Journal of Algebraic Systems, Vol. 14(No. 1): (2026), pp 135-143.
https://doi.org/10.22044/JAS.2024.13252.1732

REMARKS ON GENERALIZED DERIVATIONS IN $*-$PRIME RINGS

R. M. Al-Omary

Abstract. Considering $(R, *)$ be a $*-$prime ring with characteristic does not equal two, a nonzero L be a square closed $*-$Lie ideal of R. The pair (F, d) be a generalized derivation where $F : R \longrightarrow R$ an additive mapping correlated with a derivation $d : R \longrightarrow R$ such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. The aim of current article be demonstrating that when L satisfies any of several identities within F, then L becomes central.

1. Introduction

Considering R be an associative ring with center $Z(R)$. At a beginning in current article it is appropriate to start with recalling some familiar concepts. A ring R is said to be n-torsion free if $nx = 0$ implies $x = 0$ for all $x \in R$. For each $x, y \in R$, the symbol $[x, y]$ will stands for the commutator that the difference of xy and yx and the symbol $x \circ y$ stands for the skew-commutator that the combined of xy and yx. A ring R is prime if $aRb = \{0\}$ implies that either $a = 0$ or $b = 0$. The concept involution on a ring R indicates to an additive mapping $x \mapsto x^*$ such that $(xy)^* = y^*x^*$ and $(x^*)^* = x$ hold for all $x, y \in R$. A left (resp. right, two sided) ideal I of R is called a left (resp. right, two sided) $*$-ideal if $I^* = I$. An ideal P of R is called $*$-prime ideal if $P(\neq R)$ is a $*$-ideal and for $*$-ideals I, J of R, $IJ \subseteq P$ implies that $I \subseteq P$ or $J \subseteq P$. An example of $*$-ideal: Let $\mathbb{Z}$ be the ring of integers. Let $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in \mathbb{Z} \right\}$. We define a map $* : R \longrightarrow R$ as follows: $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}^* = \begin{pmatrix} c & -b \\ 0 & a \end{pmatrix}$. It is easy to check that $I = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \mid b \in \mathbb{Z} \right\}$ is a $*-$ideal of R. Now we give an example of $*-$ prime ideal: Let F be any field and $R = F[x]$ be the polynomial ring over F. Let $* : R \longrightarrow R$ be a map defined by $(f(x))^* = f(-x)$ for all $f(x) \in R$. Then it is easy to check that xR is a $*-$prime ideal of R. It should be noted that an ideal I of R may be not a $*-$ideal: Let $\mathbb{Z}$ be the ring of integers and $R = \mathbb{Z} \times \mathbb{Z}$. As

Published online: 25 October 2025
MSC(2020): Primary: 16W25; Secondary: 17N60, 16W10.
Keywords: $*$-Lie ideal; Involution; $*$-Prime ring; Derivation and generalized derivation.
Received: 11 June 2023, Accepted: 26 April 2024.

an example of consider a mapping $* : R \longrightarrow R$ defined by $(a,b)^* = (b,a)$ for all $a,b \in R$. For an ideal $I = \mathbb{Z} \times \{0\}$ of R, I is not a $*-$ideal of R since $I^* = \{0\} \times \mathbb{Z} \neq I$. A ring R equipped with an involution $*$ is said to be a $*$-prime ring if for any $a,b \in R$, $aRb = aRb^* = \{0\}$ implies $a = 0$ or $b = 0$. Obviously, every prime ring equipped with involution $*$ is $*$-prime. The converse need not be true in general. Of course, if R° denotes the opposite ring of a prime ring R, then $R \times R^\circ$ equipped with the exchange involution $*_{ex}$, defined by $*_{ex}(x,y) = (y,x)$, is $*_{ex}-$prime but not prime. The set of symmetric and skew symmetric elements of R will be denoted by $S_*(R)$ i.e.; $S_*(R) = \{x \in R \mid x^* = \pm x\}$. Let R be a $*-$prime ring, $a \in R$ and $aRa = \{0\}$. This implies that $aRaRa^* = \{0\}$ also. Certainly $*-$primness of R insures that $a = 0$ or $aRa^* = \{0\}$. $aRa^* = \{0\}$ together with $aRa = \{0\}$ gives us $a = 0$. Thus we conclude that every $*-$prime ring is a semiprime.

