Journal of Algebraic Systems
Vol. 11, No. 1, (2023), pp 137-147

LOCATION OF SOLID BURST WITHIN TWO ADJACENT SUB-BLOCKS

P. KUMAR DAS

ABSTRACT. The paper studies the existence of linear codes that locate solid burst errors, which may be confined to one sub-block or spread over two adjacent sub-blocks. An example of such a code is also given. Comparisons on the number of parity check digits required for such linear codes with solid burst detecting and correcting codes are also provided.

1. Introduction

In coding theory, once it is known that a particular type of error occurs in a communication channel, codes are constructed taking care of the specific type of error only rather than general type of error by default. This will save the time and improve the efficiency of the system. It is found that in certain memory systems (e.g. some spacecraft memories and supercomputer storage systems [2, 7]), the most common error is solid burst error, i.e., an error in consecutive bits that are stored physically adjacent in the memory. A solid burst may be defined as follows:

Definition 1.1. A solid burst of length b is a vector with nonzero entries in some b consecutive positions and zero elsewhere.

Detection and correction of solid burst was studied in [3]. In [4], location of solid burst occurring within a single sub-block was

DOI: 10.22044/JAS.2022.11136.1552.
MSC(2010): 94B05, 94B65.
Keywords: Parity check matrix; Solid burst; Error pattern-syndrome; EL-code.
Received: 24 August 2021, Accepted: 6 September 2022.

studied. The concept of location of errors was introduced by Wolf and Elspas [10] where code is divided into smaller sub-blocks and identification of corrupted sub-block is possible. Such codes are referred to as Error-Locating codes (EL-codes). In [4], Das considered the situation when solid burst is confined to one sub-block only. As solid burst may start anywhere, it may not be confined to one sub-block only, but may extend to the next adjacent sub-block. In the case of recorded data on a continuous surface, defects or dust particles produce solid bursts, which may affect two adjacent sub-blocks. This motivates us to work on solid bursts affecting two adjacent sub-blocks. Similar work in this direction for burst error can be found in [5]. However, in semiconductor memory, the data are stored in RAM chips, and a data fragment (called a byte) is stored in each chip which are separated and independent. In such memory, the presence of error or fault in a chip does not extend to the adjacent chips [6].

Consider a linear code whose length $n = mt$ is subdivided into m mutually exclusive sub-blocks of length t. Let

$\boldsymbol{ASB_b - EL}$ **code :** *Linear code locating solid bursts of length up to* b $(\leq t)$, *which may spread over two adjacent sub-blocks.*
$\boldsymbol{E_i}$ **:** *Set of all solid bursts which are confined within the* i^{th} *sub-block.*
$\boldsymbol{E_{j,j+1}}$ **:** *Set of all solid bursts which are spread over adjacent* j *and* $j+1$ *sub-blocks.*
$\boldsymbol{H}$ **:** *Parity check matrix of an* $ASB_b - EL$ *code.*

For an $(n = mt, n-r)$ ASB_b-EL code, the following three conditions need to be satisfied.

(1) $eH^T \neq 0$ for all $e \in E_i \cup E_{j,j+1}$ for any i, j.
(2) $e_1H^T \neq e_2H^T$ for any $e_1 \in E_i$ $(E_{i,i+1})$, $e_2 \in E_j$ $(E_{j,j+1})$; $i \neq j$.
(3) $e_1H^T \neq e_2H^T$ for any $e_1 \in E_i$, $e_2 \in E_{j,j+1}$.

Note that for $b = 1$, solid burst can not spread over the adjacent sub-block, so the $ASB_b - EL$ codes coincides with usual solid burst locating codes [3]. Therefore, we consider $b \geq 2$ for the $ASB_b - EL$ codes.

The paper is organized in the following way. Section 1 is the introduction, which gives the basic definition, background and importance of the study. In Section 2, we give the necessary and sufficient conditions for the existence of $ASB_b - EL$ code along with an example.

In Section 3, we make comparisons among the number of parity check digits of such EL-codes with solid burst detecting and correcting codes.

