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A GENERALIZATION OF PRIME HYPERIDEALS

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Abstract. Let R be a multiplicative hyperring. In this paper, we introduce and study the concept of n-absorbing hyperideal which is a generalization of prime hyperideal. A proper hyperideal I of R is called an n-absorbing hyperideal of R if whenever $\alpha_1 o \dots o \alpha_{n+1} \subseteq I$ for $\alpha_1, \dots, \alpha_{n+1} \in R$, then there are n of the $\alpha_i^,$s whose product is in I.

1. Introduction

The theory of algebraic hyperstructures was first initiated by Marty in 1934 [13] when he defined the hypergroups. Since then, several books and hundreds of papers have been written on this topic. A short review of the theory of hyperstructures appears in [5, 6, 9, 14, 15, 19].

The hyperrings were introduced and studied by different researchers. Contrary to classical algebra, in hyperstructure theory, there are various kinds of hyperrings. One important class of hyperrings was introduced by Rota in 1982, where the multiplication is a hyperoperation, while the addition is an operation, which is called multiplicative hyperrings [16]. Moreover, there exists a general type of hyperrings that both the addition and multiplication are hyperoperations. This type of hyperrings can be found in [20]. For more study on other types of hyperrings, we refer to [9].

The notion of prime ideal, which is a generalization of the notion of prime number in the ring of integers, plays a prominent role in the theory of rings. Badawi [3] and later Anderson and Badawi [4]

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introduced the concepts of 2-absorbing ideals and n-absorbing ideals which are two generalizations of prime ideals. The concept of prime and primary hyperideals in a multiplicative hyperring was introduced by Dasgupta [7]. Afterward, the notion was investigated by Sevim et al. [17]. Ghiasvand [2] introduced the concept of 2-absorbing hyperideal in a multiplicative hyperring which is a generalisation of prime hyperideal. Several authors have extended and generalized this concept in several ways [11, 12, 18]. Let R be a multiplicative hyperring. A proper hyperideal I of R is siad to be a 2-absorbing hyperideal of R if $xoyoz \subseteq I$ for $x, y, x \in R$ then $xoy \subseteq I$ or $xoz \subseteq I$ or $yoz \subseteq I$.

In this paper, we introduce and study the concept of n-absorbing hyperideal in a multiplicative hyperring and obtain their basic properties.

The paper is organized as follows. In Section 2, we give some definitions and notions from some references which we need to develop our paper. In Section 3, we introduce the notion of n-absorbing hyperideal. In Section 4, we study many properties of n-absorbing hyperideas. Finally, in Section 5, we study the stability of n-absorbing hyperideals with respect to various hyperring-theoric constructions.

2. Preliminaries

In this section we give some definitions and results of the hyperstructure which we need to develop our paper.

A triple $(R, +, o)$ is called a multiplicative hyperring if

(1) $(R, +)$ is an abelian group;

(2) (R, o) is semihypergroup;

(3) for all $a, b, c \in R$, we have $ao(b+c) \subseteq aob + aoc$ and $(b+c)oa \subseteq boa + coa$;

(4) for all $a, b \in R$, we have $ao(-b) = (-a)ob = -(aob)$.

If in (2) the equality holds, then we say that the multiplicative hyperring is strongly distributive. We assume throughout this paper that all multiplicative hyperrings are strongly distributive. For any two nonempty subsets A and B of R and $x \in R$, we define

$$AoB = \bigcup_{a \in A,\ b \in B} aob, \qquad Aox = Ao\{x\}$$

A non-empty subset I of R is a hyperideal of R if

(1) $a, b \in I$, then $a - b \in I$;

(2) $x \in I$ and $r \in R$, then $rox \subseteq I$.

Definition 2.1. [7] A proper hyperideal P of R is called a prime hyperideal of R if $\alpha o \beta \subseteq P$ for $\alpha, \beta \in R$ implies that $\alpha \in P$ or $\beta \in P$. The intersection of all prime hyperideals of R containing I is called the prime radical of I, being denoted by $r(I)$. If the multiplicative hyperring R does not have any prime hyperideal containing I, we define $r(I) = R$.

Definition 2.2. [7] A proper hyperideal Q of R is called a primary hyperideal of R if $\alpha o \beta \subseteq Q$ for $\alpha, \beta \in R$ implies that $\alpha \in Q$ or $\beta^n \subseteq Q$ for some $n \in \mathbb{N}$. We refer to the prime hyperideal $P = r(Q)$ as the associated prime hyperideal of Q and on the other hand Q is referred to as a P-primary hyperideal of R.

Definition 2.3. [7] Let $\mathbf{C}$ be the class of all finite products of elements of R i.e. $\mathbf{C} = \{r_1 o r_2 o ... o r_n \mid r_i \in R, n \in \mathbb{N}\} \subseteq P^*(R)$. A hyperideal I of R is said to be a $\mathbf{C}$-hyperideal of R, if whenever $A \cap I \neq \varnothing$ for any $A \in \mathbf{C}$, then $A \subseteq I$.

Theorem 2.4. [7, Proposition 3.2] *Let I be a hyperideal of R. Then, $D \subseteq r(I)$ where $D = \{r \in R \mid r^n \subseteq I \ for \ some \ n \in \mathbb{N}\}$. The equality holds when I is a $\mathbf{C}$-hyperideal of R.*

In this paper, we assume that all hyperideals are $\mathbf{C}$-hyperideal.

Definition 2.5. [8] Let $\mathfrak{U} = \{\sum_{i=1}^n A_i \mid A_i \in \mathbf{C}, n \in \mathbb{N}\}$ and $\mathbf{C} = \{r_1 o r_2 o ... o r_n \mid r_i \in R, n \in \mathbb{N}\}$. A hyperideal I of R is called a strong $\mathbf{C}$-hyperideal of R if whenever $E \cap I \neq \varnothing$ for any $E \in \mathfrak{U}$, then $E \subseteq I$.

Definition 2.6. [9] Let $(R_1, +_1, o_1)$ and $(R_2, +_2, o_2)$ be multiplicative hyperrings. A mapping f from R_1 into R_2 is said to be a good homomorphism if for all $a, b \in R_1$, $f(a +_1 b) = f(a) +_2 f(b)$ and $f(a o_1 b) = f(a) o_2 f(b)$.

