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ON THE CLASS OF ARRAY-BASED APM-LDPC CODES

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Abstract. In this paper, an explicit class of affine permutation matrix low-density parity-check (APM-LDPC) codes is presented. This class is constructed based on the array parity-check matrix by using two affine maps $f(x) = x-1$ and $g(x) = 2x-1$ on $\mathbb{Z}_m$, where m is an odd prime number, with girth 6 and flexible row (column)-weights. Simulation results justify well performance, minimum distances and cycle distribution of these codes in comparison of the array-QC LDPC, structured QC-LDPC and APM-LDPC codes.

1. Introduction

Low-density parity-check (LDPC) codes were first introduced by Robert Gallager [5] in his PhD thesis, and have been forgotten for a long time and were again raised by Neal and MacKay [11]. But, today LDPC codes have become one of the interesting topic in coding theory [1] because of their performance close to Shannon limit over additive white Guassian noise (AWGN) channels [4].

Each LDPC code can be defined by a sparse parity-check matrix H. The numbers of nonzero elements in a row (column) of parity-check matrix H is called row (column)-weight. By a (J, L)-regular LDPC code, we mean an LDPC code whose parity-check matrix has row and column weights L and J, respectively.

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One of the particularly important tools for knowing and reviewing LDPC codes is Tanner graph [14]. In fact, Tanner graph is a bipartite graph, which has two sets of vertices, check nodes and bit nodes correspond to the rows and columns of the parity-check matrix $H = (h_{i,j})$, respectively, such that $h_{i,j} = 1$, if and only if the corresponding check and bit nodes are adjacent. The girth of LDPC codes is the length of the shortes cycles in the Tanner graph of the corresponding parity-check matrix H. The performance of LDPC codes under iterative decoding algorithms is related to the girth of Tanner graph [11], the minimum distance [14] and the row (column)-weight distributions [7] of parity-check matrix H. Especially, a 4-cycle, a cycle of length 4, has a bad influence on the performance of decoding algorithm [4].

LDPC codes based on the construction are divided in two methods, random codes [5, 11] and structured codes [2, 3, 9]. Random construction has an excellent BER performance, but, on the other hand, it needs more memory to store the nonzero elements of a random parity-check matrix, against, structured LDPC codes have simple implementations [4].

Quasi-cyclic LDPC (QC-LDPC) codes are an important class of algebraically constructed LDPC codes based on circulant permutation matrices (CPM) having good performance among the class of LDPC codes because the required memory for storing corresponding parity-check matrices can be reduced [13].

Array-QC LDPC codes are a class of structured LDPC codes with good performance, first are introduced by Fan [3]. Fan has shown the beauty of algebra in array-LDPC codes.

A class of structured LDPC codes based on affine permutation matrices (APM), called briefly APM-LDPC codes [12], have been proposed recently which have some advantages other than QC-LDPC codes in performance, cycle distribution and minimum distance [6]. Anti quasi-cyclic (AQC) LDPC codes are a special case of APM-LDPC codes [8], constructed by circulant and anti-circulant permutation matrices.

In this paper, we assume two affine maps $f(x) = x - 1$ and $g(x) = 2x - 1$ on $\mathbb{Z}_m$, where m is an odd prime number, and define H as a parity-check matrix of an LDPC code based on $f(x)$, $g(x)$ and composition of these functions based on array parity-check matrix. We show that the girth of Tanner graph H is 6. The presented method is very convenient and the constructed codes have flexible row (column)-weights. Simulation results show that the constructed codes outperform the array-QC LDPC codes [3], structured QC-LDPC codes [2], APM-LDPC codes [6] and AQC LDPC codes [8].

2. Preliminaries

In this section, we present the basic concepts used in the paper.

