \rho_d(G)$*, then* $\mathrm{Det}(G) \leq \frac{n}{2}$.

(ii) *If $d = 2$, then* $\mathrm{Det}(G) \leq \frac{n}{2}$.

We shall show that for any positive integer m, there exists a graph G with $D(G) = d$ such that $|\mathrm{Det}(G) - \rho_d(G)| = m$. To do this we consider a friendship graph and compute its cost and determining number. The friendship graph F_n $(n \geq 2)$ can be constructed by joining n copies of the cycle graph C_3 with a common vertex (see Figure 1). The distinguishing number of friendship graphs is as follows:

Theorem 2.4. [3] *The distinguishing number of the friendship graph F_n $(n \geq 2)$ is*

$$D(F_n) = \lceil \frac{1 + \sqrt{8n+1}}{2} \rceil.$$

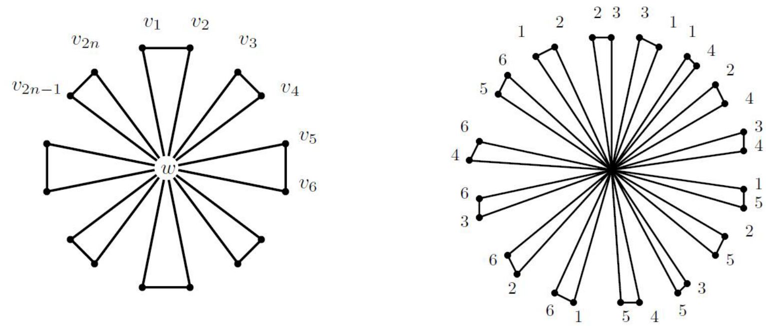

FIGURE 1. Friendship graph F_n and the vertex labeling of F_{15}, respectively.

Lemma 2.5. *Let $k_j = \min\{i : \; D(F_i) = j\}$ for any $j \geq 3$. Then*

(i) *For any $j \geq 3$,* $k_j = \lfloor \frac{j^2-3j+2}{2} \rfloor + 1$.

(ii) *For all i, $0 \leq i \leq j-2$, $D(F_{k_j+i}) = j$ and $D(F_{k_j+j-1}) = j+1$.*

Proof. (i) Suppose that $D(F_i) = j$. By Theorem 2.4, we have $\lceil \frac{1+\sqrt{8i+1}}{2} \rceil = j$ and so $j - 1 < \frac{1+\sqrt{8i+1}}{2} \leq j$ which implies that $\frac{(2j-3)^2-1}{8} < i \leq \frac{(2j-1)^2-1}{8}$ and so we have the result.

(ii) Regarding to the proof of Part (i), for every natural number m in the interval $(\frac{j^2-3j+2}{2}, \frac{j^2-j}{2}]$, $D(F_m) = j$. It is obvious that $m = k_j + i$, where $0 \leq i \leq j-2$. So we have the results. □

Theorem 2.6. *Let $j \geq 3$ and $k_j = \min\{i : D(F_i) = j\}$. Then $\rho_j(F_{k_j+i}) = i+1$ where $0 \leq i \leq j-2$.*