Definition 2.7. [1] Let R be a multiplicative hyperring and I, J be hyperideals of R with scalar identity 1. We said that I, J are coprime (comaximal) if $I + J = R$.

Let I, J be two hyperideals of R. We define

$$(I :_R J) = \{\alpha \in R \mid \alpha o J \subseteq I\}.$$

3. ON n-ABSORBING HYPERIDEALS OF MULTIPLICATIVE HYPERRINGS

Definition 3.1. Let R be a multiplicative hyperring. A proper hyperideal I of R is called an n-absorbing hyperideal of R if whenever $\alpha_1 o \dots o \alpha_{n+1} \subseteq I$ for $\alpha_1, \dots, \alpha_{n+1} \in R$, then there are n of the α_i's whose product is in I.

Example 3.2. Let $(\mathbb{Z}, +, \cdot)$ be the ring of integers. We define the hyperoperation $a \odot b = \{2ab, 4ab\}$, for all $a, b \in \mathbb{Z}$. Then $(\mathbb{Z}, +, \odot)$ is a multiplicative hyperring. In this multiplicative hyperring, $15\mathbb{Z} = \{15n \mid n \in \mathbb{Z}\}$ is an n-absorbing hyperideal for $n \geq 2$ and $105\mathbb{Z} = \{105n \mid n \in \mathbb{Z}\}$ is an n-absorbing hyperideal for $n \geq 3$.

Example 3.3. Consider the ring $(\mathbb{Z}_6, \oplus, \odot)$ that for all $\bar{x}, \bar{y} \in \mathbb{Z}_6$, $\bar{x} \oplus \bar{y}$ and $\bar{x} \odot \bar{y}$ are the remainder of $\frac{x+y}{6}$ and $\frac{x.y}{6}$, respectively, which $+$ and $\cdot$ are ordinary addition and multiplication, and $x, y \in \mathbb{Z}$. We define the hyperoperation $\bar{x} \boxdot \bar{y} = \{\overline{xy}, \overline{2xy}, \overline{3xy}, \overline{4xy}, \overline{5xy}\}$. Then $(\mathbb{Z}_6, \oplus, \boxdot)$ is a commutative multiplicative hyperring and $\{\bar{0}\}$ is an n-absorbing hyperideal of $\mathbb{Z}_6$ for $n \geq 2$.

Theorem 3.4. *If $P_1, \ldots, P_n$ are prime hyperideals of R, then $P_1 \cap \cdots \cap P_n$ is an n-absorbing hyperideal of R.*

Proof. It is routine. □

Example 3.5. In the multiplicative hyperring of integers $\mathbb{Z}_A$ with $A = \{7, 11\}$, $\langle 2 \rangle$, $\langle 3 \rangle$ and $\langle 5 \rangle$ are prime hyperideals (see [7, Proposition 4.3]). Hence, $\langle 2 \rangle \cap \langle 3 \rangle \cap \langle 5 \rangle$ is a 3-absorbing hyperideal of $\mathbb{Z}_A$, by Theorem 3.4.

Theorem 3.6. *Let I be an n-absorbing hyperideal of R. Then $r(I)$ is an n-absorbing hyperideal of R and $x^n \subseteq I$ for all $x \in r(I)$.*

Proof. Let $x \in r(I)$. Then $x^m \subseteq I$ for some $m \in \mathbb{N}$. If $m \leq n$, we are done. If $m > n$, by using the n-absorbing property on products $x^n o x^k$, we conclude that $x^n \subseteq I$. Now, let $x_1 o \ldots o x_{n+1} \subseteq r(I)$ for $x_1, \ldots, x_{n+1} \in R$. Then $(x_1 o \ldots o x_{n+1})^n = x_1^n o \ldots o x_{n+1}^n \subseteq I$. Since I is an n-absorbing hyperideal of R, we may assume that $x_1^n o \ldots o x_n^n \subseteq I$. Thus $(x_1 o \ldots o x_n)^n \subseteq I$, and so $x_1 o \ldots o x_n \subseteq r(I)$, which implies $r(I)$ is an n-absorbing hyperideal of R. □

Let I be a proper hyperideal of R. It is clear that an n-absorbing hyperideal is also an k-absorbing hyperideal for all integers $k \geq n$. If I is an n-absorbing hyperideal of R for some $n \in \mathbb{N}$, then define $Abs(I) = min\{n \mid I \text{ is an } n\text{-absorbing hyperideal of } R\}$, otherwise, set $Abs(I) = \infty$ (we will just write $Abs(I)$ when the context is clear). We define $Abs(R) = 0$. Hence for any hyperideal I of R, we get $Abs(I) \in \mathbb{N} \cup \{0, \infty\}$ with $Abs(I) = 1$ if and only if I is a prime hyperideal of R and $Abs(I) = 0$ if and only if $I = R$. Thus $Abs(I)$ measures, in some sense, how far I is from being a prime hyperideal of R.

Lemma 3.7. *Let $I \subseteq P$ be a hyperideal of R, where P is a prime hyperideal. Then the following conditions are equivalent:*

(1) *P is a minimal prime hyperideal of I.*
(2) *For each $x \in P$, there is a $y \notin P$ and a non-negative integer i such that $yox^i \subseteq I$.*

Proof. ($\Longrightarrow$) Let P be a minimal prime hyperideal of I and Q_i's be other minimal prime hyperideals of I. Then $r(I) = P \cap (\bigcap_{Q_i \in Min(I)} Q_i)$. Suppose that $x \in P$ but $x \notin r(I)$. We may assume that $x \in P \cap (\bigcap_{i=1}^{t} Q_i)$ such that $x \notin \bigcup_{i \geq t+1} Q_i$. Take any $w \in \bigcap_{i \geq t+1} Q_i \backslash P$. Hence we have $wox \subseteq P \cap (\bigcap_{i=1}^{t} Q_i) \cap (\bigcap_{i \geq t+1} Q_i)$, that is $wox \subseteq r(x)$. It implies that $(wox)^n = w^n o x^n \subseteq I$. Now we take $y \in w^n$. Therefore $yox^n \subseteq I$.
($\Longleftarrow$) We assume that P is not a mimimal prime hyperideal of I and look for a contradiction. Our assumption means that we have $I \subseteq Q \subseteq P$ for some prime hyperideal Q of R. Let $x \in P \backslash Q$. Hence we have $yox^n \subseteq I \subseteq Q$ for some $n \in \mathbb{N}$. This is a contradiction, since $x, y \notin Q$. □