2.1. **APM-LDPC Codes.**

Let m be a non-negetive integer. The ring of integers in modular m is shown by $\mathbb{Z}_m = \{0, 1, \dots, m-1\}$ and $\mathbb{Z}_m^* = \{i \in \mathbb{Z}_m \mid \gcd(i, m) = 1\}$. Now, corresponding to each affine map $f(x) = ax + b \pmod{m}$, $(a, b) \in \mathbb{Z}_m^* \times \mathbb{Z}_m$, define the affine permutation matrix (APM) If by the $m \times m$ binary matrix $(e_{i,j})$ in which $e_{i,j} = 1$ if and only if $f(i) = j$ in modulus of m. If $a = 1$, then I^f is a CPM. We denote the $m \times m$ zero matrix with $I^{\emptyset}$, for empty function $\emptyset$ on $\mathbb{Z}_m$. The following propositions give some of properties of APMs [6].

Proposition 2.1. *Let f, f_1 and f_2 be bijective functions on $\mathbb{Z}_m$. Then we have,*

(1) $I^{f_1} \times I^{f_2} = I^{f_2 \circ f_1}$, *where $f_2 \circ f_1$ is the composition of f_2 and f_1.*
(2) $I^{f^{-1}} = (I^f)^{-1} = (I^f)^T$, *where $(I^f)^T$ is the transpose of I^f.*
(3) $I^{f^n} = \underbrace{I^f \times \cdots \times I^f}_{n}$, *where f^n means the compositions n times of f.*

Proposition 2.2. *Let f be an affine map on $\mathbb{Z}_m$. If two first rows (columns) of I^f are given, then f can be uniquely determined.*

Proof. The assertion follows easily by solving of a system of two equations in two unknowns. $\square$

Let J and L, $J < L$, be some positive integers. A (J, L) function matrix $F = (f_{i,j})_{J \times L}$ means a $J \times L$ array of $f_{i,j}$ which $f_{i,j}$'s are some affine or empty functions on $\mathbb{Z}_m$. Corresponding to a function matrix F, a (J, L) APM-LDPC code of APM-size m, length mL and rate at least $1 - \frac{J}{L}$ can be defined by the following parity-check matrix $H = H(F)$.

$$H = \begin{pmatrix} I^{f_{1,1}} & \dots & I^{f_{L,1}} \\ \vdots & \ddots & \vdots \\ I^{f_{1,J}} & \dots & I^{f_{L,J}} \end{pmatrix}, \tag{2.1}$$

such that the corresponding function matrix is as follows:

$$F = \begin{pmatrix} f_{1,1} & \cdots & f_{L,1} \\ \vdots & \ddots & \vdots \\ f_{1,J} & \cdots & f_{L,J} \end{pmatrix}. \tag{2.2}$$

A (J, L) APM-LDPC code is called *conventional*, if H does not contain the zero block, i.e., the associated function matrix F does not contain the empty function, otherwise it is called *unconventional*.

The following lemma gives a necessary and sufficient condition for H, that the Tanner graph of H has a $2l$-cycle.

Lemma 2.3. [12] *There is a $2l$-cycle in Tanner graph H in (2.1), by a chain $(j_0, l_0); (j_1, l_1); \cdots; (j_{n-1}, l_{n-1}); (j_n, l_n) = (j_0, l_0), (j_k, l_k) \neq (j_{k+1}, l_{k+1})$ that $0 \leq k \leq n-1$, if the function*

$$\mathcal{F} = f_{j_n,l_n} \circ f^{-1}_{j_n,l_{n-1}} \circ \cdots \circ f_{j_1,l_1} \circ f^{-1}_{j_1,l_0}$$

has a fixed point, it means that there is $a \in \mathbb{Z}_m$, such that $\mathcal{F}(a) = a$.

QC-LDPC codes are a class of APM-LDPC codes interested for well performances and simple hardware implementations. In [6], some conditions are discussed which examine whether an APM-LDPC code is equivalent with a QC-LDPC code or not.