Proof. To obtain the value of the cost of j-distinguishing of this friendship graph, we should consider its j-distinguishing labeling. In any j-distinguishing labeling of F_{k_j+i} with labels $\{1,\dots,j\}$, each of the 2-sets consisting of vertex of degree two and its neighbor of degree two (two vertices on the base of triangles in friendship graph) must have a different 2-subset of labels $\{1,\dots,j\}$. Consider the friendship graph in Figure 1. As stated in the proof of Theorem 2.2 of [3], the function that maps v_1 to v_2 and v_2 to v_1 and fixes the rest of vertices, is a non-trivial automorphism. Thus the labels v_1 and v_2 should be different. We assign the vertex v_1 the label 1 and the vertex v_2 the label 2. Similarly, the function that maps v_3 to v_4 and v_4 to v_3 and fixes the rest, is a non-trivial automorphism. Thus the labels v_3, v_4 should be distinct. Let assign the vertex v_3 the label 2 and the vertex v_4 the label 3. We continue this method to label all vertices of friendship graph (see the label of F_{15} in Figure 1). Note that the label of vertex w is 1. Hence this method gives a distinguishing vertex labeling with the minimum number of labels. Since $k_j = \min\{i : D(F_i) = j\}$, all 2-subsets of $\{1,\dots j\}$ have been used for any distinguishing labeling of F_{k_j-1}. Thus without loss of generality, we can assume that the number of label p which is used for labeling of vertex set of F_{k_j-1}, say $n_p(F_{k_j-1})$, is $n_p(F_{k_j-1}) = j-2$ for $2 \leq p \leq j-1$ and $n_1(F_{k_j-1}) = j-1$ (the central vertex w is labeled with label 1). If we assign the 2-sets $\{v_{2q-1}, v_{2q}\}$, where $k_j \leq q \leq k_j + i$, the 2-subsets $\{i+1, j\}$ of labels, then we obtain a distinguishing labeling for F_{k_j+i} with labels $1,\dots,j$ such that $n_j(F_{k_j+i}) = i+1$. Thus $\rho_j(F_{k_j+i}) \leq i+1$. On the other hand, we have $n_p(F_{k_j-1}) \geq j-2$, for any $2 \leq p \leq j-1$, so since $n_p(F_{k_j+i}) \geq n_p(F_{k_j-1}) \geq j-2$ and $0 \leq i \leq j-2$, the number of label j used for the labeling of vertex set of F_{k_j+i}, $n_j(F_{k_j+i})$ is equal to $\min\{n_1(F_{k_j+i}),\dots,n_j(F_{k_j+i})\}$. Now since the label j have been used only for vertices v_q, where $2k_j - 1 \leq q \leq 2k_j + 2i$, and since the 2-subsets of labels related to the 2-sets $\{v_{2q-1}, v_{2q}\}$ and $\{v_{2q'-1}, v_{2q'}\}$ must be different for any $q, q' \in \{k_j, k_j+1,\dots,k_j+i\}$ where $q \neq q'$, so $n_j(F_{k_j+i}) = i+1$, and therefore $\rho_j(F_{k_j+i}) = i+1$. $\square$

Theorem 2.7. *For any $n \geq 2$, $\mathrm{Det}(F_n) = n$.*

Proof. Let the vertices of F_n be as shown in Figure 1. It can be easily seen that the set $A = \{v_1, v_3,\dots,v_{2n-1}\}$ is a determining set for F_n. On the other hand, if B is a determining set of F_n with $|B| \leq n-1$, then there exists $i \in \{1,\dots,n\}$ such that $v_{2i-1}, v_{2i} \notin B$. Hence there exists a non-identity automorphism f of F_n with $f(x) = x$ for all $x \in B$, $f(v_{2i-1}) = v_{2i}$ and $f(v_{2i}) = v_{2i-1}$, which is a contradiction to that B is a determining set. Therefore $\mathrm{Det}(F_n) = n$. □

Now we end this section by the following theorem:

Theorem 2.8. *For any positive integer m, there exists a graph G with $D(G) = d$ such that $|\mathrm{Det}(G) - \rho_d(G)| = m$.*

Proof. By Theorems 2.6 and 2.7, it can be concluded that for every m, there exists some suitable n such that the friendship graph F_n satisfies $|\mathrm{Det}(F_n) - \rho_d(F_n)| = m$. □

3. The cost and determining number of corona product

In this section, we shall study the cost number and the determining number of corona product of graphs. The *corona product* $G \circ H$ of two graphs G and H is defined as the graph obtained by taking one copy of G and $|V(G)|$ copies of H and joining the i-th vertex of G to every vertex in the i-th copy of H. The distinguishing number of corona product of graphs have been studied by the authors in [3]. Before presenting our results, we explain the relationship between the automorphism group of the graph $G \circ H$ with the automorphism groups of two connected graphs G and H such that $G \neq K_1$. Note that there is no vertex in the copies of H which has the same degree as a vertex in G. Because if there exists a vertex w in one of the copies of H and a vertex v in G such that $\deg_{G\circ H}(v) = \deg_{G\circ H}(w)$, then $\deg_G(v) + |V(H)| = \deg_H(w) + 1$. So we have $\deg_H(w) + 1 > |V(H)|$, which is a contradiction. Here we like to give an automorphism for $G \circ H$ as stated in the proof of Theorem 3.2 of [3]. Let the vertex set of G be $\{v_1, \ldots, v_{|V(G)|}\}$ and the vertex set of i-th copy of H, H_i, be $\{w_{i1}, \ldots, w_{i|V(H)|}\}$. Since there is no vertex in copies of H which has the same degree as a vertex in G, for every $f \in \mathrm{Aut}(G \circ H)$ and for every copy of H, we have $f|_H \in \mathrm{Aut}(H)$ and $f|_G \in \mathrm{Aut}(G)$. In addition, for any $i, j \in \{1, \ldots, |V(G)|\}$ we have