Theorem 3.8. *Let I be a n-absorbing hyperideal of R. Then there are at most n prime hyperideals of R that are minimal over I. Moreover, $|Min_R(I)| \leq Abs(I)$*

Proof. Assume that $P_1, \ldots, P_{n+1}$ are distinct prime hyperideals of R minimal over I. Hence we get $\alpha_i \in P_i \backslash ((\bigcup_{j \neq i} P_j) \cup P_{n+1})$, for $1 \leq i \leq n$. By Lemma 3.7, we have $\beta_i \in R \setminus P_i$ for $1 \leq i \leq n$, such that $\beta_i o \alpha_i^{n_i} \subseteq I$ for some $n_i \in \mathbb{N}$. Since $I \subseteq P_{n+1}$ and $\alpha_i \notin P_{n+1}$ for $1 \leq i \leq n$, then for $1 \leq i \leq n$, $\beta_i o \alpha_i^{n-1} \subseteq I$, which implies $(\beta_1 + \cdots + \beta_n) o \alpha_1^{n-1} o \ldots o \alpha_n^{n-1} \subseteq I$. Since $\alpha_i \in P_i \setminus (\bigcup_{j \neq i} P_j)$ and $\beta_i o \alpha_i^{n-1} \subseteq I \subseteq P_1 \cap \cdots \cap P_n$ for $1 \leq i \leq n$, then for $1 \leq i \leq n$, $\beta_i \in (\bigcap_{j \neq i} P_j) \setminus P_i$, which means $\beta_1 + \cdots + \beta_n \notin P_i$ for $1 \leq i \leq n$. We have $(\beta_1 + \cdots + \beta_n) \prod_{j \neq i} \alpha_j^{n-1} \nsubseteq P_i$ for $1 \leq i \leq n$, hence $(\beta_1 + \cdots + \beta_n) \prod_{j \neq i} \alpha_j^{n-1} \nsubseteq I$ for $1 \leq i \leq n$. Since I is an n-absorbing hyperideal of R, $\alpha_1^{n-1} o \ldots o \alpha_n^{n-1} \subseteq I \subseteq P_{n+1}$. It implies that $\alpha_i \in P_{n+1}$ for some $1 \leq i \leq n$, which is a contradiction. Thus, there are at most n prime hypeideals of R minimal over I. The last assertion is obvious. □

The converse of Theorem 3.8 is not true in general, as is shown in the following example.

Example 3.9. Let $(\mathbb{Z}, +, \cdot)$ be the ring of integers. We define the hyperoperation $a \star b = \{2ab, 3ab\}$, for all $a, b \in \mathbb{Z}$. Then $(\mathbb{Z}, +, \star)$ is a multiplicative hyperring. In the hyperring, $12\mathbb{Z} = \{12n \mid n \in \mathbb{Z}\}$ is not a 2-absorbing hyperideal of $\mathbb{Z}$. However, $2\mathbb{Z}$ and $3\mathbb{Z}$ are minimal prime hyperideals over $12\mathbb{Z}$.

Lemma 3.10. *Let $P_1, \dots, P_n$ be incomparable prime hyperideals of R, and let I be an n-absorbing hyperideal of R such that $I \subseteq P_1 \cap \cdots \cap P_n$. If $x_1^{t_1} o \dots o x_n^{t_n} \subseteq I$ for $x_i \in P_i \backslash (\bigcup_{j \neq i} P_j)$ and for positive integers t_i, then $x_1 o \dots o x_n \subseteq I$.*

Proof. Since I is an n-absorbing hyperideal of R, then there exist integers $s_1, \dots, s_n$ with $0 \leq s_i \leq t_i$ and $s_1 + \cdots + s_n = n$ such that $x_1^{s_1} o \dots o x_n^{s_n} \subseteq I$. Assume that for one of s_is, say s_1, we have $s_1 = 0$. Therefore $x_2^{t_2} o \dots o x_n^{t_n} \subseteq I$, that is $x_2^{t_2} o \dots o x_n^{t_n} \subseteq P_1$, which is a contradiction. Hence $x_1 o \dots o x_n \subseteq I$. □

Theorem 3.11. *Let I be an n-absorbing strong* **C***-hyperideal of R such that exactly n prime hyperideals $P_1, \dots, P_n$ of R are mimimal over I. Let $u_j \in P_j \backslash (\bigcup_{s \neq j} P_s)$ for every $j \neq i$ with $1 \leq i, j \leq n$. Then $P_i o \prod_{j \neq i} u_j \subseteq I$.*

Proof. Suppose that $x \in P_i$. If $x \in P_i$ but $x \notin \bigcup_{j \neq i} P_j$, then by Theorem 3.6 and Lemma 3.10, we obtain $xo \prod_{j \neq i} u_j \subseteq I$. Let $x \in P_i \cap (\bigcup_{j \neq i} P_j)$ and $z \in P_j \backslash (\bigcup_{j \neq i} P_i)$. Now, we want to show that there exists an element $y \in R$ such that for every $v \in yoz$, $v + x \in P_i \backslash (\bigcup_{i \neq j} P_j)$. Suppose that $S = \{t \mid x \in P_t, 1 \leq t \leq n, t \neq i\}$ and $T = \{t \mid x \notin P_t, 1 \leq t \leq n\}$. We assume that $y \in \prod_{s \in T} u_s$. Since $zo \prod_{s \in T} u_s \subseteq P_t$ and $x \notin P_t$ for every $t \in T$, we conclude that $v + x \notin P_t$ for every $v \in yoz$ and $t \in T$. Also, since $zo \prod_{s \in T} u_s \nsubseteq P_t$ for every $t \in T$ and $x \in P_t$ for every $t \in S$, we infer $v + x \notin P_t$ for every $v \in yoz$ and $t \in S$. Hence $v + x \in P_i \backslash (\bigcup_{j \neq i} P_j)$ for every $v \in yoz$. On the other hand, by Theorem 3.6 and Lemma 3.10, we have $(v + x) o \prod_{j \neq i} u_j \subseteq I$ for every $v \in yoz$ and $zo \prod_{j \neq i} u_j \subseteq I$. Hence we get $(v + x) o \prod_{j \neq i} u_j \subseteq (vo \prod_{j \neq i} u_j) + (xo \prod_{j \neq i} u_j) \subseteq (yozo \Pi_{j \neq i} u_j) + (xo \Pi_{j \neq i} u_j)$. Since I is an n-absorbing strong **C**-hyperideal of R and $(v + x) o \prod_{j \neq i} u_j \subseteq I$, then we have $(yozo \prod_{j \neq i} u_j) + (xo \prod_{j \neq i} u_j) \subseteq I$. Since $yozo \prod_{j \neq i} u_j \subseteq I$, then we have $xo \prod_{j \neq i} u_j \subseteq I$. Consequently, $P_i o \prod_{j \neq i} u_j \subseteq I$. □

