Journal of Algebraic Systems, Vol. 14(No. 1): (2026), pp 145-155.
https://doi.org/10.22044/JAS.2024.13556.1754

ON α-SHORT TYPE MODULES

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Abstract. We introduce and study the concept of α-short type modules. Using this concept we extend some of the basic results of α-short modules to α-short type modules. We observe that if M is an α-short type module, then the Noetherian dimension of M is less than or equal to $\omega_1 + \alpha + 1$, where ω_1 is the first uncountable ordinal number.

1. Introduction

Lemonnier [18] has introduced the concept of deviation (resp., codeviation) of an arbitrary poset, which in particular, when applied to the lattice of all submodules of a module M_R give the concept of Krull dimension, see [9], [10] and [20] (resp., the concept of dual Krull dimension of M. The dual Krull dimension in [7], [8], [11], [12], [13], [14], [15], [16] and [17] is called Noetherian dimension and in [5] is called N-dimension. This dimension is called Krull dimension in [21]. The name of dual Krull dimension is also used by some authors, see [1], [2] and [3]). The Noetherian dimension of an R-module M is denoted by $n\text{-}\dim M$ and by $k\text{-}\dim M$ we denote the Krull dimension of M. We recall that if an R-module M has Noetherian dimension and α is an ordinal number, then M is called α-atomic if $n\text{-}\dim M = \alpha$ and $n\text{-}\dim N < \alpha$, for all proper submodule N of M. An R-module M is called atomic if it is α-atomic for some ordinal α (note, atomic modules are also called conotable, dual critical and N-critical in some other articles; see for example [2], [5] and [19]). We introduced and extensively investigated uncountably generated Krull dimension and uncountably generated Noetherian dimension of an R-module M, see [6]. The uncountably generated Noetherian dimension (resp., uncountably generated Krull dimension), which is denoted by $ucn\text{-}\dim M$ (resp., $uck\text{-}\dim M$) is defined to be the codeviation (resp., deviation) of the poset of the uncountably generated submodules of M. We recall that an R-module M is called α-atomic type, where α is an ordinal, if

Published online: 25 October 2025
MSC(2020): Primary: 16P60; Secondary: 16P20, 16P40.
Keywords: α-Short modules; α-Almost Noetherian modules; α-Short type modules; α-Almost Noetherian type modules; Uncountably generated Noetherian dimension.
Received: 1 September 2023, Accepted: 26 April 2024.

ucn-dim $M = \alpha$ and ucn-dim $N < \alpha$ for any proper uncountably generated submodule N of M. M is said to be atomic type if it is α-atomic type for some α. Bilhan and Smith have introduced and extensively investigated short modules and almost Noetherian modules, see [4]. Later Davoudian, Karamzadeh and Shirali undertook a systematic study of the concepts of α-short modules and α-almost Noetherian modules, see [8]. We recall that an R-module M is called an α-short module, if for each submodule N of M, either n-dim $N \leq \alpha$ or n-dim $\frac{M}{N} \leq \alpha$ and α is the least ordinal number with this property. We shall call an R-module M to be α-short type, if for each uncountably generated submodule N of M, either ucn-dim $N \leq \alpha$ or ucn-dim $\frac{M}{N} \leq \alpha$ and α is the least ordinal number with this property. Using this concept, we show that each α-short type module M has Noetherian dimension and $\alpha \leq n$-dim $M \leq \omega_1 + \alpha + 1$. We also recall that an R-module M is called α-almost Noetherian, if for each proper submodule N of M, n-dim $N < \alpha$ and α is the least ordinal number with this property, see [8]. We shall call an R-module M to be α-almost Noetherian type if for each proper uncountably generated submodule N of M, ucn-dim $N < \alpha$ and α is the least ordinal number with this property. In section 2, of this paper we investigate some basic properties of α-almost Noetherian type and α-short type modules. We show that if M is an α-short type module (resp., α-almost Noetherian type module), then ucn-dim $M = \alpha$ or ucn-dim $M = \alpha + 1$ (resp., ucn-dim $M \leq \alpha$). Thus we observe that if M is an α-short type module, then M has Noetherian dimension and $\alpha \leq n$-dim $M \leq \omega_1 + \alpha + 1$. In the last section we also investigate some properties of α-almost Noetherian type and α-short type modules.

2. α-short type modules and α-almost quasi Noethrian modules

First we recall the following definition from [6].

Definition 2.1. Let M be an R-module. The uncountably generated Noetherian dimension of M (briefly, uc-Noetherian dimension), denoted by ucn-dim M is defined by transfinite recursion as follows: If M does not have any uncountably generated submodule, then ucn-dim $M = -1$. If α is an ordinal number and ucn-dim $M \not< \alpha$, then ucn-dim $M = \alpha$ provided that there is no infinite ascending chain of uncountably generated submodules of M such as $M_0 \subseteq M_1 \subseteq M_2 \subseteq \dots$ such that for each $i \geq 1$ we have ucn-dim $\frac{M_{i+1}}{M_i} \not< \alpha$. In otherwise ucn-dim $M = \alpha$, if ucn-dim $\not< \alpha$ and for each infinite ascending chain of uncountably generated submodules of M such as $M_0 \subseteq M_1 \subseteq M_2 \subseteq \dots$ there exists an integer t, such that for each $i \geq t$, we have $ucn\text{-}\dim \frac{M_{i+1}}{M_i} < \alpha$. A ring R has uncountably generated Noetherian dimension, if as an R-module has uncountably generated Noetherian dimension. It is possible that there is no ordinal α such that $ucn\text{-}\dim M = \alpha$, in this case we say M does not have uncountably generated Noetherian dimension.