A term Lie ideal L of R will be refer to an additive subgroup of R with property $[L,R] \subseteq L$. In case a nonzero $[L,L]$ then a Lie ideal L will be said to be non-commutative. It is well known that $[R[L,L]R,R] \subseteq L$ with considering L be a non-commutative Lie ideal of R (for the proof see [[8], Lemma 1.3]. Since L is a non-commutative Lie ideal, we have $0 \neq [I,R] \subseteq L$ for a nonzero ideal of L: $I = R[L,L]R$. A Lie ideal becomes a $*-$Lie ideal if $L^* = L$. A Lie (resp. $*-$Lie) ideal L of R is said to be a square closed Lie (resp. $*-$ Lie) ideal of R if $x^2 \in L$ for all $x \in L$.

A derivation d will be refers to an additive mapping $d : R \longrightarrow R$ satisfies that $d(xy) = d(x)y + xd(y)$ for all $x,y \in R$. In particular, for a fixed $a \in R$, the mapping $I_a : R \longrightarrow R$ given by $I_a(x) = [a,x]$ is a derivation which is said to be an inner derivation. A generalized inner derivation F will be refers to an additive mapping $F : R \longrightarrow R$ if satisfying $F(x) = ax + xb$ for fixed $a,b \in R$. For such a mapping F, it is easy to see that

$$F(xy) = F(x)y + x[y,b] = F(x)y + xI_b(y), \text{ for all } x,y \in R.$$

The prior observation leads to the following definition, an additive mapping F: $R \to R$ is said to be a generalized derivation associated with a derivation d if $F(xy) = F(x)y + xd(y)$ holds for all $x,y \in R$.

Derivations and generalized inner derivations be familiar examples of generalized derivations, also the latter includes left multipliers. Since the sum of two generalized derivations is a generalized derivation too, every mapping of the form $F(x) = cx + d(x)$, for a fixed element c of R and d is a derivation of R, is a generalized derivation; and in case R be a ring with unity (that is R has multiplicative identity 1), then all generalized derivations have this form.

The Second Theorem of Posner proved that if the nonzero derivation d on a prime ring R is a centralizing on R, then R is commutative [14]. Mayne generalized Posner's theorem when a ring R has an automorphism or a nonzero centralizing derivation on some ideal $U \neq 0$, concluding that R is commutative [10].

Over the last four decades, several authors have been demonstrated commutativity theorems for prime rings of R and the behavior of special mappings on that ring (see, [3], [6] and [11]). Recently, some well-known results concerning prime rings have been proved for $*-$prime rings (see, [1], [2], [5], [4], [7] and [[13] - [15]], for partial bibliography). In the year 2015, Rehman et al. [16], discussed when $L \subseteq Z(R)$ such that R is a $*-$prime ring with generalized derivations (F, d) and (G, g) satisfying several conditions. Enthusiastic by the above results, we shall discuss when L is central such that R is a $*-$prime ring in which the generalized derivation (F, d) and derivation g satisfies any one of the properties: (1) $[g(x), F(y)] = [x, y]$, (2) $[g(x), F(y)] = x \circ y$, (3) $[F(x), d(y)] = F(x) \circ d(y)$, (4) $F^2(x) = [F(x), x]$, (5) $F[x, y] = [F(x), y] + [d(y), x]$, (6) $F(x \circ y) = F(x) \circ y - d(y) \circ x$, (7) $F(x)[x, y] = [F(x), y]$, (8) $F(x)[x, y] = F(x) \circ y$, for all $x, y \in L$.

2. Preliminary results

We shall use without explicit mention the following basic identities to make access easier to the proofs of our theorems, that hold for any $x, y, z \in R$:

(1) $[xy, z] = x[y, z] + [x, z]y$
(2) $[x, yz] = y[x, z] + [x, y]z$
(3) $x \circ (yz) = (x \circ y)z - y[x, z] = y(x \circ z) + [x, y]z$
(4) $(xy) \circ z = x(y \circ z) - [x, z]y = (x \circ z)y + x[y, z]$

We begin our discussion with the following Facts which are essential for developing the proof of our main results. For the proof of Fact A can be seen in [8] while that of Facts $B-$ E can be found in [16].

