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# SHEFFER STROKE BE-ALGEBRAS BASED ON THE SOFT SET ENVIRONMENT

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Abstract. In this paper, we introduce the concept of soft Sheffer stroke BE-algebras, offering a novel perspective on their algebraic properties within the framework of soft set theory. These algebras provide a flexible and adaptable approach to logical operations, allowing the seamless integration of fuzzy and crisp information. Moreover, we delve into the concept of soft Sheffer stroke sub-BE-algebras and thoroughly examine the properties of these structures. Through our analysis, we uncover interesting connections between soft sets, Sheffer stroke operations, and the foundational BE-algebraic structure. The outcomes presented in this paper enhance our understanding of algebraic structures that revolve around Sheffer stroke operations in the realm of soft sets. This research a Mathematical foundation that holds promise for applications in decision making, information fusion, and uncertain reasoning.

## 1. Introduction

When a structure is established as a mathematical model, we must throw off redundant statements at first. In accordance with this purpose, we aim to provide equivalent statements with minimal axioms or operations and so on. Tarski achieved explain Abelian groups by providing the least number of axioms necessary, focusing on the divisor operator [28]. Sheffer demonstrated that all Boolean functions could be expressed using only the Sheffer stroke operation [27]. Building upon these ideas, McCune et al. axiomatized Boolean algebras exclusively using the Sheffer stroke operation in 2002 [21].

The practical applications of the Sheffer stroke operation extend to computer systems, particularly in chip technology. By employing the Sheffer stroke operation, it becomes feasible to utilize a uniform set of diodes on a chip, simplifying the design and reducing costs compared to employing distinct diodes for other logical connectives like conjunction, disjunction and negation [1].

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The significance of logical algebraic structures transcends classical logic and finds relevance in various disciplines, including information sciences, artificial intelligence, quantum logics, computer sciences, and probability theory. Boolean algebras serve as the foundation for classical logic, while MV-algebras, inspired by [3] and corresponding to Łukasiewicz logics [18], are utilized for non-classical logic. Chajda et al. introduced basic algebras as a unifying generalization of orthomodular lattices and MV-algebras [2]. Moreover, Oner and Senturk proposed Sheffer stroke basic algebras as a means of reducing basic algebras to a single operation [25]. These advancements lay the groundwork for effective reasoning methods and establish a fundamental algebraic structure applicable to numerous fields.

In order to address challenges in fields such as economics, engineering, and the environment, classical methods prove inadequate due to the inherent uncertainties associated with these problems. To handle uncertainties effectively, three major mathematical theories have become useful tools: probability theory, fuzzy set theory, and interval mathematics. However, each of these theories comes with its own difficulties and limitations.

Traditional Mathematical tools face a big challenge when dealing with uncertainties. But there are many theories available to tackle this problem, such as probability theory, fuzzy set theory (including intuitionistic fuzzy set theory and vague set theory), interval mathematics, and rough set theory. It is worth noting that each of these theories faces its own set of challenges, as highlighted in [22]. In the realm of handling the difficulties associated with uncertainties, Molodtsov [22] and Maji et al. [20] have proposed that one of the contributing factors is the limitation of parametrization tools in existing theories. In order to overcome these challenges, Molodtsov [22] introduced the concept of soft sets as a novel mathematical tool that effectively handles uncertainties without encountering the obstacles encountered in traditional approaches. Molodtsov also outlined various potential applications of soft sets. Then, there is rapid progress in the field of soft set theory. Maji et al. [20] demonstrated the application of soft set theory to a decision-making problem, while Maji et al. [19] investigated several operations within the framework of soft sets. Chen et al. [4] introduced a new definition of soft set parametrization reduction and compared it to the concept of attribute reduction in rough set theory. Feng and Li examined soft subsets and soft product operations [6]. Also, Jun investigated soft set concept on BCK/BCI-algebras [8].

Furthermore, researchers have explored the algebraic structure of set theories designed for managing uncertainties. The theory of fuzzy sets, pioneered by Zadeh [29], is considered the most suitable framework for handling uncertainties. The authors, along with colleagues, have applied fuzzy set theory to various algebraic structures, including BCK-algebras [15, 14], BC-algebras [5], B-algebras [12], hyper BCK-algebras [13], MTL-algebras [16], hemirings [11], implicative algebras [10], lattice implication algebras [7], incline algebras [9], class of BCK-algebras [23] and states in Sheffer stroke basic algebras [26]. These investigations demonstrate the versatility and broad applicability of fuzzy set theory in modeling uncertainties across diverse domains.

In this paper, we introduce a novel construction of soft Sheffer stroke BE-algebras and soft Sheffer stroke sub-BE-algebras, presenting their key properties and significant results. We also explore the connections between fuzzy structures and soft sets, showcasing their potential applications in diverse fields, including artificial intelligence, computer science, information sciences, quantum logics, probability theory, and beyond. The findings of our research provide a solid foundation for researchers in various scientific disciplines who seek to employ the notions of soft sets and fuzzy structures in their work. In Section 2, we provide a comprehensive review of essential notions, fundamental definitions, lemmas, and their corresponding results pertaining to Sheffer stroke BE-algebras. Furthermore, we revisit the definition of soft sets, a crucial concept in the context of this study. Additionally, we present a concise overview of fundamental fuzzy concepts that form the basis of our investigation. In Section 3, we introduce the definition of a soft Sheffer stroke BE-algebra. We establish a correspondence between fuzzy Sheffer stroke BE-algebras and soft Sheffer stroke BE-algebras, identifying the conditions under which this correspondence holds. Additionally, we introduce the concept of restriction within this algebraic structure. We provide a proof demonstrating that the intersection or union of two soft Sheffer stroke BE-algebras remains a soft Sheffer stroke BE-algebra. Furthermore, we illustrate that a soft Sheffer stroke BE-algebra maintains its properties under homomorphism. We also discuss the concept of nullification within soft Sheffer stroke BE-algebras and explore its relationship under homomorphism. In Section 4, we introduce the concept of a soft Sheffer stroke sub-BE-algebra. We establish that the intersection or union of two soft Sheffer stroke BE-algebras forms a soft Sheffer stroke sub-BE-algebra. Moreover, we demonstrate that a soft Sheffer stroke sub-BE-algebra retains its properties under homomorphism, thus remaining a soft Sheffer stroke sub-BE-algebra.

## 2. Preliminaries

In this section, we begin with some definitions, lemmas and theorem that will be needed throughout this paper.

**Definition 2.1** ([27]). Let $\mathcal{A} := (A, |)$ be a groupoid. Then the operation "$|$" is said to be *Sheffer stroke* or *Sheffer operation* if it satisfies:
(s1) $(\forall \mathfrak{a}, \mathfrak{b} \in A)$ $(\mathfrak{a}|\mathfrak{b} = \mathfrak{b}|\mathfrak{a})$,
(s2) $(\forall \mathfrak{a}, \mathfrak{b} \in A)$ $((\mathfrak{a}|\mathfrak{a})|(\mathfrak{a}|\mathfrak{b}) = \mathfrak{a})$,
(s3) $(\forall \mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in A)$ $(\mathfrak{a}|((\mathfrak{b}|\mathfrak{c})|(\mathfrak{b}|\mathfrak{c})) = ((\mathfrak{a}|\mathfrak{b})|(\mathfrak{a}|\mathfrak{b}))|\mathfrak{c})$,
(s4) $(\forall \mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in A)$ $((\mathfrak{a}|((\mathfrak{a}|\mathfrak{a})|(\mathfrak{b}|\mathfrak{b})))|(\mathfrak{a}|((\mathfrak{a}|\mathfrak{a})|(\mathfrak{b}|\mathfrak{b}))) = \mathfrak{a})$.

**Definition 2.2** ([17]). By a *Sheffer stroke BE-algebra* we mean a structure $(X, |, 0)$ with the constant 1 and the Sheffer stroke "$\uparrow$" that satisfies:
(sBE1) $\mathfrak{a} \uparrow (\mathfrak{a} \uparrow \mathfrak{a}) = 1$,
(sBE2) $\mathfrak{a} \uparrow ((\mathfrak{b} \uparrow (\mathfrak{c} \uparrow \mathfrak{c})) \uparrow (\mathfrak{b} \uparrow (\mathfrak{c} \uparrow \mathfrak{c}))) = \mathfrak{b} \uparrow ((\mathfrak{a} \uparrow (\mathfrak{c} \uparrow \mathfrak{c})) \uparrow (\mathfrak{a} \uparrow (\mathfrak{c} \uparrow \mathfrak{c})))$
for all $\mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in X$.