We recall that an R-module M is called α-almost Noetherian, if for each proper submodule N of M, $n\text{-}\dim N < \alpha$ and α is the least ordinal number with this property. In the following definition we consider a related concept.

Definition 2.2. An R-module M is called α-almost Noetherian type if for each proper uncountably generated submodule N of M, $ucn\text{-}\dim N < \alpha$ and α is the least ordinal number with this property.

It is manifest that if M is an α-almost Noetherian type, then each submodule and each factor module of M is β-almost Noetherian type for some ordinal number $\beta \leq \alpha$ (note, see [6, Lemmas 3.4, 3.5]).

In view of [6, Lemma 3.7], we have the next three trivial, but useful facts.

Lemma 2.3. *If M is an α-almost Noetherian type module, then M has uncountably generated Noetherian dimension and $ucn\text{-}\dim M \leq \alpha$. In particular, $ucn\text{-}\dim M = \alpha$ if and only if M is α-atomic type.*

Lemma 2.4. *If M is a module with $ucn\text{-}\dim M = \alpha$, then either M is α-atomic type, in which case it is α-almost Noetherian type, or it is $\alpha+1$-almost Noetherian type.*

Lemma 2.5. *If M is an α-almost Noetherian type module, then either M is α-atomic type or $\alpha = ucn\text{-}\dim M + 1$. In particular, if M is α-almost Noetherian type module, where α is a limit ordinal, then M is α-atomic type.*

Proposition 2.6. *An R-module M has uncountably generated Noetherian dimension if and only if M is α-almost Noetherian type for some ordinal α.*

In view of Lemma 2.3 and [6, Theorem 3.11], we have the following result.

Corollary 2.7. *Let M be a quotient finite dimensional module. If R-module M is α-almost Noetherian type, then M has Noetherian dimension and $n\text{-}\dim M \leq \omega_1 + \alpha$, where ω_1 is the first uncountable ordinal number.*

Next we give our definition of α-short type modules.

Definition 2.8. An R-module M is called α-short type, if for each uncountably generated submodule N of M, either $ucn\text{-}\dim N \leq \alpha$ or $ucn\text{-}\dim \frac{M}{N} \leq \alpha$ and α is the least ordinal number with this property.

In view of [6, Corollary 3.6], we have the following results.

Remark 2.9. If M is an R-module with $ucn\text{-}\dim M = \alpha$, then M is β-short type for some ordinal number $\beta \leq \alpha$.

The following example shows that the concepts of α-short type module and α-short module are different.

Example 2.10. In view of the last part of [8, Section 1], for each ordinal number α there exists an α-short module. Let α be a countable ordinal number and M be an α-short module. In view of [8, Proposition 1.12], we infer that $n\text{-}\dim M = \alpha$ or $n\text{-}\dim M = \alpha + 1$. Hence $ucn\text{-}\dim M = 0$, see [6]. This implies that M is -1-short type, but it is α-short.

Remark 2.11. If M is an α-short type module, then each submodule and each factor module of M is β-short type for some ordinal number $\beta \leq \alpha$.

We cite the following result from [6, Lemma 3.9].

Lemma 2.12. *If M is an R-module and for each uncountably generated submodule N of M, either N or $\frac{M}{N}$ has uncountably generated Noetherian dimension, then so does M.*

The previous result and Remark 2.9, immediately yield the next result.

Corollary 2.13. *Let M be an α-short type module. Then M has uncountably generated Noetherian dimension and $\alpha \leq ucn\text{-}\dim M$.*

The following is now immediate.

Proposition 2.14. *An R-module M has uncountably generated Noetherian dimension if and only if M is α-short type for some ordinal α.*

Proposition 2.15. *If M is an α-short type R-module, then either $ucn\text{-}\dim M = \alpha$ or $ucn\text{-}\dim M = \alpha + 1$.*