2. Conditions for $ASB_b - EL$ codes

In this section, we derive necessary and sufficient conditions required for the existence of an $ASB_b - EL$ code. Firstly, we give the necessary condition. The proof is based on the technique used in Theorem 4.13, Peterson and Weldon [8].

Theorem 2.1. *For an $(n = mt, n - r)$ $ASB_b - EL$ code over $GF(q)$, the number of check digits r required is at least*

$$\begin{cases} loq_q\big[m(b+1)\big] & \text{for } q = 2 \\ loq_q\Big[1 + m(q-1)\sum_{i=0}^{b-1}\Big(\lfloor\frac{q-1}{2}\rfloor\Big)^i + (m-1)\sum_{j=2}^{b}\Big(\lfloor\frac{q-1}{2}\rfloor\Big)^j\Big] & \text{for } q \neq 2, \end{cases}$$

where $\lfloor y \rfloor$ means the greatest integer less than or equal to y.

Proof. We proceed for the proof by counting the total number of syndromes according to Condition $(1) - (3)$ and comparing with the maximum number of possible syndromes q^r.

For binary case:

Let X be the set of all vectors whose all the first i $(i \leq b)$ positions of one sub-block are the nonzero element of the field and the rest are zero. From Condition (1), we deduce that the elements of X should be in different cosets due to Condition (1). Again from Condition (2), the syndromes produced by solid bursts lying in different single sub-block must be distinct. As the number of elements of X is b and there are m sub-blocks, so the number of distinct syndromes, excluding the all zero syndrome, is mb.

Again let Y be the set consisting of the vector such that the last component of the first sub-block and the first component of the second sub-block of any two adjacent sub-blocks are the nonzero element of the field and the rest are zero. Then applying Condition (1)-(2), the syndrome produced by the element of Y should be nonzero and syndromes produced by solid bursts lying in different adjacent sub-blocks must be distinct. Since there is one element in Y and $m-1$ two adjacent sub-blocks, the number of distinct syndromes resulting from two adjacent sub-blocks is $m - 1$.

Further, from Condition (3), syndromes produced by the vectors of X and Y should be distinct. Therefore, the total number of distinct syndromes of solid bursts (whether confined to one sub-block or spread over two adjacent sub-blocks) for the $ASB_b - EL$ code, excluding the zero syndrome, is

$$mb + m - 1.$$

Hence, we must have

$$q^r \geq 1 + mb + m - 1 = m(b+1). \tag{2.1}$$

For non-binary case:

Let us consider X to be a set of all those vectors such that the first position of the initial b consecutive components of any one sub-block is any nonzero field element and its immediately following i consecutive components $(0 \leq i \leq b-1)$ are any nonzero element belonging to $\{1, 2, \ldots, \lfloor \frac{q-1}{2} \rfloor\}$ and zero elsewhere.

In this case by Condition (1)-(2), the elements of X would be in different cosets and the nonzero distinct syndromes produced by the elements of X is $(q-1)\sum_{i=0}^{b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i$ corresponding to vectors in any single sub-block. As there are m sub-blocks in all, the number of distinct syndromes, excluding the zero syndrome, is at least

$$m(q-1)\sum_{i=0}^{b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i.$$

Again let Y be the set of those vectors such that the last position of the first sub-block and the first i consecutive components of the second sub-block of any two adjacent sub-blocks $(1 \leq i \leq b-1)$ are any nonzero element belonging to $\{1, 2, \ldots, \lfloor \frac{q-1}{2} \rfloor\}$ and zero elsewhere. Like previous cases applying Condition (1)-(2), the number of distinct syndromes resulting from the elements of Y, excluding the zeros syndrome, is given by

$$(m-1)\sum_{j=2}^{b}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^j.$$

Then by Condition (3), the total number of distinct syndromes of solid bursts (whether confined to single sub-block or spread over two adjacent sub-blocks) for the $ASB_b - EL$ code is

$$m(q-1)\sum_{i=0}^{b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i + (m-1)\sum_{j=2}^{b}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^j.$$

Therefore, we must have

$$q^r \geq 1 + m(q-1)\sum_{i=0}^{b-1}\left(\left\lfloor\frac{q-1}{2}\right\rfloor\right)^i + (m-1)\sum_{j=2}^{b}\left(\left\lfloor\frac{q-1}{2}\right\rfloor\right)^j. \quad (2.2)$$

This completes the theorem. □

Remark 2.2. It is worth noticing that the result obtained in Theorem 2.1 is free from t, the length of the sub-block. Thus the bound obtained in Theorem 2.1 remains valid for all t, so long as $b \leq t$ and $n = mt$.