Let $(X, |, 0)$ be a Sheffer stroke BE-algebra. Define a relation "$\leq_X$" on $X$ by

$$(\forall \mathfrak{a}, \mathfrak{b} \in X)(\mathfrak{a} \leq_X \mathfrak{b} \iff \mathfrak{a} \uparrow (\mathfrak{b} \uparrow \mathfrak{b}) = 1). \tag{2.1}$$

The relation "$\leq_X$" is not a partial order on $X$. It is only a reflexive relation on $X$ (see [17]).

**Proposition 2.3** ([17]). *Every Sheffer stroke BE-algebra* $(X, |, 0)$ *satisfies:*

$$(\forall \mathfrak{a} \in X)(\mathfrak{a} \uparrow (1 \uparrow 1) = 1), \tag{2.2}$$

$$(\forall \mathfrak{a} \in X)(1 \uparrow (\mathfrak{a} \uparrow \mathfrak{a}) = \mathfrak{a}), \tag{2.3}$$

**Definition 2.4** ([17]). A Sheffer stroke BE-algebra $(X, |, 0)$ is said to be *self-distributive* if it satisfies:

$$\mathfrak{a} \uparrow ((\mathfrak{b} \uparrow (\mathfrak{c} \uparrow \mathfrak{c})) \uparrow (\mathfrak{b} \uparrow (\mathfrak{c} \uparrow \mathfrak{c}))) = (\mathfrak{a} \uparrow (\mathfrak{b} \uparrow \mathfrak{b})) \uparrow ((\mathfrak{a} \uparrow (\mathfrak{c} \uparrow \mathfrak{c})) \uparrow (\mathfrak{a} \uparrow (\mathfrak{c} \uparrow \mathfrak{c}))) \tag{2.4}$$

for all $\mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in X$.

**Definition 2.5** ([17]). Let $(X, |, 0)$ be a Sheffer stroke BE-algebra. A subset $K$ of $X$ is called a *Sheffer stroke BE-subalgebra* of $(X, |, 0)$ if it satisfies:

$$(\forall \mathfrak{a}, \mathfrak{b} \in X)(\mathfrak{a}, \mathfrak{b} \in K \Rightarrow \mathfrak{a} \uparrow (\mathfrak{b} \uparrow \mathfrak{b}) \in K). \tag{2.5}$$

**Definition 2.6** ([24])**.** Let $(X, |, 0)$ be a Sheffer stroke BE-algebra. A fuzzy set $f$ in $X$ is called a *fuzzy Sheffer stroke BE-subalgebra* of $(X, |, 0)$ if it satisfies:

$$(\forall \mathfrak{a}, \mathfrak{b} \in X)(f(\mathfrak{a} \uparrow (\mathfrak{b} \uparrow \mathfrak{b})) \geq \min f(\mathfrak{a}), f(\mathfrak{b})\}). \tag{2.6}$$

**Definition 2.7** ([17])**.** Let $(X, \uparrow_X, 1_X)$ and $(Y, \uparrow_Y, 1_Y)$ be Sheffer stroke BE-algebras. A mapping $f : X \to Y$ is called a *homomorphism* if it satisfies:

$$f(\uparrow_X) = \uparrow_Y, \tag{2.7}$$

$$(\forall x, \mathfrak{a} \in X)(f(x \uparrow_X \mathfrak{a}) = f(x) \uparrow_Y f(\mathfrak{a})). \tag{2.8}$$

Let $U$ and $\tilde{E}$ denote an initial universe set and a set of parameters, respectively.

**Definition 2.8** ([22])**.** Given a subset $C$ of $\tilde{E}$, a pair $(\mathfrak{g}, C)$ is called a *soft set* over $U$, where $\mathfrak{g}$ is a mapping given by

$$\mathfrak{g} : C \to 2^U.$$

In other words, a soft set over $U$ is a parameterized family of subsets of the universe $U$. For $\alpha \in C$, $\mathfrak{g}(\alpha)$ may be considered as the set of $\alpha$-approximate elements of the soft set $(\mathfrak{g}, C)$. Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [22].

**Definition 2.9** ([19])**.** Given two soft sets $(\mathfrak{g}, B)$ and $(\mathfrak{h}, C)$ over a common universe $U$, the *intersection* of $(\mathfrak{g}, B)$ and $(\mathfrak{h}, C)$ is defined to be the soft set $(\mathfrak{f}, D)$ satisfying the following conditions:

(i) $D = B \cap C$,
(ii) $(\forall \gamma \in D)$ $(\mathfrak{f}(\gamma) = \mathfrak{g}(\gamma)$ or $\mathfrak{h}(\gamma)$, (as both are same set)).

In this case, we write $(\mathfrak{g}, B) \widetilde{\cap} (\mathfrak{h}, C) = (\mathfrak{f}, D)$.

**Definition 2.10** ([19])**.** Given two soft sets $(\mathfrak{g}, B)$ and $(\mathfrak{h}, C)$ over a common universe $U$, the *union* of $(\mathfrak{g}, B)$ and $(\mathfrak{h}, C)$ is defined to be the soft set $(\mathfrak{f}, D)$ satisfying the following conditions:

(i) $D = B \cup C$,
(ii) for all $\gamma \in D$,

$$\mathfrak{f}(\gamma) = \begin{cases} \mathfrak{g}(\gamma) & \text{if } \gamma \in B \setminus C, \\ \mathfrak{h}(\gamma) & \text{if } \gamma \in C \setminus B, \\ \mathfrak{g}(\gamma) \cup \mathfrak{h}(\gamma) & \text{if } \gamma \in B \cap C. \end{cases}$$

In this case, we write $(\mathfrak{g}, B) \widetilde{\cup} (\mathfrak{h}, C) = (\mathfrak{f}, D)$.

**Definition 2.11** ([19])**.** If $(\mathfrak{g}, B)$ and $(\mathfrak{h}, C)$ are two soft sets over a common universe $U$, then "$(\mathfrak{g}, B)$ *AND* $(\mathfrak{h}, C)$" denoted by $(\mathfrak{g}, B)\widetilde{\wedge}(\mathfrak{h}, C)$ is defined by $(\mathfrak{g}, B)\widetilde{\wedge}(\mathfrak{h}, C) = (\mathfrak{f}, B \times C)$, where $\mathfrak{f}(\alpha, \beta) = \mathfrak{g}(\alpha) \cap \mathfrak{h}(\beta)$ for all $(\alpha, \beta) \in B \times C$.

**Definition 2.12** ([19])**.** If $(\mathfrak{g}, B)$ and $(\mathfrak{h}, C)$ are two soft sets over a common universe $U$, then "$(\mathfrak{g}, B)$ *OR* $(\mathfrak{h}, C)$" denoted by $(\mathfrak{g}, B)\widetilde{\vee}(\mathfrak{h}, C)$ is defined by $(\mathfrak{g}, B)\widetilde{\vee}(\mathfrak{h}, C) = (\mathfrak{f}, B \times C)$, where $\mathfrak{f}(\alpha, \beta) = \mathfrak{g}(\alpha) \cup \mathfrak{h}(\beta)$ for all $(\alpha, \beta) \in B \times C$.

## 3. Soft Sheffer stroke BE-algebras

In this section, we introduce the definition of a soft Sheffer stroke BE-algebra. We establish the correspondence between fuzzy Sheffer stroke BE-algebras and soft Sheffer stroke BE-algebras, highlighting the conditions under which this correspondence holds. Additionally, we propose the concept of restriction within this algebraic structure. We provide a proof demonstrating that the intersection or union of two soft Sheffer stroke BE-algebras remains a soft Sheffer stroke BE-algebra. Furthermore, we establish that a soft Sheffer stroke BE-algebra preserves its properties under homomorphism. We also explore the concept of nullification in soft Sheffer stroke BE-algebras and examine its relationship under homomorphism.