$$(f(v_i) = v_j) \Longleftrightarrow (f(H_i) = H_j).$$

Conversely, let $\varphi \in \mathrm{Aut}(G)$ and $\phi \in \mathrm{Aut}(H)$ such that $\varphi(v_i) = v_{j_i}$, where $i, j_i \in \{1, \ldots, |V(G)|\}$. Now we define the following automorphism h of $G \circ H$:

$$h : \begin{cases} v_i \mapsto \varphi(v_i) = v_{j_i} & i, j_i \in \{1, \ldots, |V(G)|\}, \\ w_{ik} \mapsto \phi(w_{j_ik}) & k \in \{1, \ldots, |V(H)|\}. \end{cases}$$

We start with the determining number of corona product of two graphs.

Theorem 3.1. *Let G and H be two connected graphs of orders $n, m \geq 2$, respectively. Then*

$$\mathrm{Det}(G \circ H) = \mathrm{Det}(G) + n\mathrm{Det}(H).$$

Proof. We denote the vertices of G in $G \circ H$ by $v_1, \ldots, v_n$, and vertices of H corresponding to the vertex v_i by $w_{i1}, \ldots, w_{im}$. Let $\mathrm{Det}(G) = k$ and $\mathrm{Det}(H) = k'$. We suppose that the sets $\{v_1, \ldots, v_k\}$ and $\{w_1, \ldots, w_{k'}\}$ are the determining sets of G and H, respectively. Clearly, the set $\{v_1, \ldots, v_k\} \cup (\bigcup_{i=1}^{n} \{w_{i1}, \ldots, w_{ik'}\})$ is a determining set of $G \circ H$. Hence $\mathrm{Det}(G \circ H) \leq \mathrm{Det}(G) + n\mathrm{Det}(H)$. On the other hand if $\mathrm{Det}(G \circ H) < \mathrm{Det}(G) + n\mathrm{Det}(H)$, then there exists a determining set Z for $G \circ H$ with $|Z| = \mathrm{Det}(G \circ H)$ such that $|Z \cap V(H_i)| < k'$ or $|Z \cap V(G)| < k$ for some $1 \leq i \leq n$, where H_i is the isomorphic copy of H corresponding to the vertex v_i in $G \circ H$. We consider two following cases:

Case 1) Let $Z \cap V(H_i) = \{w_{ij_1}, \ldots, w_{ij_t}\}$ where $t < k'$ for some i, $1 \leq i \leq n$. Since $t < k'$, it can be concluded that there exists a non-identity automorphism f of H such that $f(w_{ij_1}) = w_{ij_1}, \ldots, f(w_{ij_t}) = w_{ij_t}$. We extend f to a non-identity automorphism $\overline{f}$ of $G \circ H$ with

$$\overline{f}(x) = \begin{cases} x & \text{if } x \in V(G), \\ f(x) & \text{if } x \in V(H_i), \\ x & \text{if } x \in V(H_{i'}), i' \neq i. \end{cases}$$

In this case, $\overline{f}$ is a non-identity automorphism of $G \circ H$ and it fixes the determining set Z, pointwise, which is a contradiction.

Case 2) Let $Z \cap V(G) = \{v_{j_1}, \ldots, v_{j_t}\}$ where $t < k$. Since $t < k$, so there exists a non-identity automorphism f of G such that $f(v_{j_1}) = v_{j_1}, \ldots, f(v_{j_t}) = v_{j_t}$. We extend f to a non-identity automorphism $\overline{f}$ of $G \circ H$ with

$$\overline{f}(x) = \begin{cases} f(x) & \text{if } x \in V(G), \\ x & \text{if } x \in V(H_i), i = 1, \ldots, n. \end{cases}$$

In this case, $\overline{f}$ is a non-identity automorphism of $G \circ H$ and it fixes the determining set Z, pointwise, which is a contradiction. □

Theorem 3.2. *If G is a connected graph of order $n \geq 2$, then* $\mathrm{Det}(G \circ K_1) = \mathrm{Det}(G)$.