In the following result, a sufficient condition required for the existence of an $ASB_b - EL$ code is derived. The proof is based on the technique used to establish Varshamov-Gilbert-Sacks bound by constructing a parity check matrix for such a code (refer Sacks [9], also Theorem 4.17, Peterson and Weldon [8]). This technique also gives a method for construction of the code.

Theorem 2.3. *For the existence of an $(n = mt, n - r)$ $ASB_b - EL$ code over $GF(q)$, the number of check digits r required for the code is given by*

$$q^r > \sum_{i=0}^{b-1}(q-1)^i\left\{1 + (m-1)t\sum_{j=1}^{b}(q-1)^j\right\}.$$

Proof. We shall prove the result by constructing an appropriate $(n-k) \times n$ parity check matrix H for the desired code. Suppose that the columns of the first $m-1$ sub-blocks of H and the first $t-1$ columns $h_{(m-1)t+1}, h_{(m-1)t+2}, \ldots, h_{n-1}$ of the m^{th} sub-block have been appropriately added. Now we lay down the conditions to add the column h_n of the m^{th} sub-block of the matrix H, satisfying Condition (1)–(3):

According to Condition (1), the n^{th} the column h_n should not be a linear sum of immediately preceding $l-1$ columns, where $l \leq b$. The number of such columns that h_n should not be equal to, including the vector of all zeros, is (refer Das [3])

$$\sum_{i=0}^{b-1}(q-1)^i. \quad (2.3)$$

According to Condition (2), the syndrome resulting from any solid burst of length up to b confined to a single sub-block or two adjacent sub-blocks must be distinct from the syndrome resulting likewise from a solid burst of length up to b within any *other* single sub-block. In this case, the column h_n can not be equal to, is (refer Das [4])

$$\sum_{i=0}^{b-1}(q-1)^i \times (m-1)\sum_{j=1}^{b}(t-j+1)(q-1)^j. \tag{2.4}$$

Further, according to Condition (3), the syndrome resulting from the occurrence of any solid burst of length up to b confined to a single sub-block or two adjacent sub-blocks must be distinct from the syndrome resulting likewise from solid burst of length up to b confined to any *other* two adjacent sub-blocks. In this case, h_n should not be a linear sum of immediately preceding $l-1$ columns ($l \leq b$), together with any linear sums of b or less consecutive columns in any two previous adjacent sub-blocks. Then, h_n can be added provided that

$$\begin{aligned} h_n \neq\ & (u_1h_{n-1} + u_2h_{n-2} + \cdots + u_{l-2}h_{n-l+2} + u_{l-1}h_{n-l+1}) \\ & +(v_ih_i + v_{i+1}h_{i+1} + \cdots + v_{i+p-1}h_{i+p-1}) \end{aligned} \tag{2.5}$$

where $l, p \leq b$, $u_i \in GF(q) \setminus \{0\}$ and $v_i \in GF(q)$ are such that the h_i's are any b or less consecutive columns spread over any two previous adjacent sub-blocks.

The number of ways in which the coefficients u_i's of (2.5) can be selected is given by (2.3) and the number of coefficients v_i's in two adjacent sub-blocks, excluding the vector of all zeros, is given by

$$\sum_{i=2}^{b}(q-1)^i + \sum_{i=3}^{b}(q-1)^i + \cdots + \sum_{i=b}^{b}(q-1)^i = \sum_{j=2}^{b}(j-1)(q-1)^j. \tag{2.6}$$