Theorem 2.4. *An APM-LDPC code with parity-check matrix H in (2.1), can be considered as a QC-LDPC code, if $I^{f_{j_1,l_1}} \times I^{f_{j_2,l_2}} = I^{f_{j_2,l_2}} \times I^{f_{j_1,l_1}}$, for each $(j_1, l_1) \neq (j_2, l_2)$, $1 \leq j_1, j_2 \leq J$ and $1 \leq l_1, l_2 \leq L$.*

Lemma 2.5. *An APM-LDPC code with function matrix F in (2.2), can be considered as a QC-LDPC code, if each two functions of F, interchanges with each other under the composition function, it means $f_{j_2,l_2} \circ f_{j_1,l_1} = f_{j_1,l_1} \circ f_{j_2,l_2}$, for each $(j_1, l_1) \neq (j_2, l_2)$, $1 \leq j_1, j_2 \leq J$ and $1 \leq l_1, l_2 \leq L$.*

Remark 2.6. If in (2.1), $a_{j,l} = 1$ for every $1 \leq j \leq J$ and $1 \leq l \leq L$, then by using Lemma 2.5, H is a parity-check matrix of a QC-LDPC code.

2.2. **Array-QC LDPC codes.** Let n, m are some positive integers, such that $n \leq m$, m is an odd prime number, a (n, m) array-QC LDPC code is defined by the following parity check matrix.

$$H_{m,n} = \begin{pmatrix} I & I & I & \cdots & I \\ I & M & M^2 & \cdots & M^{m-1} \\ I & M^2 & M^4 & \cdots & M^{2(m-1)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ I & M^{n-1} & M^{(n-1)2} & \cdots & M^{(n-1)(m-1)} \end{pmatrix}, \tag{2.3}$$

where M denotes a $m \times m$ permutation matrix of the form

$$M = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}. \tag{2.4}$$

Array based QC-LDPC codes are among regular LDPC codes whose Tanner graphs are free of 4-cycles, having an acceptable bit-error rate performance [3]. They have been proposed for a number of applications, magnetic recording and including digital subscriber lines.

It is interesting that the matrix M is equivalent to an affine map $f(x) = x - 1$ in modular m. Then, $H_{m,n}$ can be considered as the parity-check matrix of an APM-LDPC code, so we have the function matrix F,

$$F = \begin{pmatrix} f(x)^0 & f(x)^0 & \cdots & f(x)^0 \\ f(x)^0 & f(x)^1 & \cdots & f(x)^{m-1} \\ f(x)^0 & f(x)^2 & \cdots & f(x)^{2(m-1)} \\ \vdots & \vdots & \cdots & \vdots \\ f(x)^0 & f(x)^{n-1} & \cdots & f(x)^{(n-1)(m-1)} \end{pmatrix}, \tag{2.5}$$

which, $f(x)^0$ means the identity function $I(x) = x$.

Structured LDPC codes have been more attention than random LDPC codes. In fact, a regular algebraic structure can guarantee to improve the minimum distances, cycle distribution and to simplify the implementation. Recently, efforts have been made to build structured LDPC codes. Therefore, we use the structure of array parity-check matrix to construct a class of APM-LDPC codes.

3. Construction

Let $f(x) = x - 1$ and $g(x) = 2x - 1$ on $\mathbb{Z}_m$, such that m is an odd prime number. Define the following function matrix F of dimension $s \times m$

$$F = \begin{pmatrix} g^0 \circ f^0 & g^0 \circ f^1 & \cdots & g^0 \circ f^{m-1} \\ g^1 \circ f^0 & g^1 \circ f^1 & \cdots & g^1 \circ f^{m-1} \\ \vdots & \vdots & \ddots & \vdots \\ g^{s-1} \circ f^0 & g^{s-1} \circ f^1 & \cdots & g^{s-1} \circ f^{m-1} \end{pmatrix}, \tag{3.1}$$