In what follows, $(X, |, 0)$ stands for a Sheffer stroke BE-algebra, unless otherwise stated. As an initial universe set, we take the set $X$ in $(X, |, 0)$, and let $\tilde{E}$ denote a set of parameters and $B$, $C$, $D$... etc are subsets of $\tilde{E}$. We use the notation $\mathfrak{g}(B, X)$ as a soft set over $X$ with parameter $B$.

**Definition 3.1.** A soft set $\mathfrak{g}(B, X)$ is called a *soft Sheffer stroke BE-algebra* of $(X, |, 0)$ if $\mathfrak{g}(\alpha)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $\alpha \in B$.

**Example 3.2.** (i) Let $X = \{0, 1, a, 1\}$ be a set with the Sheffer stroke "$\uparrow$" given by Table 1.

Table 1. Cayley table for the Sheffer stroke "$\uparrow$"

| $\uparrow$ | 0 | 1 | $a$ | 1 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | $a$ | 1 | $a$ |
| $a$ | 1 | 1 | 1 | 1 |
| 1 | 1 | $a$ | 1 | 0 |

Then $(X, |, 0)$ is a Sheffer stroke BE-algebra (see [17]). Let $\mathfrak{g}(B, X)$ be a soft set where $B = X$ and $\mathfrak{g}$ is given as follows: $\mathfrak{g}(0) = \{1\}$, $\mathfrak{g}(1) = \{1, 1\}$,

$\mathfrak{g}(a) = \{1, a\}$, and $\mathfrak{g}(1) = \{1, 1, a\}$. Then $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$.

(ii) Given a self-distributive Sheffer stroke BE-algebra $(X, |, 0)$, consider a set-valued function:

$$\mathfrak{g} : X \times X \to 2^X,\ (x, y) \mapsto U_x^y$$

where $U_x^y = \{z \in X \mid x \uparrow ((y \uparrow (z|z)) \uparrow (y \uparrow (z|z))) = 1\}$. Then $\mathfrak{g}(x, y) = U_x^y$ is a Sheffer stroke BE-filter (see [17, Theorem 29]), and so it is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $(x, y) \in X \times X$ (see [17, Lemma 37]). Therefore $\mathfrak{g}(X \times X, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$.

**Theorem 3.3.** *Given a fuzzy Sheffer stroke BE-subalgebra $f$ of $(X, |, 0)$, consider a mapping:*

$$\mathfrak{g} : [0, 1] \to 2^X,\ t \mapsto U(f, t)$$

*where $U(f, t) := \{x \in X \mid f(x) \geq t\}$. Then $\mathfrak{g}([0, 1], X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$.*

*Proof.* Let $x, y \in U(f, t)$. Then $f(x) \geq t$ and $f(y) \geq t$. Hence

$$f(x|(y|y)) \geq \min\{f(x), f(y)\} \geq t,$$

and so $x|(y|y) \in U(f, t)$. Thus $\mathfrak{g}(t) = U(f, t)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $t \in [0, 1]$, and therefore $\mathfrak{g}([0, 1], X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. □

**Theorem 3.4.** *Given a fuzzy set $f$ in $X$, let $\mathfrak{g}(B, X)$ be a soft set with $B = \text{Im}(f)$ and $\mathfrak{g}(t) = U(f, t)$ for all $t \in B$. If $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$, then $f$ is a fuzzy Sheffer stroke BE-subalgebra of $(X, |, 0)$.*

*Proof.* If $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$, then $\mathfrak{g}(t) = U(f, t)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $t \in B$. Assume that $f(\mathfrak{a} \uparrow (\mathfrak{a}|(\mathfrak{b}|\mathfrak{b}))) < \min\{f(\mathfrak{a}), f(\mathfrak{b})\}$ for some $\mathfrak{a}, \mathfrak{b} \in X$, and take $t := \min\{f(\mathfrak{a}), f(\mathfrak{b})\}$. Then $\mathfrak{a}, \mathfrak{b} \in U(f, t)$ and $\mathfrak{a} \uparrow (\mathfrak{a}|(\mathfrak{b}|\mathfrak{b})) \notin U(f, t)$. This is a contradiction, and thus $f(x|(y|y)) \geq \min\{f(x), f(y)\}$ for all $x, y \in X$. Therefore $f$ is a fuzzy Sheffer stroke BE-subalgebra of $(X, |, 0)$. □

**Question 3.5.** Given a fuzzy set $f$ in $X$, consider the soft set $\mathfrak{g}(B, X)$ where $B \subseteq [0, 1]$ and $\mathfrak{g}$ is given as follows:

$$\mathfrak{g} : B \to 2^X,\ t \mapsto \{x \in X \mid f(x) > 1 - t\}. \tag{3.1}$$

Is $\mathfrak{g}(B, X)$ a soft Sheffer stroke BE-algebra of $(X, |, 0)$?

The example below gives a negative answer to the Question 3.5.

**Example 3.6.** Let $X = \{0, 1, a, b, c, 1\}$ be a set with the Sheffer stroke "$\uparrow$" given by Table 2.

TABLE 2. Cayley table for the Sheffer stroke "$\uparrow$"

| $\uparrow$ | $0$ | $1$ | $a$ | $b$ | $c$ | $1$ |
|---|---|---|---|---|---|---|
| $0$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| $1$ | $1$ | $c$ | $b$ | $1$ | $1$ | $c$ |
| $a$ | $1$ | $b$ | $b$ | $1$ | $1$ | $b$ |
| $b$ | $1$ | $1$ | $1$ | $a$ | $1$ | $a$ |
| $c$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| $1$ | $1$ | $c$ | $b$ | $a$ | $1$ | $0$ |

Then $(X, |, 0)$ is a Sheffer stroke BE-algebra (see [17]). Let

$$f : X \to [0, 1]$$

be a fuzzy set defined by $f(0) = 0.32$, $f(1) = 0.27$, $f(a) = 0.79$, $f(b) = 0.31$, $f(c) = 0.62$, and $f(1) = 0.86$. If we take

$$B = \text{Im}(f) = \{0.27, 0.31, 0.32, 0.62, 0.79, 0.86\},$$

then

$$\mathfrak{g}(t) = \{x \in X \mid f(x) > 1 - t\} = \{1, a, c\}$$

for $t = 0.62$. Since $a \uparrow (c \uparrow c) = a \uparrow 1 = b \notin \mathfrak{g}(t)$, we know that $\mathfrak{g}(t)$ is not a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for $t = 0.62$. Hence $\mathfrak{g}(B, X)$ is not a soft Sheffer stroke BE-algebra of $(X, |, 0)$.

**Theorem 3.7.** *Given a fuzzy set $f$ in $X$, let $\mathfrak{g}([0, 1], X)$ be a soft set where $\mathfrak{g}$ is given as follows:*

$$\mathfrak{g} : [0, 1] \to 2^X, \; t \mapsto \{x \in X \mid f(x) > 1 - t\}. \tag{3.2}$$

*Then $\mathfrak{g}([0, 1], X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$ if and only if $f$ is a fuzzy Sheffer stroke BE-subalgebra of $(X, |, 0)$.*

*Proof.* Suppose that $\mathfrak{g}([0, 1], X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. Then $\mathfrak{g}(t)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $t \in [0, 1]$. Let $\mathfrak{b}, \mathfrak{c} \in X$ be such that $f(\mathfrak{b} \uparrow (\mathfrak{c} \uparrow \mathfrak{c})) < \min\{f(\mathfrak{b}), f(\mathfrak{c})\}$. Then

$$f(\mathfrak{b} \uparrow (\mathfrak{c} \uparrow \mathfrak{c})) \leq 1 - t < \min\{f(\mathfrak{b}), f(\mathfrak{c})\}$$

for some $t \in [0, 1]$, which implies that $f(\mathfrak{b}) > 1 - t$ and $f(\mathfrak{c}) > 1 - t$. Thus $\mathfrak{b}, \mathfrak{c} \in \mathfrak{g}(t)$. Since $\mathfrak{g}(t)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$, it follows that $\mathfrak{b} \uparrow (\mathfrak{c} \uparrow \mathfrak{c}) \in \mathfrak{g}(t)$. Thus $f(\mathfrak{b} \uparrow (\mathfrak{c} \uparrow \mathfrak{c})) > 1 - t$, which is a contradiction. Hence $f(y \uparrow (z \uparrow z)) \geq \min\{f(y), f(z)\}$ for all $y, z \in X$. Therefore, $f$ is a fuzzy Sheffer stroke BE-subalgebra of $(X, |, 0)$.