Corollary 3.12. *Let $P_1, \dots, P_n$ are incomparable prime hyperideals of R such that $x \in P_i$ for some $1 \leq i \leq n$. Then there exists $y \in R$ and $z \in P_i \backslash (\bigcup_{j \neq i} P_j)$ such that for every $v \in yoz$, $v + x \in P_i \backslash (\bigcup_{j \neq i} P_j)$*

Theorem 3.13. *Let I be an n-absorbing strong* **C***-hyperideal of R. If I has exactly n mimimal prime hyperideals, then $P_1 o \dots o P_n \subseteq I$.*

Proof. Let $P_1, \dots, P_n$ be exactly n mimimal prime hyperideals over I. Suppose that for each $1 \leq j \leq n$, $x_j \in P_j$. By Lemma 3.11, we have $x_1 o \prod_{2 \leq j \leq n} u_j \subseteq I$ for some $u_j \in P_j \backslash (P_1 \cup (\bigcup_{i \neq j} P_i))$ with $2 \leq j \leq n$. Now, we assume that $(x_1 o \dots o x_s) \prod_{s+1 \leq j \leq n} u_j \subseteq I$ for some $1 \leq s \leq n-1$ and $u_j \in P_j \backslash (P_1 \cup (\bigcup_{i \neq j} P_i))$ with $s+1 \leq i \leq n$. We prove that $(x_1 o \ldots o x_s o x_{s+1}) \prod_{s+2 \leq j \leq n} u_j \subseteq I$ for every $u_j \in P_j \backslash (P_1 \cup (\bigcup_{i \neq j} P_i))$ with $s+2 \leq i \leq n$. There exist elements $y_{s+1} \in R$ and $z_{s+1} \in P_{s+1} \backslash (\bigcup_{i \neq s+1} P_i)$ such that for every $v_{s+1} \in y_{s+1} o z_{s+1}$, we have $v_{s+1} + a_{s+1} \in P_{s+1} \backslash (\bigcup_{i \neq s+1} P_i)$, by Corollary 3.12. Thus we get

$(x_1 o \ldots o x_s) o (v_{s+1} + x_{s+1}) o \prod_{s+2 \leq j \leq n} u_j$
$\subseteq ((x_1 o \ldots o x_s) o v_{s+1} o \prod_{s+2 \leq j \leq n} u_j) + (x_1 o \ldots o x_s o x_{s+1} o \prod_{s+2 \leq j \leq n} u_j)$.

Let $u_{s+1} = v_{s+1} + x_{s+1}$. Since I is an n-absorbing strong $\mathbf{C}$-hyperideal of R and $(x_1 o \ldots o x_s) o \prod_{s+1 \leq j \leq n} u_j \subseteq I$, then we have

$$(x_1 o \ldots o x_s) o v_{s+1} o \prod_{s+2 \leq j \leq n} u_j + (x_1 o \ldots o x_s o x_{s+1} o \prod_{s+2 \leq j \leq n} u_j) \subseteq I.$$

Since $(x_1 o \ldots o x_s) o v_{s+1} o \prod_{s+2 \leq j \leq n} u_j \subseteq I$, then we obtain

$$(x_1 o \ldots o x_s o x_{s+1}) o \prod_{s+2 \leq j \leq n} u_j \subseteq I.$$

Now, let $s = n-1$, then $(x_1 o \ldots o x_{n-1}) o (v_n + x_n) \subseteq I$ for every $v_n \in y_n o z_n$. It means that $x_1 o \ldots o x_n \subseteq I$. Consequently, $P_1 o \ldots o P_n \subseteq I$. □

Theorem 3.14. *Let $P_1, \ldots, P_n$ be prime hyperideals of a hyperring R that are pairwise coprime. Then $I = P_1 o \ldots o P_n$ is an n-absorbing hyperideal of R. Moreover, $Abs(I) = n$.*

Proof. Since $P_1, \ldots, P_n$ are pairwise coprime, then we have

$$I = P_1 o \ldots o P_n = P_1 \cap \cdots \cap P_n.$$

Hence I is an n-absorbing hyperideal of R. Also, since $P_{1,n}$ are incomparable, we choose $\alpha_i \in P_i \setminus \bigcup_{j \neq i} P_j$ for each $1 \leq i \leq n$. Then $\alpha_1 o \ldots o \alpha_n \subseteq P_1 \cap \cdots \cap P_n$, but no proper subproduct of the $\alpha_i^{,}$s is in $P_1 \cap \cdots \cap P_n$. Hence $Abs(P_1 \cap \cdots \cap P_n) = Abs(P_1 o \ldots o P_n) \geq n$. On the other hand, we have $Abs(P_1 \cap \cdots \cap P_n) = Abs(P_1 \cap \cdots \cap P_n) \leq n$. Thus $Abs(I) = Abs(P_1 o \ldots o P_n) = n$. □

Let $M_1, \ldots, M_n$ are distinct maximal hyperideals of R. Then $I = M_1 o \ldots o M_n$ is an n-absorbing hyperideal of R by Theorem 3.14. Now, we show that M^n is an n-absorbing hyperideal of R for any maximal hyperideal M of R. We show that the product of any n maximal hyperideals of R is an n-absorbing hyperideal of R.