where s is the order of 2 in the field of $\mathbb{Z}_m$. Now, for the given function matrix F, let H be the corresponding parity-check matrix of a (s, m)-regular APM-LDPC as follows:

$$H = \begin{pmatrix} I & I^{f} & \cdots & I^{f^{m-1}} \\ I^{g} & I^{g\circ f} & \cdots & I^{g\circ f^{m-1}} \\ \vdots & \vdots & \ddots & \vdots \\ I^{g^{s-1}} & I^{g^{s-1}\circ f} & \cdots & I^{g^{s-1}\circ f^{m-1}} \end{pmatrix}, \tag{3.2}$$

in which $I^{g^i\circ f^j}$ is the APM matrix of size m corresponding to the function $g^i \circ f^j(x) = 2^i x - 2^i j - (2i-1)$ in modulus of m, where $1 \leq i, j \leq m-1$.

By the definition of APM based on an affine map,

$$I^{g} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 \end{pmatrix}, \tag{3.3}$$

and matrix representation of $I^{g^i\circ f^j}$'s are depended on $\mathbb{Z}_m$. For instance, $I^{g^2\circ f^2}$ in $\mathbb{Z}_5$ as follows:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3.4}$$

and in $\mathbb{Z}_7$ as follows:

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.5}$$

The data rate of the parity-check matrix H in (3.2) is $R \geq 1 - \frac{m(m-s)}{m^2}$. Hence, by increasing m, R tends to 1. Notice, we can take a part of matrix H in (2.5), as parity-check matrix of an APM-LDPC code, thus in this case, the data rate will be change.

Example 3.1. For $m = 7$, the order of 2 in the field $\mathbb{Z}_7$ is 3, thus in this case the column-weight of parity-check matrix H can be equal or less than three. But for $m = 11$, the order of 2 in the field $\mathbb{Z}_{11}$ is 10. So we can consider (J, L)-regular APM-LDPC code such that $1 \leq J \leq 10$ and $1 \leq L \leq 11$.

Proposition 3.2. *The given matrix H in (3.2) is not a parity-check matrix of a QC-LDPC code.*

Proof. If H is a parity-check matrix of a QC-LDPC code, then by Lemma 2.5, every two elements of F in (3.1), should be interchanged. Without loss of generality, we assume two elements of $F(H)$, the $(2,1)$th array and $(2,2)$th array, i.e., $g(x)$ and $g(x) \circ f(x)$, respectively. Therefore with a simple calculation, $g(x) \circ (g(x) \circ f(x)) = 4x + 7(m-1)$ and $(g(x) \circ f(x)) \circ g(x) = 4x + 5(m-1)$ in modular m. This is clear that $-7 \neq -5 \pmod m$. Therefore $g(x) \circ (g(x) \circ f(x)) \neq (g(x) \circ f(x)) \circ g(x)$. Accordingly, H in (3.2), is not a parity-check matrix of a QC-LDPC code. Thus, the constructed code is not a QC-LDPC code. □

Theorem 3.3. *Let the given matrix H in (3.2) be a parity-check matrix of an APM-LAPC code. Then the girth of Tanner graph of H is 6.*

Proof. First we show that there is no 4-cycle in Tanner graph H in (3.2). Otherwise, by Lemma 2.3, a chain of 4-cycle is (j_0, l_0), (j_0, l_1), (j_1, l_1), (j_1, l_0), $0 \leq j_0, j_1 \leq s-1$ and $0 \leq l_0, l_1 \leq m-1$, $(j_0, l_0) \neq (j_1, l_1)$ and we have,

$$\mathcal{F} = (g^{j_0} \circ f^{l_0}) \circ (g^{j_0} \circ f^{l_1})^{-1} \circ (g^{j_1} \circ f^{l_1}) \circ (g^{j_1} \circ f^{l_0})^{-1}. \tag{3.6}$$