Conversely, assume that $f$ is a fuzzy Sheffer stroke BE-subalgebra of $(X, |, 0)$. Let $x, y \in \mathfrak{g}(t)$ for every $t \in [0, 1]$. Then $f(x) > 1 - t$ and $f(y) > 1 - t$. Hence

$$f(x|(y|y)) \geq \min\{f(x), f(y)\} > 1 - t,$$

and so $x|(y|y) \in \mathfrak{g}(t)$. This shows that $\mathfrak{g}(t)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $t \in [0, 1]$. Thus $\mathfrak{g}([0, 1], X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. □

**Lemma 3.8.** *Let $C$ be a subset of $B$. If $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$, then so is $\mathfrak{g}|_C(C, X)$ where $\mathfrak{g}|_C$ is the restriction of $\mathfrak{g}$ to $C$.*

*Proof.* It is straightforward. □

The next example shows that the converse of Lemma 3.8 may not be true.

**Example 3.9.** Consider the Sheffer stroke BE-algebra $(X, |, 0)$ described in Example 3.6. Let $\mathfrak{g}(B, X)$ be a soft set where $B = X$ and $\mathfrak{g}$ is given by

$$\mathfrak{g} : B \to 2^X, \ \alpha \mapsto \begin{cases} \{1, a, c\} & \text{if } \alpha = 0, \\ \{1, 1, b\} & \text{if } \alpha = 1, \\ \{1, a, b, 0\} & \text{if } \alpha = a, \\ \{0, 1, a\} & \text{if } \alpha = b, \\ \{1, 1, b, c\} & \text{if } \alpha = c, \\ \{1, b, c\} & \text{if } \alpha = 1. \end{cases}$$

If we take $C := \{1, a, c, 1\}$, then $\mathfrak{g}|_C(C, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. But $\mathfrak{g}(0) = \{1, a, c\}$ is not a Sheffer stroke BE-subalgebra of $(X, |, 0)$ since $a \uparrow (c \uparrow c) = a \uparrow 1 = b \notin \mathfrak{g}(0)$. Also, $\mathfrak{g}(b) = \{0, 1, a\}$ is not a Sheffer stroke BE-subalgebra of $(X, |, 0)$ since $1 \uparrow (a \uparrow a) = 1 \uparrow b = 1 \notin \mathfrak{g}(b)$. Hence $\mathfrak{g}(B, X)$ is not a soft Sheffer stroke BE-algebra of $(X, |, 0)$.

**Theorem 3.10.** *Given a soft set $\mathfrak{g}(B, X)$, let $\{B_i \mid i \in \Lambda\}$ be a partition of $B$ and let $\mathfrak{g} : B \to 2^X$ be a set-valued function given by $\mathfrak{g}(\alpha) = C_i(\neq \emptyset) \in 2^X$ for all $\alpha \in B_i$ and $i \in \Lambda$. Then $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$ if and only if $\mathfrak{g}_{B_i}(B_i, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$ for all $i \in \Lambda$.*

*Proof.* Suppose that $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. Then $\mathfrak{g}_{B_i}(B_i, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$ for all $i \in \Lambda$ by Lemma 3.8. The sufficiency is obvious because $\{B_i \mid i \in \Lambda\}$ is a partition of $B$. □

**Theorem 3.11.** *Given disjoint subsets $B$ and $C$ of $\tilde{E}$, if $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$, then so is their intersection $\mathfrak{g}(B, X) \,\widetilde{\cap}\, \mathfrak{h}(C, X)$.*

*Proof.* Assume that $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$. Let $\mathfrak{f}(D, X) = \mathfrak{g}(B, X) \,\widetilde{\cap}\, \mathfrak{h}(C, X)$. Then $D = B \cap C$, and $\mathfrak{f}(z) = \mathfrak{g}(z)$ or $\mathfrak{f}(z) = \mathfrak{h}(z)$ for all $z \in D$. Since $B$ and $C$ are disjoint, if $z \in D$ then $z \in B \setminus C$ or $z \in C \setminus B$. Thus $\mathfrak{f}(z) = \mathfrak{g}(z)$ and $\mathfrak{f}(z) = \mathfrak{h}(z)$ are Sheffer stroke BE-subalgebras of $(X, |, 0)$ for all $z \in B \setminus C$ and $z \in C \setminus B$, respectively. Consequently, we know that $\mathfrak{g}(B, X) \,\widetilde{\cap}\, \mathfrak{h}(C, X) = \mathfrak{f}(D, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. □

**Theorem 3.12.** *Let $B$ and $C$ be disjoint subsets of $\tilde{E}$. If $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$, then so is their union $\mathfrak{g}(B, X) \,\widetilde{\cup}\, \mathfrak{h}(C, X)$.*

*Proof.* Let $\mathfrak{f}(D, X) = \mathfrak{g}(B, X) \,\widetilde{\cup}\, \mathfrak{h}(C, X)$. Then $D = B \cup C$, and

$$\mathfrak{f}(\mathfrak{c}) = \begin{cases} \mathfrak{g}(\mathfrak{c}) & \text{if } \mathfrak{c} \in B \setminus C, \\ \mathfrak{h}(\mathfrak{c}) & \text{if } \mathfrak{c} \in C \setminus B, \\ \mathfrak{g}(\mathfrak{c}) \cup \mathfrak{h}(\mathfrak{c}) & \text{if } \mathfrak{c} \in B \cap C \end{cases}$$

for all $\mathfrak{c} \in D$. Since $B$ and $C$ are disjoint, either $\mathfrak{c} \in B \setminus C$ or $\mathfrak{c} \in C \setminus B$ for all $\mathfrak{c} \in D$. If $\mathfrak{c} \in B \setminus C$, then $\mathfrak{f}(\mathfrak{c}) = \mathfrak{g}(\mathfrak{c})$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ since $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. If $\mathfrak{c} \in C \setminus B$, then $\mathfrak{f}(\mathfrak{c}) = \mathfrak{h}(\mathfrak{c})$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ since $\mathfrak{h}(C, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. Therefore $\mathfrak{f}(D, X) = \mathfrak{g}(B, X) \,\widetilde{\cup}\, \mathfrak{h}(C, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. □

In Theorem 3.12, if $B$ and $C$ are not disjoint, then the result is not valid as shown in example below.

**Example 3.13.** Consider the Sheffer stroke BE-algebra $(X, |, 0)$ described in Example 3.6. For subsets $B := \{0, 1, a\}$ and $C := \{a, b, c\}$ of $\tilde{E} := X$,
let $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ be soft sets given as follows:

$$\mathfrak{g} : B \to 2^X, \ \alpha \mapsto \{1, c\}$$

and

$$\mathfrak{h} : C \to 2^X,\ \beta \mapsto \{1, 0\},$$

respectively. Then $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$. The union $\mathfrak{f}(D, X) = \mathfrak{g}(B, X) \mathbin{\widetilde{\cup}} \mathfrak{h}(C, X)$ of $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ is calculated as follows:

$$\mathfrak{f} : D = B \cup C \to 2^X,\ \gamma \mapsto \begin{cases} \{1, c\} & \text{if } \gamma \in \{0, 1\}, \\ \{1, 0\} & \text{if } \gamma \in \{b, c\}, \\ \{1, 0, c\} & \text{if } \gamma = a. \end{cases}$$

We can observe that $\mathfrak{f}(a) = \{1, 0, c\}$ is not a Sheffer stroke BE-subalgebras of $(X, |, 0)$ since $c \uparrow (0 \uparrow 0) = c \uparrow 1 = 1 \notin \mathfrak{f}(a)$. Therefore $\mathfrak{f}(D, X) = \mathfrak{g}(B, X) \mathbin{\widetilde{\cup}} \mathfrak{h}(C, X)$ is not a soft Sheffer stroke BE-algebra of $(X, |, 0)$.