Fact A In any 2-torsion free semiprime ring R, and for any Lie ideal L satisfying $[L, L] = 0$ must be central.

Fact B In any 2-torsion free semiprime ring R, and for any non-central Lie ideal L, the subring $< L >$ generated by L contains the nonzero ideal $I = R[L, L]R$. Thus, when L is $*-$invariant then I is a nonzero $*-$ideal. In this case we will simply write $I \subseteq < L >$.

Fact C In any $*-$prime ring R, any nonzero $*-$ideal I has no left (right) annihilator and $[r, I] = 0$ implies that $r \in Z(R)$.

Fact D For R semiprime and L a Lie ideal, $aLb = 0$ implies that $a < L > b = 0$. Thus, for L a non-central $*-$Lie ideal is in a $*-$prime ring R with $char(R) \neq 2$, $aIb = 0$, where I be an ideal of R.

Fact E Every $*-$prime ring R is semiprime.

3. Main Results

In the context of this paper, the pair (F, d) will be representing to a generalized derivation associated with a derivation d.

Theorem 3.1. *Consider (F, d) be a nonzero generalized derivation and a derivation g on a $*$-prime ring R with characteristic not two and L be a nonzero square closed $*$-Lie ideal of R, such that*

(1) *$[g(x), F(y)] = [x, y]$ for all $x, y \in L$, or*
(2) *$[g(x), F(y)] = x \circ y$ for all $x, y \in L$,*

then either $d = 0$ or $L \subseteq Z(R)$.

Proof. (1) Suppose that L is not central. We have

$$[g(x), F(y)] = [x, y] \text{ for all } x, y \in L. \tag{3.1}$$

For any $z \in L$, replacing y by $2yz$ in (3.1), using it with the fact that $char(R) \neq 2$, we get

$$F(y)[g(x), z] + y[g(x), d(z)] + [g(x), y]d(z) = y[x, z]. \tag{3.2}$$

Again, replacing z by $2zg(x)$ in (3.2) and using it, we get

$$yz[g(x), d(g(x))] + y[g(x), z]d(g(x)) + [g(x), y]zd(g(x)) = yz[x, g(x)]. \tag{3.3}$$

Now, replacing y by $2my$ in (3.3) and using it, we get

$$[g(x), m]yzd(g(x)) = 0 \text{ for all } x, y, z, m \in L. \tag{3.4}$$

That is, $[g(x), L]L^2d(g(x)) = 0$ for all $x \in L$. But L is not central, using Fact D, we get $[g(x), L]L = 0$ or $d(g(x)) = 0$ for all $x \in L$. Since L is not central so if $[g(x), L]L = 0$, then $g(x) \in Z(R)$ by Facts B and C. Let $U = \{x \in L \mid g(x) \in Z(R)\}$ and $V = \{x \in L \mid d(g(x)) = 0\}$. Thus, U and V are additive subgroups of L such that $U \cup V = L$, so the Brauer's trick shows that either $U = L$ or $V = L$. If $U = L$, then $g(L)$ is central, hence by (3.1), we get $[L, L] = 0$, so $L \subseteq Z(R)$ by Fact A, this is a contradiction with our initial hypothesis. On the other hand, if $V = L$, then $d(g(x)) = 0$ for all $x \in L$. Replacing x by $2xg(y)$ in the last equation and using it, we get