Proof. It is clear that each determining set of G is a determining set of $G \circ K_1$, and so $\mathrm{Det}(G \circ K_1) \leq \mathrm{Det}(G)$. Set $\mathrm{Det}(G) = k$, $V(G) = \{v_1, \ldots, v_n\}$, and denote the vertex of K_1 adjacent to the vertex v_i, by w_i. Assume by contrary that $t = \mathrm{Det}(G \circ K_1) < k$. Then, there exists a determining set Z of $G \circ K_1$ such that $Z = \{v_1, \ldots, v_{t_1}, w_{j_1}, \ldots, w_{j_{t-t1}}\}$ where $t_1 \leq t < k$. We show that $\{v_1, \ldots, v_{t_1}, v_{j_1}, \ldots, v_{j_{t-t1}}\}$ is a determining set of G with less than k elements, which is a contradiction. Before it, we note that since Z is a determining set, so $\{1, \ldots, t_1\} \cap \{j_1, \ldots, j_{t-t_1}\} = \emptyset$, otherwise if $j_x \in \{1, \ldots, t_1\} \cap \{j_1, \ldots, j_{t-t_1}\}$, then $Z' = Z - \{w_{j_x}\}$ is a determining set of $G \circ K_1$ with $|Z'| < |Z|$, which is a contradiction. If f is a non-identity automorphism of G with $f(v_i) := v_{\sigma(i)}$, where σ is a non-identity permutation of $1, \ldots, n$, fixing the vertices of $\{v_1, \ldots, v_{t_1}, v_{j_1}, \ldots, v_{j_{t-t_1}}\}$, pointwise, then we can extend f to the non-identity automorphism $\overline{f}$ of $G \circ K_1$ with definition $\overline{f}(v_i) := v_{\sigma(i)}$ and $\overline{f}(w_i) = w_{\sigma(i)}$ for every $1 \leq i \leq n$. Thus $\overline{f}$ fixes the vertices of Z pointwise, which is a contradiction. Thus the vertices of $\{v_1, \ldots, v_{t_1}, v_{j_1}, \ldots, v_{j_{t-t_1}}\}$ is a determining set of G. □

Theorem 3.3. *Let G and H be two connected graphs of orders $n, m \geq 2$, respectively, with $D(G) = k$ and $D(H) = k'$. If $k'' = \max\{k, k'\}$ and $D(G \circ H) = k''$, then*

$$\rho_{k''}(G \circ H) \leq \rho_k(G) + n\rho_{k'}(H).$$

Proof. We present a distinguishing labeling for $G \circ H$ with k'' labels such that the minimum size of a distinguishing class in this k''-distinguishing labeling is $\rho_k(G) + n\rho_{k'}(H)$. For this purpose, we label the vertices of G distinguishingly with k labels $1, \ldots, k$ such that the distinguishing class 1 has the minimum size among others. Then we label each of copies of H distinguishingly with k' labels $1, \ldots, k'$ such that the distinguishing class 1 has the minimum size among the remaining distinguishing classes of H. This labeling of $G \circ H$ is a k''-distinguishing labeling. In fact, if f is an automorphism of $G \circ H$ preserving the labeling, then since the restriction of f to G and each copy of H is an automorphism of G and H, respectively, and since the vertices of G and each copy of H have been labeled distinguishingly, so these restrictions are identity, and hence f is the identity automorphism of $G \circ H$ (see explanations before Theorem 3.1). Since the distinguishing class 1 has the minimum size $\rho_k(G) + n\rho_{k'}(H)$ among the remaining distinguishing classes of $G \circ H$, so the result follows. □

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عدد متمایزکننده گراف G، $D(G)$ عبارتست از کوچکترین عدد صحیح d به‌طوری که گراف G دارای یک رنگ‌آمیزی راسی با d رنگ است که تنها تحت خودریختی همانی حفظ می‌شود. به کمترین اندازه کلاس رنگ‌ها در این رنگ‌آمیزی با $D(G) = d$، عدد ارزشی d متمایزکننده گراف می‌گویند و آن‌را با نماد $\rho_d(G)$ نشان می‌دهند. مجموعه $S \subseteq V(G)$ را مجموعه تعیین‌کننده گراف G گوییم، هرگاه هر خودریختی روی G به‌طور یکتا توسط عمل خود روی S مشخص گردد. عدد تعیین‌کننده گراف G، $Det(G)$ کمترین اندازه مجموعه‌های تعیین‌کننده گراف G است. در این مقاله عدد ارزشی و عدد تعیین‌کننده را برای گراف‌های دوستانه و ضرب کرونای دو گراف محاسبه می‌کنیم.

کلمات کلیدی: عدد متمایزکننده، مجموعه تعیین کننده، عدد ارزشی، گراف.