**Theorem 3.14.** *If $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$, then so is $\mathfrak{g}(B, X) \mathbin{\widetilde{\wedge}} \mathfrak{h}(C, X)$.*

*Proof.* We can say $\mathfrak{f}(B \times C, X) = \mathfrak{g}(B, X) \mathbin{\widetilde{\wedge}} \mathfrak{h}(C, X)$ by Definition 2.11 where $\mathfrak{f}(\alpha, \beta) = \mathfrak{g}(\alpha) \cap \mathfrak{h}(\beta)$ for all $(\alpha, \beta) \in B \times C$. Since $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$, the intersection $\mathfrak{g}(\alpha) \cap \mathfrak{h}(\beta)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$. Hence $\mathfrak{f}(\alpha, \beta)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $(\alpha, \beta) \in B \times C$, and therefore $\mathfrak{g}(B, X) \mathbin{\widetilde{\wedge}} \mathfrak{h}(C, X)$ is a soft Sheffer stroke BE-algebras of $(X, |, 0)$. □

**Question 3.15.** If $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$, then is $\mathfrak{g}(B, X) \mathbin{\widetilde{\vee}} \mathfrak{h}(C, X)$ a soft Sheffer stroke BE-algebra of $(X, |, 0)$?

The answer to Question 3.15 may not be true as seen in the following example.

**Example 3.16.** Consider the Sheffer stroke BE-algebra $(X, |, 0)$ described in Example 3.6. For $B := \{0, 1\}$ and $C := \{a, b, c\}$, let $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ be soft sets given as follows:

$$\mathfrak{g} : B \to 2^X,\ \alpha \mapsto \begin{cases} \{1, 0\} & \text{if } \alpha = 0, \\ \{1, b, c\} & \text{if } \alpha = 1, \end{cases}$$

and

$$\mathfrak{h} : C \to 2^X,\ \beta \mapsto \begin{cases} \{1, c\} & \text{if } \beta = a, \\ \{1, a, b\} & \text{if } \beta = b, \\ \{1, 1, c, 0\} & \text{if } \beta = c, \end{cases}$$

respectively. It is routine to verify that $\mathfrak{g}(B, X)$ and $\mathfrak{h}(C, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$. The “OR” operation “$\mathfrak{g}(B, X)$ OR $\mathfrak{h}(C, X)$”, denoted by $\mathfrak{g}(B,X) \,\widetilde{\vee}\, \mathfrak{h}(C,X)$, of $\mathfrak{g}(B,X)$ and $\mathfrak{h}(C,X)$ is given by $\mathfrak{g}(B,X) \,\widetilde{\vee}\, \mathfrak{h}(C,X) = \mathfrak{f}(B\times C, X)$, and it is calculated as follows:

$$\mathfrak{f} : B\times C \to 2^X,\ (\alpha,\beta) \mapsto \begin{cases} \{1,0,c\} & \text{if } (\alpha,\beta)=(0,a),\\ \{1,0,a,b\} & \text{if } (\alpha,\beta)=(0,b),\\ \{1,0,1,c\} & \text{if } (\alpha,\beta)=(0,c),\\ \{1,b,c\} & \text{if } (\alpha,\beta)=(1,a),\\ \{1,a,b,c\} & \text{if } (\alpha,\beta)=(1,b),\\ \{1,0,1,b,c\} & \text{if } (\alpha,\beta)=(1,c).\end{cases}$$

Then $\mathfrak{f}(1,c) = \{1,0,1,b,c\}$ is not a Sheffer stroke BE-subalgebra of $(X, |, 0)$ since $b\uparrow(0\uparrow 0) = b\uparrow 1 = a \notin \mathfrak{f}(1,c)$. Therefore,

$$\mathfrak{g}(B,X) \,\widetilde{\vee}\, \mathfrak{h}(C,X) = \mathfrak{f}(B\times C, X)$$

is not a soft Sheffer stroke BE-algebra of $(X, |, 0)$.

Let $f : X \to Y$ be a mapping of Sheffer stroke BE-algebras $(X, \uparrow_X, 1_X)$ and $(Y, \uparrow_Y, 1_Y)$. If $\mathfrak{g}(B,X)$ is a soft set, then $f(\mathfrak{g})(B,Y)$ is also a soft set where

$$f(\mathfrak{g}) : B \to 2^Y,\ \alpha \mapsto f(\mathfrak{g}(\alpha)).$$

**Question 3.17.** Let $f : X \to Y$ be a mapping of Sheffer stroke BE-algebras $(X, \uparrow_X, 1_X)$ and $(Y, \uparrow_Y, 1_Y)$. If $\mathfrak{g}(B,X)$ is a soft Sheffer stroke BE-algebra of $(X, \uparrow_X, 1_X)$, then is $f(\mathfrak{g})(B,Y)$ a soft Sheffer stroke BE-algebra of $(Y, \uparrow_Y, 1_Y)$?

The following example, gives a negative answer to the question 3.17.

**Example 3.18.** Consider the Sheffer stroke BE-algebra $(X, |, 0)$ described in Example 3.6. Let $B := \{\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3\}$ and define a soft set $\mathfrak{g}(B,X)$ as follows:

$$\mathfrak{g} : B \to 2^X,\ \alpha \mapsto \begin{cases} \{1,1,b\} & \text{if } \alpha=\mathfrak{a}_1,\\ \{1,c\} & \text{if } \alpha=\mathfrak{a}_2,\\ \{1,a,b,0\} & \text{if } \alpha=\mathfrak{a}_3.\end{cases}$$

Then $\mathfrak{g}(B,X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. Let $Y = \{0,1,u,v\}$ be a set with the Sheffer stroke "|" given by Table 3.
Then $(Y, |, 1)$ is a Sheffer stoke BE-algebra (see [17, Example 9]). Let $f : X \to Y$ be a mapping defined by $f(0) = 1 = f(c)$, $f(1) = u = f(a)$, $f(b) = v$, and $f(1) = 0$. Then

$$f(\mathfrak{g}) : B \to 2^Y,\ \alpha \mapsto \begin{cases} \{0,u,v\} & \text{if } \alpha=\mathfrak{a}_1,\\ \{0,1\} & \text{if } \alpha=\mathfrak{a}_2,\\ Y & \text{if } \alpha=\mathfrak{a}_3.\end{cases}$$

TABLE 3. Cayley table for the Sheffer stroke "|"

| $\mid$ | $0$ | $u$ | $v$ | $1$ |
|---|---|---|---|---|
| $0$ | $1$ | $1$ | $1$ | $1$ |
| $u$ | $1$ | $v$ | $1$ | $v$ |
| $v$ | $1$ | $1$ | $u$ | $u$ |
| $1$ | $1$ | $v$ | $u$ | $0$ |

We know that $f(\mathfrak{g})(\mathfrak{a}_2) = \{0, 1\}$ and $f(\mathfrak{g})(\mathfrak{a}_3) = Y$ are soft Sheffer stroke BE-subalgebras of $(Y, |, 1)$. But $f(\mathfrak{g})(\mathfrak{a}_1) = \{0, u, v\}$ is not a soft Sheffer stroke BE-subalgebra of $(Y, |, 1)$ since $u|(u|u) = u|v = 1 \notin f(\mathfrak{g})(\mathfrak{a}_1)$. Hence $f(\mathfrak{g})(B, Y)$ is not a soft Sheffer stroke BE-algebra of $(Y, \uparrow_Y, 1_Y)$.

We can know that the mapping $f$ in Example 3.18 is not a homomorphism since $f(1) = 0 \neq 1$ and/or $f(1 \uparrow b) = f(1) = 0 \neq 1 = u|v = f(1)|f(b)$. Therefore, the fact that $f$ is not a homomorphism is one factor that makes the answer to Question 3.17 negative. This suggests that the fact that $f$ is a homomorphism may be a necessary condition to elicit a positive answer to the Question 3.17. In fact, we have the following theorem.