Since there are $m-1$ adjacent sub-blocks, therefore number of v_i's is

$$(m-1)\sum_{j=2}^{b}(j-1)(q-1)^j. \tag{2.7}$$

So, the number of linear combinations on R.H.S. of (2.5) is

$$\sum_{i=0}^{b-1}(q-1)^i(m-1)\sum_{j=2}^{b}(j-1)(q-1)^j. \tag{2.8}$$

Thus for detection and location of solid burst of length up to b, the number of nonzero columns that h_n can not be equal to is

$$Expr.(2.3) + Expr.(2.4) + Expr.(2.8)$$

i.e.

$$\sum_{i=0}^{b-1}(q-1)^i + \sum_{i=0}^{b-1}(q-1)^i(m-1)\sum_{j=1}^{b}(t-j+1)(q-1)^j$$
$$+\sum_{i=0}^{b-1}(q-1)^i(m-1)\sum_{j=2}^{b}(j-1)(q-1)^j$$
$$=\sum_{i=0}^{b-1}(q-1)^i\left\{1+(m-1)\left[\sum_{j=1}^{b}(t-j+1)(q-1)^j+\sum_{j=2}^{b}(j-1)(q-1)^j\right]\right\}$$
$$=\sum_{i=0}^{b-1}(q-1)^i\left\{1+(m-1)t\sum_{j=1}^{b}(q-1)^j\right\}.$$

Thus, for the required code, h_n can not be this many combinations. Therefore, h_n can be added to the m^{th} sub-block of H provided that

$$q^r > \sum_{i=0}^{b-1}(q-1)^i\left\{1+(m-1)t\sum_{j=1}^{b}(q-1)^j\right\}.$$

This completes the theorem. □

Example 2.4. Consider a (16, 10) binary code with the 6×16 matrix H by taking $m=4, t=4, b=2, q=2$ in Theorem 2.3.

$$H=\begin{bmatrix} 1&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&1&0&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&1&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&1&0&1&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&1 \end{bmatrix}$$

The above code can locate any solid burst of length up to 2. It can be verified from Error Pattern-Syndrome Table 1 that all the syndromes of solid bursts of length up to 2, whether confined to one sub-block or two adjacent sub-blocks are nonzero and distinct.

Table 1: **Error Pattern-Syndrome**

Error Patterns	Syndromes	Error Patterns	Syndromes
1^{st} sub-block		3^{rd} sub-block	
1000 0000 0000 0000	100000	0000 0000 1000 0000	000010
0100 0000 0000 0000	010000	0000 0000 0100 0000	000001
0010 0000 0000 0000	100000	0000 0000 0010 0000	000010
0001 0000 0000 0000	010000	0000 0000 0001 0000	000001
1100 0000 0000 0000	110000	0000 0000 1100 0000	000011
0110 0000 0000 0000	110000	0000 0000 0110 0000	000011

Contd...

Table 1 – **Error Pattern-Syndrome**

Error Patterns	Syndromes	Error Patterns	Syndromes
0011 0000 0000 0000	110000	0000 0000 0011 0000	000011
2nd sub-block		4rd sub-block	
0000 1000 0000 0000	001000	0000 0000 0000 1000	101000
0000 0100 0000 0000	000100	0000 0000 0000 0100	010100
0000 0010 0000 0000	001000	0000 0000 0000 0010	001010
0000 0001 0000 0000	000100	0000 0000 0000 0001	000101
0000 1100 0000 0000	001100	0000 0000 0000 1100	111100
0000 0110 0000 0000	001100	0000 0000 0000 0110	011110
0000 0011 0000 0000	001100	0000 0000 0000 0011	001111
1st and 2nd sub-blocks		2st and 3rd sub-blocks	
0001 1000 0000 0000	011000	0000 0001 1000 0000	000110
3st and 4nd sub-blocks			
0000 0000 0001 1000	101001		