$g^k(x) = 2^k x - (2k-1)$, $g^{-k}(x) = (2^k)^{-1}(x + 2k - 1)$, $f^k(x) = x - k$ and $f^{-k}(x) = x + k \pmod m$, where k is a positive integer, thus

$$\mathcal{F}(x) = x + 2^{j_1 - j_0}(l_0 - l_1) + (l_1 - l_0) \pmod{m}$$

If $\mathcal{F}(a) = a$ for $a \in \mathbb{Z}_m$, then,

$$2^{j_1 - j_0}(l_0 - l_1) + (l_1 - l_0) = 0 \pmod{m} \tag{3.7}$$

Set $l_0 - l_1 = y$ and $j_1 - j_0 = x$, Equation (3.7) is equal to

$$2^x y = y \pmod{m} \tag{3.8}$$

We have two choices

(1) $y = 0$, thus $l_0 = l_1$, this is a contradiction.
(2) $2^x = 1 \pmod m$, it means x is the order of 2 in the field $\mathbb{Z}_m$, besides, the maximum column-weight of parity-check matrix H in (3.2) is s, thus the maximum amount of x is $s-1$, so $x \neq s$. As regarding to the definition of s, $2^x \neq 1$.

Therefore, the equation (3.8) has no solution and there is no 4-cycle in Tanner graph H in (3.2), and accordingly the girth is at least 6. Then again, we may assume the block sequence $(0,0)$, $(0,1)$, $(1,1)$, $(1,r)$, $(2,r)$, $(2,0)$, where $r = -2^{-1} (\mathrm{mod}\ m)$, coincides to a cycle of length 6. This completes the proof.

□

4. Numerical and Simulation Results

Tabel 1 provides some comparisons between the $(6,8)$-cycle multiplicities of (J,L) constructed codes, and cycle distributions 6 and 8 of the 4-cycle free APM-LDPC codes [6], array-QC LDPC codes [3] and structured QC-LDPC codes [2] with the same length.

As Table 1 shows, the constructed codes have remarkably smaller cycles rather than the 4-cycle free APM-LDPC [6], array-QC LDPC [3] and QC-LDPC [2] codes.

Table 2 presents the minimum distance of constructed codes against APM-LDPC [6], array-QC LDPC [3] and structured QC-LDPC [2] codes with girth 6, denoted by d, d_{APM}, d_{Array} and d_{QC}, respectively. As the outputs show, the constructed codes have better minimum distance than APM-LDPC in [6], array-QC LDPC in [3] and QC-LDPC codes in [2].

Figure 1 displays a binary performance comparison between the constructed code, on one hand, and an APM-LDPC code [6], a structured QC-LDPC code [2], an array-QC LDPC code [3] and a structured AQC-LDPC code [8] with girth 6, rate 0.875 and length 32672, on the other hand. We use Sum-product algorithm [10] on the AWGN channel for decoding of these codes with maximum iteration 20. The simulation result shows that the constructed code outperforms APM-LDPC code, QC-LDPC code, array-QC LDPC code and AQC LDPC code.

Notice that the constructed APM-LDPC codes in this paper have an explicit structure in terms of a and a random structure in terms of b.

5. Conclusion

In this work, a class of APM-LDPC codes are presented whose parity-check matrices are based on parity-check matrix of array-LDPC codes. A constructed codes have some benefits rather than APM, structured QC and array-QC LDPC codes in terms of the cycle distribution, minimum distances and error-rate performance. In addition, the presented codes have flexible row and column-weights. We show the constructed codes have the new class of APM-LDPC and not the class of QC-LDPC codes. Moreover, we present the girth of these codes have 6. As Table 1 shows, the constructed codes have better cycle distributions rather than APM, array-QC and structured QC-LDPC codes with the same girth. Tables 2 illustrates that the constructed codes have bigger minimum distances rather than APM, array-QC and structured QC-LDPC codes. In the last, simulation result shows that the binary constructed codes perform better than APM, array-QC, structured QC and AQC LDPC codes with the same girth, rates and lengths.