$d(x)g^2(y) + xd(g^2(y)) = 0$ for all $x, y \in L$. Again, replacing x by $2zx$ in the last equation and using it, we get

$$d(z)Lg^2(y) = 0 \text{ for all } z, y \in L. \tag{3.5}$$

Since L is not central, then the Fact D forces that either $d(z) = 0$ or $g^2(y) = 0$. Let $U_1 = \{z \in L \mid d(z) = 0\}$ and $V_1 = \{y \in L \mid g^2(y) = 0\}$. Thus U_1 and V_1 are additive subgroups of L such that $U_1 \cup V_1 = L$, so the Brauer's trick shows that either $U_1 = L$ or $V_1 = L$. If $U_1 = L$, then $d(L) = 0$ which means $d(< L >) = 0$, and hence $d(I) = 0$, therefore $d(R) = 0$. On the other hand, if $V_1 = L$, then $g^2(y) = 0$ for all $y \in L$. Replacing y by $2xy$ in the last equation and using it, we get $g^2(2xy) = 4g(x)g(y) = 0$ using the fact that $char(R) \neq 2$, we get $g(x)g(y) = 0$ for all $x, y \in L$. Again, replacing y by $2yx$ in the last equation and using it, we get $g(x)Lg(x) = 0$ for all $x \in L$, again Fact D gives us $g(x) = 0$ for all $x \in L$. Hence (3.1), leads to $[L, L] = 0$, therefore by Fact A, we find that $L \subseteq Z(R)$. Once again we face a contradiction.

(2) Suppose that L is not central. We have

$$[g(x), F(y)] = x \circ y \text{ for all } x, y \in L. \tag{3.6}$$

For any $z \in L$, replacing y by $2yz$ in (3.6), using it and the fact $char(R) \neq 2$, we get

$$F(y)[g(x), z] + y[g(x), d(z)] + [g(x), y]d(z) = -y[x, z]. \tag{3.7}$$

Again, replacing z by $2zg(x)$ in (3.7) and using it, we get

$$yz[g(x), d(g(x))] + y[g(x), z]d(g(x)) + [g(x), y]zd(g(x)) = -yz[x, g(x)]. \tag{3.8}$$

Now, replacing y by $2my$ in (3.8) and using it, we get

$$[g(x), m]yzd(g(x)) = 0 \text{ for all } x, y, z, m \in L. \tag{3.9}$$

Using the same arguments and techniques as after Equation (3.4), we obtain the desired conclusion.

□

Theorem 3.2. *Consider (F, d) be a nonzero generalized derivation on a $*$-prime ring R with characteristic not two and L be a nonzero square closed $*$-Lie ideal of R such that $[F(x), d(y)] = F(x) \circ d(y)$ for all $x, y \in L$, then either $d = 0$ or $L \subseteq Z(R)$.*

Proof. Assume that L is not central. We have

$$[F(x), d(y)] = F(x) \circ d(y) \text{ for all } x, y \in L. \tag{3.10}$$

This can be rewritten as $F(x)d(y) - d(y)F(x) = F(x)d(y) + d(y)F(x)$ for all $x, y \in L$. Since $char(R) \neq 2$, then the previous equation becomes $d(y)F(x) = 0$ for any $x, y \in L$. Replacing x by $2xy$ in the last expression and using it, we get $d(y)Ld(y) = 0$ for all $y \in L$, so Fact D gives $d(y) = 0$ for all $y \in L$. Hence, $d = 0$. □

Theorem 3.3. *Consider (F, d) be a nonzero generalized derivation on a $*$-prime ring R with characteristic not two and L be a nonzero square closed $*$-Lie ideal of R such that $F^2(x) = [F(x), x]$ for all $x \in L$, then either $d = 0$ or $L \subseteq Z(R)$.*

Proof. Assume that L is not central. We have $F^2(x) = [F(x), x]$ for all $x \in L$. By linearizing the last expression, we find

$$[F(x), y] + [F(y), x] = 0 \text{ for all } x, y \in L. \tag{3.11}$$

Replacing, y by $2yx$ in (3.11) and using it with the fact that $char(R) \neq 2$, we get

$$y[d(x), y] + [y, x]d(x) + y[F(x), x] = 0. \tag{3.12}$$

Again, replacing y by $2zy$ in (3.12) and using it with the fact that $char(R) \neq 2$, we get