Lemma 3.15. *Let M be a maximal hyperideal of R and n be a positive integer. Then M^n is an n-absorbing hyperideal of R such that $Abs(M^n) \leq n$. Moreover, if $M^{n+1} \subset M^n$ then $Abs(M^n) = n$.*

Proof. Let $\alpha_1 o \dots o \alpha_{n+1} \subseteq M^n$ for $\alpha_1, \dots, \alpha_{n+1} \in R$. If $\alpha_1, \dots, \alpha_{n+1} \in M$, then we are done. We may assume that $\alpha_{n+1} \notin M$. Hence $(M^n, \alpha_{n+1}) = R$, so there exist $\beta \in M^n$ and $\gamma \in R$ such that $1 \in \beta + \alpha_{n+1} o \gamma$. Hence

$$\alpha_1 o \dots o \alpha_n \subseteq (\alpha_1 o \dots o \alpha_n) o 1 \subseteq (\alpha_1 o \dots o \alpha_n) o \beta + (\alpha_1 o \dots o \alpha_{n+1}) \gamma \subseteq M^n.$$

Thus M^n is an n-absorbing hyperideal of R. Now, we assume that $M^{n+1} \subset M^n$. Then there are $\alpha_1, \dots, \alpha_n \in M$ such that $\alpha_1 o \dots o \alpha_n \subseteq M^n \setminus M^{n+1}$. Hence all products of $n-1$ of the $\alpha_i^{,}$s are not in M^n, since otherwise $\alpha_1 o \dots o \alpha_n \subseteq M^{n+1}$, and this is a contradiction. Thus M^n is not an $(n-1)$-absorbing hyperideal of R. Since M^n is an n-absorbing hyperideal of R, then $Abs(M^n) = n$. □

Theorem 3.16. *Let $M_1, \dots, M_n$ are maximal hyperideals of R. Then $I = M_1 o \dots o M_n$ is an n-absorbing hyperideal of R. Moreover, $Abs(I) \leq n$.*

Proof. Suppose that $M_1, \dots, M_n$ are distinct maximal hyperideals of R and $n_1, \dots, n_k$ are positive integers such that $n = n_1 + \dots + n_k$. We show that $I = M_1^{n_1} o \dots o M_k^{n_k}$ is an n-absorbing hyperideal of R. By Lemma 3.15, for all $1 \leq i \leq k$, $M_i^{n_i}$ is an n_i-absorbing hyperideal of R. Hence $I = M_1^{n_1} o \dots o M_k^{n_k} = M_1^{n_1} \cap \dots \cap M_k^{n_k}$ is an n-absorbing hyperideal of R □

4. Some properties of n-absorbing hyperideals

In this section, we study some properties of n-absorbing hyperideals.

Theorem 4.1. *Let P be a prime hyperideal of R, and let I be a P-primary hyperideal of R such that $P^n \subseteq I$ for some positive integer n. Then I is an n-absorbing hyperideal of R with $Abs(I) \leq n$. In particular, if P^n is a P-primary hyperideal of R, then P^n is an n-absorbing hyperideal of R with $Abs(P^n) \leq n$. Moreover, if $P^{n+1} \subset P^n$ then $Abs(P^n) = n$.*

Proof. Suppose that $\alpha_1 o \dots o \alpha_{n+1} \subseteq I$ for $\alpha_1, \dots, \alpha_{n+1} \in R$. Assume that one of the $\alpha_i^{,}$s is not in P. Since I is a P-primary hyperideal of R, then we conclude that the product of the other $\alpha_i^{,}$s is in I . Hence, we may assume that $\alpha_i \in P$ for every $1 \leq i \leq n$. We get $\alpha_1 o \dots o \alpha_n \subseteq I$ since $P^n \subseteq I$. Hence I is an n-absorbing hyperideal of R. The rest of the proof is obvious.

□

Theorem 4.2. *Let I be an n-absorbing hyperideal of R. Then $I_\alpha = (I :_R \alpha)$ is an n-absorbing hyperideal of R containing I for all $\alpha \in R \setminus I$. Moreover, $Abs(I_\alpha) \leq Abs(I)$ for all $\alpha \in R$*

Proof. Suppose that $\alpha_1 o \ldots o\alpha_{n+1} \subseteq I_\alpha$ for $\alpha_1, \ldots, \alpha_{n+1} \in R$. Thus $(\alpha o\alpha_1)o\alpha_2 o \ldots o\alpha_{n+1} \subseteq I$ which implies either the product of $\alpha o\alpha_1$ with $n-1$ of the $\alpha_i^{,}$s for $2 \leq i \leq n+1$ is in I or $\alpha_2 o \ldots o\alpha_{n+1} \subseteq I$. Then there is a product of n of the $\alpha_i^{,}$s that is in I_α. Hence I_α is an n-absorbing hyperideal of R. It is clear that $I \subseteq I_\alpha$. If $\alpha \in I$, then $I_\alpha = R$, and then $Abs(I_\alpha) = o \leq Abs(I)$. The last assertion is obvious. □

Theorem 4.3. *Let $I \subset r(I)$ be an n-absorbing strong* **C***-hyperideal of R for $n \geq 2$. If $k \geq 2$ is the least positive integer such that $\alpha^k \subseteq I$ for $\alpha \in r(I) \setminus I$, then $I_{\alpha^{k-1}} = (I :_R \alpha^{k-1})$is an $(n-k+1)$-absorbing hyperideal of R containing I.*

Proof. Let I be an n-absorbing strong **C**-hyperideal of R. Since $2 \leq k \leq n$, then we have $n-k+1 \geq 1$. It is clear that $I \subseteq I_{\alpha^{k-1}}$. Suppose that $c_1 o \ldots oc_{n-k+2} \subseteq I_{\alpha^{k-1}}$ for $c_1, \ldots, c_{n-k+2} \in R$. Since I is an n-absorbing hyperideal of R and $\alpha^{k-1} oc_1 o \ldots oc_{n-k+2} \subseteq I$, either $\alpha^{k-2} oc_1 o \ldots oc_{n-k+2} \subseteq I$ or the product of α^{k-1} with some $n-k+1$ of the $c_i^{,}$s is in I. In the second case, we are done. Thus suppose that the product of α^{k-1} with any $n-k+1$ of the $c_i^{,}$s is not in I. Hence $\alpha^{k-2} oc_1 o \ldots oc_{n-k+2} \subseteq I$. Since I is an strong **C**-hyperideal of R and $\alpha o\alpha^{k-2} oc_1 o \ldots oc_{n-k+1} o(c_{n-k+2} + \alpha) \subseteq I$, then we get

$$\alpha^{k-2} oc_1 o \ldots oc_{n-k+1}(c_{n-k+2} + \alpha) \subseteq A + B \subseteq I,$$

where $A = \alpha^{k-2} oc_1 o \ldots oc_{n-k+2}$ and $B = \alpha^{k-1} oc_1 o \ldots oc_{n-k+1}$. Since $\alpha^{k-2} oc_1 o \ldots oc_{n-k+2} \subseteq I$, we get $\alpha^{k-1} oc_1 o \ldots oc_{n-k+1} \subseteq I$. It is a contradiction, since the product of α^{k-1} with any $n-k+1$ of the $c_i^{,}$s is not in I. Hence the product of α^{k-1} with some $n-k+1$ of the $c_i^{,}$s is in I, which implies that $I_{\alpha^{k-1}}$ is an $(n-k+1)$-absorbing hyperideal of R containing I. □