**Theorem 3.19.** *Let $f : X \to Y$ be a homomorphism of Sheffer stroke BE-algebras $(X, \uparrow_X, 1_X)$ and $(Y, \uparrow_Y, 1_Y)$. If $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, \uparrow_X, 1_X)$, then $f(\mathfrak{g})(B, Y)$ is a soft Sheffer stroke BE-algebra of $(Y, \uparrow_Y, 1_Y)$.*

*Proof.* Assume that $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, \uparrow_X, 1_X)$. Then $\mathfrak{g}(\alpha)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $\alpha \in B$. Let $\alpha \in B$ and $\mathfrak{a}, \mathfrak{b} \in f(\mathfrak{g})(\alpha) = f(\mathfrak{g}(\alpha))$. Then $f(x) = \mathfrak{a}$ and $f(y) = \mathfrak{b}$ for some $x, y \in \mathfrak{g}(\alpha)$. Hence

$$\mathfrak{a} \uparrow_Y (\mathfrak{b} \uparrow_Y \mathfrak{b}) = f(x) \uparrow_Y (f(y) \uparrow_Y f(y)) = f(x \uparrow_X (y \uparrow_X y)) \in f(\mathfrak{g}(\alpha)),$$

and so $f(\mathfrak{g})(\alpha) = f(\mathfrak{g}(\alpha))$ is a Sheffer stroke BE-subalgebra of $(Y, \uparrow_Y, 1_Y)$. Therefore $f(\mathfrak{g})(B, Y)$ is a soft Sheffer stroke BE-algebra of $(Y, \uparrow_Y, 1_Y)$. □

A soft Sheffer stroke BE-algebra $\mathfrak{g}(B, X)$ of $(X, |, 0)$ is said to be *nullified* (resp., *entire*) if $\mathfrak{g}(\alpha) = \{1\}$ (resp., $\mathfrak{g}(\alpha) = X$) for all $\alpha \in B$.

**Theorem 3.20.** *Let $f : X \to Y$ be a homomorphism of Sheffer stroke BE-algebras $(X, \uparrow_X, 1_X)$ and $(Y, \uparrow_Y, 1_Y)$, and let $\mathfrak{g}(B, X)$ be a soft Sheffer stroke BE-algebra of $(X, \uparrow_X, 1_X)$. If $\mathfrak{g}(\alpha) \subseteq Ker(f) := \{x \in X \mid f(x) = 1\}$ for all $\alpha \in B$, then $f(\mathfrak{g})(B, Y)$ is a nullified soft Sheffer stroke BE-algebra of*

$(Y, \uparrow_Y, 1_Y)$. *Also, if* $f$ *is onto and* $\mathfrak{g}(B, X)$ *is an entire soft Sheffer stroke BE-algebra of* $(X, \uparrow_X, 1_X)$*, then* $f(\mathfrak{g})(B, Y)$ *is an entire soft Sheffer stroke BE-algebra of* $(Y, \uparrow_Y, 1_Y)$.

*Proof.* Let $\mathfrak{g}(B, X)$ be a soft Sheffer stroke BE-algebra of $(X, \uparrow_X, 1_X)$.Then $f(\mathfrak{g})(B, Y)$ is a soft Sheffer stroke BE-algebra of $(Y, \uparrow_Y, 1_Y)$ by Theorem 3.19. Assume that $\mathfrak{g}(\alpha) \subseteq Ker(f)$ for all $\alpha \in B$. Then $f(\mathfrak{g})(\alpha) = f(\mathfrak{g}(\alpha)) = \{1\}$ for all $\alpha \in B$, and so $f(\mathfrak{g})(B, Y)$ is a nullified soft Sheffer stroke BE-algebra of $(Y, \uparrow_Y, 1_Y)$. Suppose that $f$ is onto and $\mathfrak{g}(B, X)$ is an entire soft Sheffer stroke BE-algebra of $(X, \uparrow_X, 1_X)$. Then $\mathfrak{g}(\alpha) = X$ for all $\alpha \in B$, and thus $f(\mathfrak{g})(\alpha) = f(\mathfrak{g}(\alpha)) = f(X) = Y$ for all $\alpha \in B$. Therefore $f(\mathfrak{g})(B, Y)$ is an entire soft Sheffer stroke BE-algebra of $(Y, \uparrow_Y, 1_Y)$. □

## 4. Soft Sheffer stroke sub-BE-algebras

In this section, we present the concept of a soft Sheffer stroke sub-BE-algebra. We prove that the intersection or union of two soft Sheffer stroke BE-algebras constitutes a soft Sheffer stroke sub-BE-algebra. Additionally, we show that a soft Sheffer stroke sub-BE-algebra preserves its properties under homomorphism, thus maintaining its status as a soft Sheffer stroke sub-BE-algebra.

**Definition 4.1.** Let $\mathfrak{h}(C, X)$ be a soft Sheffer stroke BE-algebras of $(X, |, 0)$. Then a soft set $\mathfrak{g}(B, X)$ is called a *soft Sheffer stroke sub-BE-algebra* of $\mathfrak{h}(C, X)$, denoted by $\mathfrak{g}(B, X) \mathrel{\widetilde{\lesssim}}_S \mathfrak{h}(C, X)$, if $B \subseteq C$ and $\mathfrak{g}(\alpha)$ is a Sheffer stroke BE-subalgebra of $\mathfrak{h}(\alpha)$ for all $\alpha \in B$.

**Example 4.2.** Let $X = \{0, 1, u, v, w, t, p, q\}$ be a set with the Sheffer stroke "$\uparrow$" given by Table 4.

Table 4. Cayley table for the Sheffer stroke "$\uparrow$"

| $\uparrow$ | $0$ | $u$ | $v$ | $w$ | $t$ | $p$ | $q$ | $1$ |
|---|---|---|---|---|---|---|---|---|
| $0$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| $u$ | $1$ | $q$ | $1$ | $1$ | $q$ | $q$ | $1$ | $q$ |
| $v$ | $1$ | $1$ | $p$ | $1$ | $p$ | $1$ | $p$ | $p$ |
| $w$ | $1$ | $1$ | $1$ | $t$ | $1$ | $t$ | $t$ | $t$ |
| $t$ | $1$ | $q$ | $p$ | $1$ | $w$ | $q$ | $p$ | $w$ |
| $p$ | $1$ | $q$ | $1$ | $t$ | $q$ | $v$ | $t$ | $v$ |
| $q$ | $1$ | $1$ | $p$ | $t$ | $p$ | $t$ | $u$ | $u$ |
| $1$ | $1$ | $q$ | $p$ | $t$ | $w$ | $v$ | $u$ | $0$ |

Then $(X, |, 0)$ is a Sheffer stroke BE-algebra (see [17, Example 47]). Let $\mathfrak{h}(C, X)$ be a soft set where $C = X$ and $\mathfrak{h}$ is given by

$$\mathfrak{h} : C \to 2^X,\ \beta \mapsto \begin{cases} \{1, p, q\} & \text{if } \beta = 0, \\ \{1, w, t, 0\} & \text{if } \beta = 1, \\ \{1, u, w, t, p, q\} & \text{if } \beta \in \{u, v\}, \\ \{1, 0\} & \text{if } \beta = w, \\ \{1, t, p\} & \text{if } \beta = t, \\ \{1, v, t, q\} & \text{if } \beta \in \{p, q\}. \end{cases}$$

Then $\mathfrak{h}(C, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$. For a subset $B = \{1, u, p\}$ of $C$, let $\mathfrak{g}(B, X)$ be a soft set defined as follows:

$$\mathfrak{g} : B \to 2^X,\ \alpha \mapsto \begin{cases} \{1, w, t, 0\} & \text{if } \alpha = 1, \\ \{1, u, w, t, p, q\} & \text{if } \alpha = u, \\ \{1, v, t, q\} & \text{if } \alpha = p. \end{cases}$$

It is easy to verify that $\mathfrak{g}(\alpha)$ is a Sheffer stroke BE-subalgebra of $\mathfrak{h}(\alpha)$ for $\alpha \in \{1, u, p\}$. Therefore $\mathfrak{g}(B, X)$ is a soft Sheffer stroke sub-BE-algebra of $\mathfrak{h}(C, X)$.

It is clear that $\mathfrak{g}(\{1\}, X) \mathrel{\widetilde{\leq}_S} \mathfrak{h}(X, X)$, and if $\mathfrak{g}(B, X)$ and $\mathfrak{h}(B, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$ that satisfies $\mathfrak{g}(x) \subseteq \mathfrak{h}(x)$ for all $x \in B$, then $\mathfrak{g}(B, X) \mathrel{\widetilde{\leq}_S} \mathfrak{h}(B, X)$.