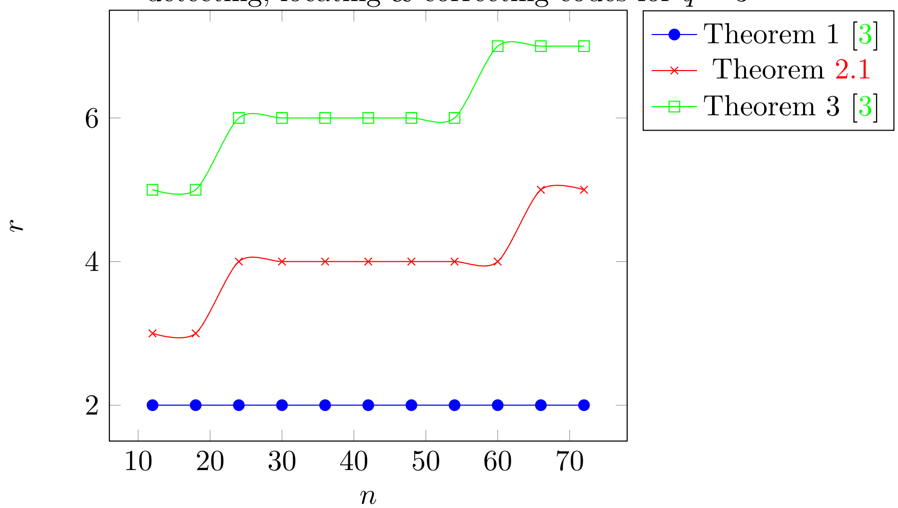
3. Comparisons of information rates

In this section, we make comparisons between the necessary and sufficient number of check digits required for $ASB_b - EL$ codes with the solid burst detecting and correcting codes [3]. First, we give a comparison among the necessary number of check digits required for a code discussed in Theorem 1 [3], Theorem 2.1 and Theorem 3 [3].

Table 1: Comparison of necessary number of check digits for solid burst detecting, locating & correcting codes for $q = 3$

m	t	b	n	r Theorem 1 [3]	r Theorem 2.1	r Theorem 3 [3]
2	6	3	12	2	3	5
3	6	3	18	2	3	5
4	6	3	24	2	4	6
5	6	3	30	2	4	6
6	6	3	36	2	4	6
7	6	3	42	2	4	6
8	6	3	48	2	4	6
9	6	3	54	2	4	7
10	6	3	60	2	4	7
11	6	3	66	2	5	7
12	6	3	72	2	5	7

Figure 1: Comparison of necessary number of check digits for solid burst detecting, locating & correcting codes for $q = 3$



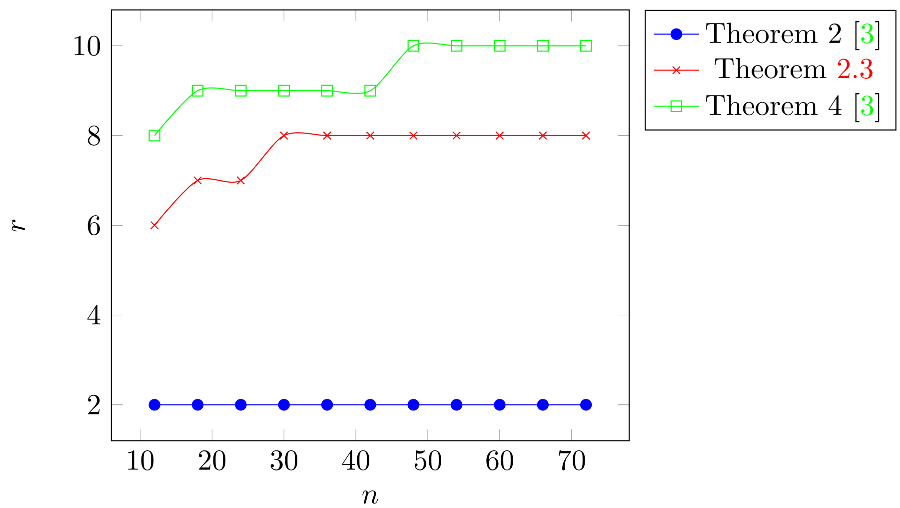
It is evident from Table 1 and Figure 1 that the necessary number of check digits required for an $ASB_b - EL$ code lie between the necessary number of check digits required for solid burst detecting and correcting linear codes.

Now, we give the comparison on the sufficient number of check digits required for a code discussed in Theorem 2 [3], Theorem 2.3 and Theorem 4 [3].