$$[z, x]Ld(x) = 0 \text{ for all } x, z \in L. \tag{3.13}$$

Since L is not central, $[z, x]Id(x) = 0$ for all $x, z \in L$, then by Fact D either $[z, x] = 0$ or $d(x) = 0$. Let $U = \{x \in L \mid [z, x] = 0 \text{ for all } z \in L\}$ and $V = \{x \in L \mid d(x) = 0\}$. Thus, U and V are additive subgroups of L and $U \cup V = L$. But $(L, +)$ is not a union of proper subgroups. Therefore, we have either $U = L$ or $V = L$. If $U = L$, then $[z, x] = 0$ for all $z, x \in L$, that is $[L, L] = 0$. Hence by Fact A, we get a contradiction. On the other hand, if $V = L$, then $d(L) = 0$ and hence $d(< L >) = 0$ which implies that $d(I) = 0$, i.e., $d(R) = 0$. □

Theorem 3.4. *Consider (F, d) be a nonzero generalized derivation on a $*$-prime ring with characteristic not two and L be a nonzero square closed $*$-Lie ideal of R such that*

1. $F[x, y] = [F(x), y] + [d(y), x]$, *for all* $x, y \in L$, *or*
2. $F(x \circ y) = F(x) \circ y - d(y) \circ x$, *for all* $x, y \in L$, *or*
3. $F(x)[x, y] = [F(x), y]$, *for all* $x, y \in L$, *or*
4. $F(x)[x, y] = F(x) \circ y$, *for all* $x, y \in L$.

If $d \neq 0$, then $L \subseteq Z(R)$.

Proof. (1) By our hypothesis, we have

$$F[x,y]=[F(x),y]+[d(y),x] \text{ for all } x,y\in L. \tag{3.14}$$

Replacing, y by $2yx$ in (3.14) and using it, we get

$$2[x,y]d(x)=y[F(x),x]+y[d(x),x] \text{ for all } x,y\in L. \tag{3.15}$$

Again, replacing, y by $2zy$ in (3.15) and using the fact that $char(R)\neq 2$, we get $[x,z]Ld(x)=0$ for all $x,z\in L$. This is identical with the relation (3.13). Arguing as in above, we get the required result.

(2) It is given that F is a generalized derivation of R such that

$$F(x\circ y)=F(x)\circ y-d(y)\circ x \text{ for all } x,y\in L. \tag{3.16}$$

Replacing, y by $2yx$ in (3.16) and using it, we get

$$(x\circ y)d(x)=-y[F(x),x]-y(d(x)\circ x)+[y,x]d(x) \text{ for all } x,y\in L. \tag{3.17}$$

Again, replacing, y by $2zy$ in (3.17) and using it with the fact that $char(R)\neq 2$, we get $[x,z]Ld(x)=0$ for all $x,z\in L$. The last expression is the same as (3.13) and thus we obtain the desired conclusion by repeating the same arguments and techniques.

(3) It is given that F is a generalized derivation of R such that $F(x)[x,y]=[F(x),y]$ for all $x,y\in L$. This can be rewrite as

$$F(x)xy-F(x)yx=F(x)y-yF(x) \text{ for all } x,y\in L. \tag{3.18}$$

Replacing y by $2yx$ in (3.18) and using the fact that $char(R)\neq 2$, we get

$$F(x)xyx-F(x)yxx=F(x)yx-yxF(x) \text{ for all } x,y\in L. \tag{3.19}$$

Multiplying (3.18) by x from the right and comparing it with (3.19), we find $yxF(x)+yF(x)x=0$ for all $x,y\in L$ so, $y(F(x)\circ x)=0$ i.e., $L(F(x)\circ x)=0$ for all $x\in L$ and hence $F(x)\circ x=0$ for all $x\in L$. Linearizing the last relation, we get $F(x)\circ y+F(y)\circ x=0$ for all $x,y\in L$. Now, replacing y by $2yx$ and using the fact that $char(R)\neq 2$, we get

$$-y[F(x),x]+(y\circ x)d(x)+y[d(x),x]=0.$$

Again, replace y by $2zy$ and use the fact that $char(R)\neq 2$, to get $[z,x]Ld(x)=0$ for all $x,z\in L$. Notice that the arguments given after equation (3.13) are still valid in the present situation and hence repeating the same process, we get the required result.