Let $\mathfrak{g}(B, X)$ be a soft set. If $\mathfrak{h}(C, X)$ is an entire soft Sheffer stroke BE-algebras of $(X, |, 0)$, then $\mathfrak{g}(B, X)$ is a soft Sheffer stroke sub-BE-algebra of $\mathfrak{h}(C, X)$ if and only if $\mathfrak{g}(B, X)$ is a soft Sheffer stroke BE-algebra of $(X, |, 0)$.

**Theorem 4.3.** *Let $\mathfrak{h}(C, X)$ be a soft Sheffer stroke BE-algebra of $(X, |, 0)$. If $\mathfrak{g}_1(B_1, X)$ and $\mathfrak{g}_2(B_2, X)$ are soft Sheffer stroke sub-BE-algebras of $\mathfrak{h}(C, X)$, then $\mathfrak{g}_1(B_1, X) \mathbin{\widetilde{\cap}} \mathfrak{g}_2(B_2, X)$ is a soft Sheffer stroke sub-BE-algebra of $\mathfrak{h}(C, X)$. Moreover, if $B_1$ and $B_2$ are disjoint, then $\mathfrak{g}_1(B_1, X) \mathbin{\widetilde{\cup}} \mathfrak{g}_2(B_2, X)$ is a soft Sheffer stroke sub-BE-algebra of $\mathfrak{h}(C, X)$.*

*Proof.* Let $\mathfrak{h}(C, X)$ be a soft Sheffer stroke BE-algebra of $(X, |, 0)$. Then $\mathfrak{h}(x)$ is a Sheffer stroke BE-subalgebra of $(X, |, 0)$ for all $x \in C$. Assume that $\mathfrak{g}_1(B_1, X)$ and $\mathfrak{g}_2(B_2, X)$ are soft Sheffer stroke sub-BE-algebras of $\mathfrak{h}(C, X)$. Let $\mathfrak{g}(B, X) := \mathfrak{g}_1(B_1, X) \mathbin{\widetilde{\cap}} \mathfrak{g}_2(B_2, X)$. Then $B = B_1 \cap B_2 \subseteq C$, and $\mathfrak{g}(x) = \mathfrak{g}_1(x)$ or $\mathfrak{g}(x) = \mathfrak{g}_2(x)$ for all $x \in B$. Let $x \in B$. Then $x \in B_i$ for $i = 1, 2$. Hence $\mathfrak{g}(x) = \mathfrak{g}_i(x)$ is a Sheffer stroke BE-subalgebra of $\mathfrak{h}(x)$ since $\mathfrak{g}_i(B_i, X) \mathrel{\widetilde{\leq}} \mathfrak{h}(C, X)$ for $i = 1, 2$. Therefore $\mathfrak{g}_1(B_1, X) \mathbin{\widetilde{\cap}} \mathfrak{g}_2(B_2, X) = \mathfrak{g}(B, X)$

$\mathrel{\widetilde{\lessapprox}}_S \mathfrak{h}(C,X)$. Now, suppose $B_1$ and $B_2$ are disjoint. Let $\mathfrak{g}(B,X) := \mathfrak{g}_1(B_1,X) \mathbin{\widetilde{\cup}} \mathfrak{g}_2(B_2,X)$. Then $B = B_1 \cup B_2$ and

$$\mathfrak{g}(\mathfrak{c}) = \begin{cases} \mathfrak{g}_1(\mathfrak{c}) & \text{if } \mathfrak{c} \in B_1 \setminus B_2, \\ \mathfrak{g}_2(\mathfrak{c}) & \text{if } \mathfrak{c} \in B_2 \setminus B_1, \\ \mathfrak{g}_1(\mathfrak{c}) \cup \mathfrak{g}_2(\mathfrak{c}) & \text{if } \mathfrak{c} \in B_1 \cap B_2 \end{cases}$$

for all $\mathfrak{c} \in B$. Since $B_1 \cap B_2 = \emptyset$, it follows that $\mathfrak{g}(\mathfrak{c}) = \mathfrak{g}_i(\mathfrak{c})$ for $i = 1, 2$, which is a Sheffer stroke BE-subalgebra of $\mathfrak{h}(\mathfrak{c})$. Hence

$$\mathfrak{g}_1(B_1,X) \mathbin{\widetilde{\cup}} \mathfrak{g}_2(B_2,X) = \mathfrak{g}(B,X) \mathrel{\widetilde{\lessapprox}}_S \mathfrak{h}(C,X).$$

This completes the proof. □

In Theorem 4.3, if $B_1$ and $B_2$ are not disjoint, then $\mathfrak{g}_1(B_1,X) \mathbin{\widetilde{\cup}} \mathfrak{g}_2(B_2,X)$ is not a soft Sheffer stroke sub-BE-algebra of $\mathfrak{h}(C,X)$ as seen in the example below.

**Example 4.4.** Consider the Sheffer stroke BE-algebra $(X, |, 0)$ in Example 4.2 and let $\mathfrak{h}(C,X)$ be the entire soft Sheffer stroke BE-algebra of $(X, |, 0)$ with $C = X$. Let $\mathfrak{g}_1(B_1,X)$ be a soft, where $B_1 := \{v, q\} \subseteq C$ and $\mathfrak{g}_1$ is a set-valued function given by

$$\mathfrak{g}_1 : B_1 \to 2^X,\ \alpha \mapsto \begin{cases} \{1, t\} & \text{if } \alpha = v, \\ \{1, u, w, t, p, q\} & \text{if } \alpha = q. \end{cases}$$

Also, we take a soft set $\mathfrak{g}_2(B_2,X)$ where $B_2 := \{0, v, p\} \subseteq C$ and $\mathfrak{g}_2$ is a set-valued function given by

$$\mathfrak{g}_2 : B_2 \to 2^X,\ \alpha \mapsto \begin{cases} \{1, w, t, 0\} & \text{if } \alpha = 0, \\ \{1, v, p\} & \text{if } \alpha = v, \\ \{1, w, t, p, q\} & \text{if } \alpha = p. \end{cases}$$

It is routine to verify that $\mathfrak{g}_1(B_1,X)$ and $\mathfrak{g}_2(B_2,X)$ are soft Sheffer stroke sub-BE-algebras of $\mathfrak{h}(C,X)$. Then $B_1$ and $B_2$ are not disjoint and the union $\mathfrak{g}(B,X) := \mathfrak{g}_1(B_1,X) \mathbin{\widetilde{\cup}} \mathfrak{g}_2(B_2,X)$ of $\mathfrak{g}_1(B_1,X)$ and $\mathfrak{g}_2(B_2,X)$ is calculated as follows:

$$\mathfrak{g} : B \to 2^X,\ \alpha \mapsto \begin{cases} \{1, w, t, 0\} & \text{if } \alpha = 0, \\ \{1, w, t, p, q\} & \text{if } \alpha = p, \\ \{1, u, w, t, p, q\} & \text{if } \alpha = q. \\ \{1, v, t, p\} & \text{if } \alpha = v. \end{cases}$$

Then $\mathfrak{g}(v) = \{1, v, t, p\}$ is not a Sheffer stroke BE-subalgebra of $\mathfrak{h}(v) = X$ since $t \uparrow (v \uparrow v) = t \uparrow p = q \notin \mathfrak{g}(v)$. Therefore, $\mathfrak{g}_1(B_1,X) \mathbin{\widetilde{\cup}} \mathfrak{g}_2(B_2,X)$ is not a soft Sheffer stroke sub-BE-algebra of $\mathfrak{h}(C,X)$.