Corollary 4.4. *Let $I \subset r(I)$ be an n-absorbing strong* **C***-hyperideal of R for $n \geq 2$. Let $\alpha \in r(I) \setminus I$ and $\alpha^n \subseteq I$ such that $\alpha^{n-1} \nsubseteq I$. Then $I_{\alpha^{n-1}} = (I :_R \alpha^{n-1})$ is a prime hyperideal of R containing $r(I)$.*

Proof. By Theorem 5.5, $I_{\alpha^{n-1}}$ is an $(n-n+1)$-absorbing hyperideal of R and then $I_{\alpha^{n-1}}$ is a prime hyperideal of R containing $r(I)$. □

Corollary 4.5. *Let I be an n-absorbing P-primary strong* **C***-hyperideal of R for some prime hyperideal P of R and $n \geq 2$. If $\alpha \in r(I) \setminus I$ and n is the least positive integer such that $\alpha^n \subseteq I$, then $I_{\alpha^{n-1}} = (I :_R \alpha^{n-1}) = P$*

Proof. Let I be an n-absorbing P-primary strong **C**-hyperideal of R. By Corollary 4.4, $P = r(I) \subseteq I_{\alpha^{n-1}}$. Assume that $\beta \in I_{\alpha^{n-1}}$, hence

$\alpha^{n-1}o\beta \subseteq I$. We have $\beta \in P$, since I is a P-primary hyperideal and $\alpha^{n-1} \nsubseteq I$. Hence $I_{\alpha^{n-1}} = P$. □

Theorem 4.6. *Let I be a P-primary hyperideal of R such that $P^n \subseteq I$ for some positive integer n, and let $\alpha \in P\backslash I$. If $\alpha^k \nsubseteq I$ for some positive integer k, then $(I :_R \alpha^k) = I_{\alpha^k}$ is an $(n-k)$-absorbing hyperideal of R.*

Proof. Since $P^n \subseteq I$, then $k \leq n$. Therefore, we have $n - k \geq 1$. It is clear that I_{α^k} is a P-primary hyperideal of R. Since $P^n \subseteq I$, we have $\alpha^k o P^{n-k} \subseteq I$. Hence $P^{n-k} \subseteq I_{\alpha^k}$. Thus I_{α^k} is an $(n-k)$-absorbing hyperideal of R by Theorem 4.1. □

5. STABILITY OF n-ABSORBING HYPERIDEALS

In this section, we will prove some theorems and corollaries generalizing well-known results about prime hyperideals.

Theorem 5.1. *Let $f : R_1 \to R_2$ be a good homomorphism of multiplicative hyperrings. Then the following statements hold.*
(i)If I_2 is a n-absorbing primary hyperideal of R_2, then $f^{-1}(I_2)$ is a n-absorbing hyperideal of R_1.
(ii)If f is an epimorphism and I_1 is an n-absorbing hyperideal of R_1 containing $Ker(f)$, then $f(I_1)$ is a n-absorbing hyperideal of R_2.

Proof. (i) Assume that $\alpha_1, \ldots, \alpha_{n+1} \in R_1$ and $\alpha_1 o \ldots o\alpha_{n+1} \subseteq f^{-1}(I_2)$. Then $f(\alpha_1 o \ldots o\alpha_{n+1}) = f(\alpha_1)o \ldots of(\alpha_{n+1}) \subseteq I_2$. Since I_2 is an n-absorbing hyperideal of R_2, then there are n of the $f(\alpha_i)$'s whose product is in I_2. Without loss of generality, we may assume that $f(\alpha_1)o \ldots of(\alpha_n) \subseteq I_2$ and hence $\alpha_1 o \ldots o\alpha_n \subseteq f^{-1}(I_2)$. Thus, $f^{-1}(I_2)$ is an n-aborbing hyperideal of R_1.
(ii) Assume that $\alpha_1', \ldots, \alpha_{n+1}' \in R_2$ and $\alpha_1' o \ldots o\alpha_{n+1}' \subseteq f(I_1)$. Then there exist $\alpha_1, \ldots, \alpha_{n+1} \in R_1$ such that $f(\alpha_1) = \alpha_1', \ldots, f(\alpha_{n+1}) = \alpha_{n+1}'$, and $f(\alpha_1 o \ldots o\alpha_{n+1}) = \alpha_1' o \ldots o\alpha_{n+1}'$. Now, take any element $u \in \alpha_1 o \ldots o\alpha_{n+1}$. Then we get $f(u) \in f(\alpha_1 o \ldots o\alpha_{n+1}) \subseteq f(I_1)$ and so $f(u) = f(w)$ for some $w \in I_1$. This implies that $f(u - w) = 0 \in (0)$, that is, $u - w \in Ker(f) \subseteq I_1$ and so $u \in I_1$. Since I_1 is a **C**-hyperideal of R_1, then we conclude that $\alpha_1 o \ldots o\alpha_{n+1} \subseteq I_1$. Since I_1 is an n-absorbing hyperideal of R_1, then there are n of the α_i's whose product is in I_1. Without loss of generality, we may assume that $\alpha_1 o \ldots o\alpha_n \subseteq I_1$. This means that $\alpha_1' o \ldots o\alpha_n' \subseteq f(I_1)$. Thus $f(I_1)$ is a n-absorbing hyperideal of R_2 □

Corollary 5.2. *Let I, J be hyperideals of a hyperring R such that $J \subseteq I$. If I is an n-absorbing hyperideal of R then $\dfrac{I}{J}$ is an n-absorbing hyperideal of $\dfrac{R}{J}$.*

Proof. Define $f : R \longrightarrow R/J$ by $f(r) = r + J$. Clearly, f is a good epimorphism. Since $Ker(f) = J \subseteq I$ and I is an n-hyperideal of R , then the claim follows from Theorem 5.1 (i). □

Corollary 5.3. *Let T is a subhyperring of R. If I is an n-absorbing hyperideal of R such that $T \nsubseteq I$, then $I \cap T$ is an n-absorbing hyperideal of T.*

Proof. Define $j : T \longrightarrow R$ by $j(t) = t$. It is clear that $j^{-1}(I) = I \cap T$. Thus $I \cap T$ an n-hyperideal of T, by 5.1 (i). □

Let R be a multiplicative hyperring. Then $M_n(R)$ denotes the set of all hypermatixes of R. Also, for all $A = (A_{ij})_{n\times n}, B = (B_{ij})_{n\times n} \in P^\star(M_n(R)), A \subseteq B$ if and only if $A_{ij} \subseteq B_{ij}$.