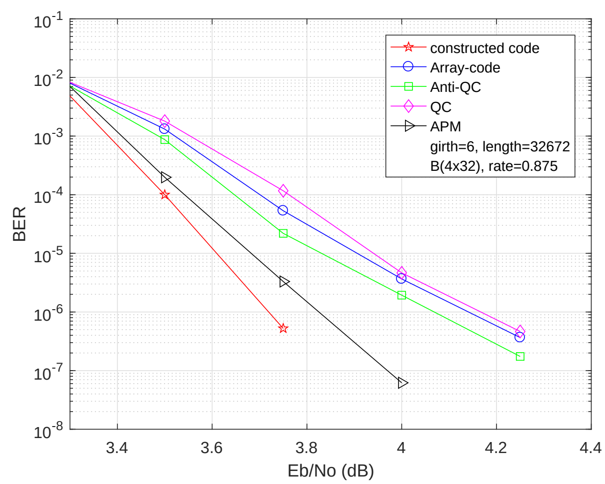
TABLE 1. A comparison between the cycle multiplicities of the (J, L) constructed codes and APM-LDPC codes [6], array-QC LDPC codes [3] and structured QC-LDPC codes [2] with girth 6 and the same block size m.

m	J	L	Cycle length	Constructed code	APM	Array	QC
5	2	3	6	0	0	0	0
			8	5	5	5	5
5	2	4	6	0	0	0	0
			8	30	30	30	30
11	3	4	6	22	22	44	66
			8	242	242	286	253
13	3	5	6	52	78	104	104
			8	767	702	858	832
17	3	6	6	102	204	456	513
			8	1938	2295	2295	2363
19	4	5	6	266	152	456	513
			8	3135	3439	4066	3857
23	4	6	6	460	667	828	1334
			8	8050	8165	10396	11109
29	4	7	6	870	754	1624	2523
			8	17197	16936	23258	27666
31	4	8	6	1488	2139	2356	3844
			8	30814	33046	40610	49352
37	5	6	6	1036	1110	2960	4292
			8	25160	25345	40108	44696
41	5	7	6	1886	2296	5084	7052
			8	50594	51455	80688	94915

TABLE 2. The minimum distance of constructed codes, in comparison with APM-LDPC codes [6], array-QC LDPC codes [3] and structured QC-LDPC codes [2] with the same girth and block size m.

m	$J \times L$	d	d_{APM}	d_{Array}	d_{QC}
13	3×5	8	8	6	6
17	3×6	8	6	6	4
19	4×5	16	16	12	4
23	4×6	16	16	12	12
29	4×7	14	14	12	12

FIGURE 1. A girth-6 constructed code against an APM-LDPC code [6], an array-QC LDPC code [3], an AQC-LDPC code [8] and a structured QC-LDPC code [2] with girth 6.



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کلاسی از کدهای خلوت آفین بر پایه کدهای آرایه‌ای

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در این مقاله، کلاس صریحی از کدهای خلوت آفین ارائه می‌شود. این کلاس بر اساس طرح ماتریس بررسی توازن کدهای آرایه‌ای با استفاده از دو نگاشت آفین $f(x) = x - ۱$ و $g(x) = ۲x - ۱$ روی حلقه پیمانه‌ای اعداد صحیح m، که در آن m یک عدد اول فرد است، مطرح می‌شود. کدهای ساخته شده دارای وزن سطری و ستونی انعطاف‌پذیر و همچنین کمر ۶ هستند. نتایج عددی نشان می‌دهد که کدهای ساخته شده دارای کمترین فاصله بیشتر، توزیع دوری بهتر و اجرای مناسب‌تری در مقایسه با کدهای آرایه‌ای، کدهای شبه‌دوری ساختاری و کدهای خلوت آفین می‌باشد.

کلمات کلیدی: کد خلوت آفین، کد آرایه‌ای، گراف تنر.