Table 2: Comparison of sufficient number of check digits for solid burst detecting, locating & correcting codes for $q = 3$

m	t	b	n	r Theorem 2 [3]	r Theorem 2.3	r Theorem 4 [3]
2	6	3	12	2	6	8
3	6	3	18	2	7	9
4	6	3	24	2	7	9
5	6	3	30	2	8	9
6	6	3	36	2	8	9
7	6	3	42	2	8	9
8	6	3	48	2	8	10
9	6	3	54	2	8	10
10	6	3	60	2	8	10
11	6	3	66	2	8	10
12	6	3	72	2	8	10

Figure 2: Comparison of sufficient number of check digits for solid burst detecting, locating & correcting codes for $q = 3$



From Table 2 and Figure 2, we find that the sufficient number of check digits required for an $ASB_b - EL$ code also lie between those of solid burst detecting and correcting linear codes.

Conclusion

The paper obtains necessary and sufficient conditions for an $ASB_b - EL$ code. It justifies that error location concept is a midway concept between error detection and correction. As the information rate of an $ASB_b - EL$ code is less than a solid burst correcting linear code, the $ASB_b - EL$ codes will be more efficient if the location of error is sufficient. This study may be extended to other types of errors as well.

References

1. N. M. Abramson, A class of systematic codes for non-independent errors, *IRE Trans. Inf. Theory*, **IT-5**(4) (1959), 150–157.
2. J. Arlat, W. C. Carter, Implementation and evaluation of a (b, k)-adjacent error-correcting/detecting scheme for supercomputer systems, *IBM J. Res. Develop.*, **28**(2) (1984), 159–169.
3. P. K. Das, Codes detecting and correcting solid burst errors, *Bull. Electr. Eng. Inform.*, **1**(3) (2012), 225–232.
4. P. K. Das, Codes detecting and locating solid burst errors, *Rom. J. Math. Comput. Sci.*, **2**(2) (2012), 1–10.
5. P. K. Das and S. Kumar, Location of burst and repeated burst error in single and adjacent sub-blocks, *TWMS J. of Apl. & Eng. Math.* **12**(3) (2022), 996–1003.

6. E. Fujiwara, *Code Design for Dependable Systems: Theory and Practical Applications*, John Wiley & Sons, Inc. 2006.
7. D. W. Jensen, *Block code to efficiently correct adjacent data and /or check bit errors*, Patent number: US 6604222 B1, Date of Patent Aug. 5, 2013.
8. W. W. Peterson and E. J. Weldon(Jr.), *Error-Correcting Codes*, 2nd edition, The MIT Press, 1972.
9. G. E. Sacks, Multiple error correction by means of parity-checks, *IRE Trans. Inform. Theory*, **IT-4** (1958), 145–147.
10. J. Wolf and B. Elspas, Error-locating codes a new concept in error control, *IEEE Trans. Inf. Theory*, **9**(2) (1963), 113–117.

Pankaj Kumar Das
Department of Mathematical Sciences, Tezpur University, Napaam, P.O. Box 784028, Sonitpur, Assam, India.
Email: `pankaj4thapril@yahoo.co.in, pankaj4@tezu.ernet.in`

Journal of Algebraic Systems

LOCATION OF SOLID BURST WITHIN TWO ADJACENT SUB-BLOCKS

P. KUMAR DAS

مکان خطای کپه‌ای در دو زیر بلاک متصل

پانکاج کومار داس

گروه علوم ریاضی، دانشگاه تزپور، نپام، آسام، هند

در این مقاله وجود کدهای خطی که مکان خطای کپه‌ای را مشخص می‌کنند، مطالعه می‌شود. این مکان به یک زیر بلوک یا دو بلوک مجاور محدود شده است. نمونه‌ای از چنین کدی نیز آورده شده است. مقایسه تعداد ارقام آزمون توازن مورد نیاز برای چنین کدهای خطی با کدهای تشخیص و تصحیح خطای کپه‌ای نیز ارائه شده است.

کلمات کلیدی: ماتریس آزمون توازن، خطای کپه‌ای، الگوی خطا، EL-کد.