(4) It is given that F is a generalized derivation of R such that $F(x)[x,y]=F(x)\circ y$ for all $x,y\in L$. This can be rewrite as

$$F(x)xy-F(x)yx=F(x)y+yF(x) \text{ for all } x,y\in L. \tag{3.20}$$

Replacing y by $2yx$ in (3.20) and using the fact that $char(R) \neq 2$, we get

$$F(x)xyx - F(x)yxx = F(x)yx + yxF(x) \text{ for all } x, y \in L. \tag{3.21}$$

Multiplying (3.20) by x from the right and comparing it with (3.21), we find $yxF(x) - yF(x)x = 0$ for all $x, y \in L$ so, $y[F(x), x] = 0$ i.e., $L[F(x), x] = 0$ for all $x \in L$ and hence $[F(x), x] = 0$ for all $x \in L$. Linearizing the last relation, we get $[F(x), y] + [F(y), x] = 0$ for all $x, y \in L$. Now, replacing y by $2yx$ and using the fact that $char(R) \neq 2$, we get $y[d(x), x] + [y, x]d(x) = 0$. Again, replace y by $2zy$ and using the fact that $char(R) \neq 2$, to get $[z, x]Ld(x) = 0$ for all $x, z \in L$. Notice that the arguments given after equation (3.13) are still valid in the present situation and hence repeating the same process, we get the required result. □

Example 3.5. Let $\mathbb{Z}$ be the ring of integers and let

$$R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in \mathbb{Z} \right\} \text{ and } L = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \mid b \in \mathbb{Z} \right\}.$$

Define

$$* : R \to R \text{ by } \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}^* = \begin{pmatrix} c & -b \\ 0 & a \end{pmatrix},$$

$$d : R \to R \text{ by } d\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & -b \\ 0 & 0 \end{pmatrix},$$

$$g : R \to R \text{ by } g\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix},$$

and

$$F : R \to R \text{ by } F\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}.$$

Then R is a ring under usual operations with involution $*$, L is a square closed $*$-Lie ideal, and it is easy to see that (F, d) is a generalized derivation of R and g is derivations of R satisfying any one of the following properties:

(1) $[g(x), F(y)] = [x, y]$,
(2) $[g(x), F(y)] = x \circ y$,
(3) $[F(x), d(y)] = F(x) \circ d(y)$,
(4) $F^2(x) = [F(x), x]$,
(5) $F[x, y] = [F(x), y] + [d(y), x]$,
(6) $F(x \circ y) = F(x) \circ y - d(y) \circ x$,
(7) $F(x)[x, y] = [F(x), y]$
and
(8) $F(x)[x, y] = F(x) \circ y$, for all $x, y \in L$, but L is not central.

Hence, in Theorems 3.1, 3.2, 3.3 and 3.4 the hypothesis of primness cannot be omitted.

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Radwan Mohammed Al-Omary
Department of Mathematics, IBB University, IBB, Yemen.
Email: raradwan959@gmail.com

Journal of Algebraic Systems

REMARKS ON GENERALIZED DERIVATIONS IN $*$–PRIME RINGS

R. M. AL–OMARY

نکاتی دربارەی مشتق‌های تعمیم یافته در حلقه‌های $*$–اول

آر. ام. العمری

گروه ریاضی، دانشگاه اب، اب، یمن

فرض کنید $(R, *)$ یک حلقەی $*$–اول با مشخصەی مخالف ۲ باشد و L یک ایده‌آل $*$–لی غیرصفر و بسته نسبت به توان دوم در R باشد. زوج (F, d) یک مشتق تعمیم یافته است که $F : R \longrightarrow R$ یک نگاشت جمعی است و با مشتق $d : R \longrightarrow R$ مرتبط می‌باشد، به طوری که برای هر $x, y \in R$ داریم $F(xy) = F(x)y + xd(y)$. هدف این مقاله این است که نشان دهد هرگاه L در هر کدام از همانی‌های مرتبط با F صدق کند، آن‌گاه مرکزی است.

کلمات کلیدی: ایده‌آل $*$–لی، تقابل، حلقەی $*$–اول، مشتق و مشتق تعمیم یافته.