Theorem 5.4. *Let R be a multiplicative hyperring with scalar identity 1 and I be a hyperideal of R. If $M_n(I)$ is an n-absorbing hyperideal of $M_n(R)$, then I is an n-absorbing hyperideal of R.*

Proof. Suppose that for $x_1, \ldots, x_{n+1} \in R$, $x_1 o \ldots o x_{n+1} \subseteq I$. Then

$$\begin{pmatrix} x_1 o \ldots o x_{n+1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \subseteq M_n(I).$$

It is clear that

$$\begin{pmatrix} x_1 o \ldots o x_{n+1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} = \begin{pmatrix} x_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \cdots \begin{pmatrix} x_{n+1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}.$$

Since $M_n(I)$ is an n-absorbing hyperideal of $M_n(R)$ then there are n of the hypermatixes whose product is in I. Without loss of generality, we may assume that

$$\begin{pmatrix} x_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \cdots \begin{pmatrix} x_n & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \subseteq M_n(I).$$

It implies that

$$\begin{pmatrix} x_1 o \ldots o x_n & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \subseteq M_n(I).$$

It implies that $x_1 o \ldots o x_n \subseteq I$. Therefore I is an n-absorbing hyperideal of R.

□

Theorem 5.5. *Let R_1 and R_2 be multiplicative hyperrings with scalar identity. Then, the following statements hold:*
1) I_1 is an n-absorbing hyperideal of R_1 if and only if $I_1 \times R_2$ is an n-absorbing hyperideal of $R_1 \times R_2$.
2) I_2 is an n-absorbing hyperideal of R_2 if and only if $R_1 \times I_2$ is an n-absorbing hyperideal of $R_1 \times R_2$.

Proof. (1) ($\Longrightarrow$) Assume that I_1 is a n-absorbing hyperideal of R_1. Let $(x_1, y_1)o \ldots$
$o(x_{n+1}, y_{n+1}) \subseteq I_1 \times R_2$ for some $x_1, \ldots, x_{n+1} \in R_1$ and $y_1, \ldots, y_{n+1} \in R_2$. Therefore $x_1 o \ldots o x_{n+1} \subseteq I_1$. Since I_1 is an n-absorbing hyperideal of R_1, then there are n of the $x_i^,$s whose product is in I_1. Without loss of generality, we may assume that $x_1 o \ldots o x_n \subseteq I_1$. This implies that $(x_1, y_1)o \ldots o(x_n, y_n) \subseteq I_1 \times R_2$. Thus $I_1 \times R_2$ is an n-absorbing hyperideal of $R_1 \times R_2$.
($\Longleftarrow$) Suppose that $I_1 \times R_2$ is an n-absorbing hyperideal of $R_1 \times R_2$. Let $x_1 o \ldots o x_{n+1} \subseteq I_1$ for some $x_1, \ldots, x_{n+1} \in R_1$. Then we get $(x_1, 1)o \ldots o(x_{n+1}, 1) \subseteq I_1 \times R_2$. Since $I_1 \times R_2$ is an n-absorbing hyperideal of $R_1 \times R_2$, then there are n of the $(x_i, 1)^,$s whose product is in $I_1 \times R_2$. Without loss of generality, we may assume that $(x_1, 1)o \ldots o(x_n, 1) \subseteq I_1 \times R_2$, which means $x_1 o \ldots o x_n \subseteq I_1$. Thus I_1 is an n-absorbing hyperideal of R_1.
(2) It is similar to (1). □

Let $(R, +, o)$ be a hyperring. We define the relation γ as follows:
$a\gamma b$ if and only if $\{a, b\} \subseteq U$ where U is a finite sum of finite products of elements of R, i.e.,

a $\gamma b \Longleftrightarrow \exists z_1, \ldots, z_n \in R$ such that $\{a, b\} \subseteq \sum_{j \in J} \prod_{i \in I_j} z_i$; $\quad I_j, J \subseteq \{1, \ldots, n\}$.

We denote the transitive closure of γ by γ^*. The relation γ^* is the smallest equivalence relation on a multiplicative hyperring $(R, +, o)$

such that the quotient R/γ^*, the set of all equivalence classes, is a fundamental ring. Let $\mathfrak{U}$ be the set of all finite sums of products of elements of R. We can rewrite the definition of γ^* on R as follows:

a$\gamma^* b \Longleftrightarrow \exists z_1, \ldots, z_n \in R$ with $z_1 = a, z_{n+1} = b$ and $u_1, \ldots, u_n \in \mathfrak{U}$ such that $\{z_i, z_{i+1}\} \subseteq u_i$ for $i \in \{1, \ldots, n\}$.

Suppose that $\gamma^*(a)$ is the equivalence class containing $a \in R$. Then, both the sum $\oplus$ and the product $\odot$ in R/γ^* are defined as follows:$\gamma^*(a) \oplus \gamma^*(b) = \gamma^*(c)$ for all $c \in \gamma^*(a) + \gamma^*(b)$ and $\gamma^*(a) \odot \gamma^*(b) = \gamma^*(d)$ for all $d \in \gamma^*(a) o \gamma^*(b)$. Then R/γ^* is a ring, which is called a fundamental ring of R (see also [19]).