**Theorem 4.5.** *Let $\mathfrak{h}_1(C_1, X)$ and $\mathfrak{h}_2(C_2, X)$ be soft Sheffer stroke BE-algebras of $(X, |, 0)$. If $\mathfrak{g}_1(B_1, X)$ and $\mathfrak{g}_2(B_2, X)$ are soft Sheffer stroke sub-BE-algebras of $\mathfrak{h}_1(C_1, X)$ and $\mathfrak{h}_2(C_2, X)$ respectively, then $\mathfrak{g}_1(B_1, X) \widetilde{\wedge} \mathfrak{g}_2(B_2, X)$ is a soft Sheffer stroke sub-BE-algebra of $\mathfrak{h}_1(C_1, X) \widetilde{\wedge} \mathfrak{h}_2(C_2, X)$.*

*Proof.* If $\mathfrak{h}_1(C_1, X)$ and $\mathfrak{h}_2(C_2, X)$ are soft Sheffer stroke BE-algebras of $(X, |, 0)$, then $\mathfrak{h}_1(C_1, X) \widetilde{\wedge} \mathfrak{h}_2(C_2, X) = \mathfrak{h}(C_1 \times C_2, X)$ is a soft Sheffer stroke BE-algebras of $(X, |, 0)$ where $\mathfrak{h}(\beta_1, \beta_2) = \mathfrak{h}_1(\beta_1) \cap \mathfrak{h}_2(\beta_2)$ for all $(\beta_1, \beta_2) \in C_1 \times C_2$ (see Theorem 3.14). Assume that $\mathfrak{g}_1(B_1, X)$ and $\mathfrak{g}_2(B_2, X)$ are soft Sheffer stroke sub-BE-algebras of $\mathfrak{h}_1(C_1, X)$ and $\mathfrak{h}_2(C_2, X)$ respectively. Then $B_i \subseteq C_i$ and $\mathfrak{g}_i(\alpha_i)$ is a Sheffer stroke BE-subalgebra of $\mathfrak{h}_i(\alpha_i)$ for $i = 1, 2$. We can say $\mathfrak{g}(B_1 \times B_2, X) = \mathfrak{g}_1(B_1, X) \widetilde{\wedge} \mathfrak{g}_2(B_2, X)$ by Definition 2.11 where $\mathfrak{g}(\alpha_1, \alpha_2) = \mathfrak{g}_1(\alpha_1) \cap \mathfrak{g}_2(\alpha_2)$ for all $(\alpha_1, \alpha_2) \in B_1 \times B_2$. It follows that $B_1 \times B_2 \subseteq C_1 \times C_2$ and $\mathfrak{g}(\alpha_1, \alpha_2)$ is a Sheffer stroke BE-subalgebra of $\mathfrak{h}(\alpha_1, \alpha_2)$ for all $(\alpha_1, \alpha_2) \in B_1 \times B_2$. Therefore,

$$\mathfrak{g}_1(B_1, X) \widetilde{\wedge} \mathfrak{g}_2(B_2, X) = \mathfrak{g}(B_1 \times B_2, X)$$

is a soft Sheffer stroke sub-BE-algebra of

$$\mathfrak{h}(C_1 \times C_2, X) = \mathfrak{h}_1(C_1, X) \widetilde{\wedge} \mathfrak{h}_2(C_2, X).$$

□

**Theorem 4.6.** *Let $f : X \to Y$ be a homomorphism of Sheffer stroke BE-algebras $(X, \uparrow_X, 1_X)$ and $(Y, \uparrow_Y, 1_Y)$, and let $\mathfrak{h}(C, X)$ be a soft Sheffer stroke BE-algebra of $(X, |, 0)$. If a soft set $\mathfrak{g}(B, X)$ is a soft Sheffer stroke sub-BE-algebra of $\mathfrak{h}(C, X)$, then $f(\mathfrak{g})(B, Y)$ is a soft Sheffer stroke sub-BE-algebra of $f(\mathfrak{h})(C, Y)$.*

*Proof.* If $\mathfrak{h}(C, X)$ is a soft Sheffer stroke BE-algebra of $(X, \uparrow_X, 1_X)$, then $f(\mathfrak{h})(C, Y)$ is a soft Sheffer stroke BE-algebra of $(Y, \uparrow_Y, 1_Y)$ by Theorem 3.19. Assume that $\mathfrak{g}(B, X) \widetilde{\leq}_S \mathfrak{h}(C, X)$. Then $B \subseteq C$ and $\mathfrak{g}(\alpha)$ is a Sheffer stroke BE-subalgebra of $\mathfrak{h}(\alpha)$ for all $\alpha \in B$. Let $\mathfrak{a}, \mathfrak{b} \in f(\mathfrak{g})(\alpha) = f(\mathfrak{g}(\alpha))$ for all $\mathfrak{a}, \mathfrak{b} \in f(\mathfrak{h})(\alpha) = f(\mathfrak{h}(\alpha))$ and $\alpha \in B$. Then there exist $x, y \in \mathfrak{g}(\alpha) \subseteq \mathfrak{h}(\alpha)$ such that $f(x) = \mathfrak{a}$ and $f(y) = \mathfrak{b}$. It follows that $x \uparrow_X (y \uparrow_X y) \in \mathfrak{g}(\alpha) \subseteq \mathfrak{h}(\alpha)$ and

$$\mathfrak{a} \uparrow_Y (\mathfrak{b} \uparrow_Y \mathfrak{b}) = f(x) \uparrow_Y (f(y) \uparrow_Y f(y)) = f(x \uparrow_X (y \uparrow_X y)) \in f(\mathfrak{g}(\alpha)),$$

Hence $f(\mathfrak{g}(\alpha)) = f(\mathfrak{g}(\alpha))$ is a Sheffer stroke BE-subalgebra of $f(\mathfrak{h}(\alpha)) = f(\mathfrak{h}(\alpha))$ for all $\alpha \in B$. Therefore, $f(\mathfrak{g})(B, Y)$ is a soft Sheffer stroke sub-BE-algebra of $f(\mathfrak{h})(C, Y)$. □

## 5. Conclusion

We investigated soft Sheffer stroke BE-algebras and sub-BE-algebras, exploring their properties, connections to fuzzy structures, and potential applications. Our findings unveil their usefulness in artificial intelligence, computer science, information science, quantum logic, and probability theory.

Key Contributions:

Introduced soft Sheffer stroke BE-algebras and established a correspondence with fuzzy Sheffer stroke BE-algebras under specific conditions.

Defined restriction within this structure and rigorously proved that intersections and unions of soft Sheffer stroke BE-algebras remain such.

Demonstrated that homomorphisms preserve the properties of soft Sheffer stroke BE-algebras.

Discussed nullification within this context and its behavior under homomorphisms.

Introduced soft Sheffer stroke sub-BE-algebras and showed that intersections and unions also form sub-BE-algebras. We further proved that these sub-BE-algebras retain their properties under homomorphisms.

Impact:

This research provides a strong foundation for researchers employing soft sets and fuzzy structures. The novel constructions and results advance the field and open doors for further exploration and application.

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# SHEFFER STROKE $BE-$ALGEBRAS BASED ON THE SOFT SET ENVIRONMENT

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# $BE-$جبرهای شفر استروک بر اساس مجموعه‌های نرم

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در این مقاله، ما مفهوم $BE-$جبرهای شفر استروک نرم را معرفی می‌کنیم که دیدگاه جدیدی در مورد خواص جبری آن‌ها در چارچوب نظری مجموعه‌های نرم ارائه می‌دهد. این جبرها رویکردی انعطاف پذیر و سازگار برای عملیات منطقی ارائه می‌دهند و امکان ادغام یکپارچه اطلاعات فازی و واضح را فراهم می‌کنند. علاوه بر این، ما به مفهوم زیر $BE-$جبرهای شفر استروک نرم می‌پردازیم و خواص این ساختارها را به طور کامل بررسی می‌کنیم. تجزیه و تحلیل ما، ارتباطات جالبی را بین مجموعه‌های نرم، عملگرهای شفر استروک و اصول ساختار $BE-$جبری آشکار می‌کند. نتایج ارائه شده در این مقاله، درک ما از ساختارهای جبری را که حول محور عملیات شفر استروک در حوزه مجموعه‌های نرم می‌چرخند، به طور قابل توجهی بهبود می‌بخشد. این تحقیق یک پایه ریاضی محکم را ایجاد می‌کند که نویدبخش کاربردهایی در تصمیم گیری، ادغام اطلاعات و استدلال تحت عدم قطعیت است.

کلمات کلیدی: $BE-$جبرهای شفر استروک، $BE-$جبرهای شفر استروک نرم، $BE-$زیرجبرهای شفر استروک نرم.