Theorem 5.6. *Let R be a multiplicative hyperring with scalar identity 1. Then the hyperideal I of R is an n-absorbing if and only if I/γ^* be an n-absorbing ideal of R/γ^*.*

Proof. ($\Longrightarrow$) Let $\alpha_1^{'} \ldots, \alpha_{n+1}^{'} \in R/\gamma^*$ and $\alpha_1^{'} \odot \cdots \odot \alpha_{n+1}^{'} \in I/\gamma^*$. Thus, there exist $\alpha_1, \ldots, \alpha_{n+1} \in R$ such that $\alpha_1^{'} = \gamma^*(\alpha_1), \ldots, \alpha_{n+1}^{'} = \gamma^*(\alpha_{n+1})$ and $\alpha_1^{'} \odot \cdots \odot \alpha_{n+1}^{'} = \gamma^*(\alpha_1) \odot \cdots \odot \gamma^*(\alpha_{n+1}) = \gamma^*(\alpha_1 o \ldots o \alpha_{n+1})$. So, $\gamma^*(\alpha_1) \odot \cdots \odot \gamma^*(\alpha_{n+1}) = \gamma^*(\alpha_1 o ... o \alpha_{n+1}) \in I/\gamma^*$, then $\alpha_1 o \ldots o \alpha_{n+1} \subseteq I$. Since I is an n-absorbing hyperideal of R, then there are n of the $\alpha_i^{,}$s whose product is in I. Without losing the generality, we may assume that $\alpha_1 o \ldots o \alpha_n \subseteq I$. Therefore $\alpha_1^{'} \odot \cdots \odot \alpha_n^{'} = \gamma^*(\alpha_1) \odot \cdots \odot \gamma^*(\alpha_n^{'}) = \gamma^*(\alpha_1 o \ldots o \alpha_n) \in I/\gamma^*$. Thus I/γ^* is an n-absorbing ideal of R/γ^*.
($\Longleftarrow$) Let $\alpha_1 o \ldots o \alpha_{n+1} \subseteq I$ for $\alpha_1, \ldots, \alpha_{n+1} \in R$. Then we obtain $\gamma^*(\alpha_1), \ldots, \gamma^*(\alpha_{n+1}) \in R/\gamma^*$ and

$$\gamma^*(\alpha_1) \odot \cdots \odot \gamma^*(\alpha_{n+1}) = \gamma^*(\alpha_1 o \ldots o \alpha_{n+1}) \in I/\gamma^*.$$

Since I/γ^* is an n-absorbing ideal of R/γ^*, then there are n of the $\gamma^*(\alpha_i)^{,}$s whose product is in I/γ^*. Without loss of generality, we may assume that $\gamma^*(\alpha_1) \odot \cdots \odot \gamma^*(\alpha_n) = \gamma^*(\alpha_1 o \ldots o \alpha_n) \in I/\gamma^*$. Hence $\alpha_1 o \ldots o \alpha_n \subseteq I$. Thus I is an n-absorbing hyperideal of R. □

Let $(R, +, o)$ be a commutative multiplicative hyperring with scalar identity 1 and S be a multiplicative closed subset of R (i.e., $1 \in S$ and $aoS = Soa = S$ for all $a \in S$). Then $(S^{-1}R, \oplus, \odot)$ with the following hyperoperations is a commutative hyperring with scalar identity.
(i) $(r_1, r_2) \oplus (r_2, s_2) = (r_1 o s_2 + r_2 o s_1, s_1 o s_2) = \{(r, s) \mid r \in r_1 o s_2 + r_2 o s_1, s \in s_1 o s_2\}$.
(ii) $(r_1, r_2) \odot (r_2, s_2) = (r_1 o r_2, s_1 o s_2) = \{(r, s) \mid r \in r_1 o r_2, s \in s_1 o s_2\}$.

Let I be a hyperideal of R, then we can define that $S^{-1}I = \{(i,s) \mid i \in I, s \in S\}$, which is a hyperideal of $S^{-1}R$. If $(r,s) \in S^{-1}I$ we don't have necessarily $r \in I$, because maybe $(r,s) = (r',s)$ with $r' \in I, r \notin I$ (see also [1]).

Theorem 5.7. *Let $P_{1,k}$ be incomparable prime hyperideals of R, $I = P_1^{n_1} o \ldots o P_k^{n_k}$ for positive integers $n_1, \ldots, n_k$ with $n = n_1 + \cdots + n_k$, and $S = R \setminus (P_1 \cup \cdots \cup P_k)$. Then $E(I) = \{\alpha \in R \mid (\alpha, 1) \in S^{-1}I\}$ is an n-absorbing hyperideal of R.*

Proof. Let $f : R \to S^{-1}R$ be the homomorphism $f(\alpha) = (\alpha, 1)$. Then $S^{-1}P_1, \ldots, S^{-1}P_k$ are maximal hyperideals of $S^{-1}R$. Hence $S^{-1}I = S^{-1}(P_1^{n_1} o \ldots o P_k^{n_k})$ is an n-absorbing hyperideal of $S^{-1}R$, by Theorem 3.16. Thus $E(I) = f^{-1}(S^{-1}(P_1^{n_1} o \ldots o P_k^{n_k}))$ is an n-absorbing hyperideal of R, by Theorem 5.1. □

6. Conclusion

The main purpose of this paper is to introduce the notion of n-absorbing hyperideal which is a generalization of 2-absorbing hyperideal. Several properties of this new notion are provided. It is clear that if I is an n-absorbing hyperideal of R, then I is an m-absorbing hyperideal of R for all $m \geq n$. The converse is not true in general. For instance, $105\mathbb{Z}$ is a 3-absorbing hyperideal of multiplicative hyperring $(\mathbb{Z}, +, \odot)$, but it is not a 2-absorbing hyperideal.

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A GENERALIZATION OF PRIME HYPERIDEALS

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تعمیمی از ابرایده‌آل‌های اول

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فرض کنید R یک ابرحلقه ضربی باشد. در این مقاله به معرفی و مطالعه ابرایده‌آل‌های n-جاذب به عنوان تعمیمی از ابرایده‌آل‌های اول پرداخته می‌شود. ابرایده‌آل I از ابرحلقه R را n-جاذب گوییم هرگاه برای $\alpha_1, \ldots, \alpha_{n+1} \in R$ اگر $\alpha_1 o \ldots o \alpha_{n+1} \subseteq I$، آن‌گاه حاصل‌ضرب n تا از α_i ها زیرمجموعه I باشد.

کلمات کلیدی: ابرایده‌آل اول، ابرایده‌آل n-جاذب، ابرایده‌آل ابتدائی، ابرحلقه.