Proof. Clearly in view of Corollary 2.13, we have $ucn\text{-}\dim M \geq \alpha$. If $ucn\text{-}\dim M \neq \alpha$, then $ucn\text{-}\dim M \geq \alpha + 1$. Now, let $M_1 \subseteq M_2 \subseteq \dots$ be any ascending chain of uncountably generated submodules of M. If there exists some k such that $ucn\text{-}\dim \frac{M}{M_k} \leq \alpha$, then

$$ucn\text{-}\dim \frac{M_{i+1}}{M_i} \leq ucn\text{-}\dim \frac{M}{M_i} = ucn\text{-}\dim \frac{M/M_k}{M_i/M_k} \leq ucn\text{-}\dim \frac{M}{M_k} \leq \alpha$$

for each $i \geq k$, see [6, Lemma 3.6]. Otherwise $ucn\text{-}\dim M_i \leq \alpha$ (M is α-short type) for each i, hence $ucn\text{-}\dim \frac{M_{i+1}}{M_i} \leq ucn\text{-}\dim M_{i+1} \leq \alpha$ for each i. Thus in any case there exists an integer k such that for each $i \geq k$, $ucn\text{-}\dim \frac{M_{i+1}}{M_i} \leq \alpha$. This shows that $ucn\text{-}\dim M \leq \alpha + 1$, i.e., $ucn\text{-}\dim M = \alpha + 1$. $\square$

In view of Proposition 2.15 and [6, Corollary 3.10], we have the following result.

Proposition 2.16. *Let M be an α-short type module. If N is a submodule of M, then G-dim $\frac{M}{N} \leq c$, where c is the first uncountable cardinal number, moreover if G-dim $\frac{M}{N} = c$, then c is not attained in M.*

In view of the previous proposition and [6, Theorem 3.11] we have the following result.

Corollary 2.17. *Let M be a quotient finite dimensioal module. If M is an α-short type R-module, then $\alpha \leq n$-dim $M \leq \omega_1 + \alpha + 1$.*

Remark 2.18. An R-module M is -1-short type if and only if it has countable Noetherian dimension or it is $\omega_1 - atomic$.

Proposition 2.19. *Let M be an R-module, with ucn-dim $M = \alpha$, where α is a limit ordinal. Then M is α-short type.*

Proof. We know that M is β-short type for some $\beta \leq \alpha$. If $\beta < \alpha$, then by Proposition 2.15, ucn-dim $M \leq \beta + 1 < \alpha$, which is a contradiction. Thus M is α-short type. □

Proposition 2.20. *Let M be an R-module and ucn-dim $M = \alpha = \beta + 1$. Then M is either α-short type or it is β-short type.*

Proof. We know that M is γ-short type for some $\gamma \leq \alpha$. If $\gamma < \beta$, then by Proposition 2.15, we have ucn-dim $M \leq \gamma + 1 < \beta + 1$, which is impossible. Hence we are done. □

Proposition 2.21. *Let M be an α-atomic type R-module, where $\alpha = \beta + 1$. Then M is a β-short type module.*

Proof. Let N be an uncountably generated submodule of M, therefore ucn-dim $N < \alpha$. This shows that for some $\beta' \leq \beta$, M is β'-short type. If $\beta' < \beta$, then $\beta' + 1 \leq \beta < \alpha$. But ucn-dim $M \leq \beta' + 1 \leq \beta < \alpha$, by Proposition 2.15, which is a contradiction. Thus $\beta' = \beta$ and we are done. □

The following remark, which is a trivial consequence of the previous fact, shows that the converse of Proposition 2.19, is not true in general.

Remark 2.22. Let M be an $\alpha + 1$-atomic type R-module, where α is a limit ordinal. Then M is an α-short type module but ucn-dim $M \neq \alpha$.

Proposition 2.23. *Let M be an R-module with ucn-*$\dim M = \alpha + 1$*. Then M is either α-short type R-module or there exists an uncountably generated submodule N of M such that ucn-*$\dim N =$ *ucn-*$\dim \frac{M}{N} = \alpha + 1$*.*

Proof. We know that M is α-short type or an $\alpha+1$-short type R-module, by Proposition 2.20. Let us assume that M is not α-short type R-module, hence there exists an uncountably generated submodule N of M such that *ucn*-$\dim N \geq \alpha + 1$ and *ucn*-$\dim \frac{M}{N} \geq \alpha + 1$. This shows that *ucn*-$\dim N = \alpha + 1$ and *ucn*-$\dim \frac{M}{N} = \alpha + 1$ and we are through. □

Proposition 2.24. *Let M be a non-zero α-short type R-module. Then either M is β-almost Noetherian type for some ordinal $\beta \leq \alpha + 1$ or there exists an uncountably generated submodule N of M with ucn-*$\dim \frac{M}{N} \leq \alpha$*.*

Proof. Suppose that M is not β-almost Noetherian type for any $\beta \leq \alpha + 1$. This means that there must exist an uncountably generated submodule N of M such that Inasmuch as M is α-short type, we infer that *ucn*-$\dim \frac{M}{N} \leq \alpha$ and we are done. □

Let us cite the next result which is in [15, Theotem 2.9], see also [11, Theorem 3.2].

Theorem 2.25. *For a commutative ring R the following statements are equivalent.*

1. *Every R-module with finite Noetherian dimension is Noetherian.*
2. *Every Artinian R-module is Noetherian.*
3. *Every R-module with Noetherian dimension is both Artinian and Noetherian.*

In view [8, Proposition 2.21], Corollary 2.17 and Corollary 2.7, we have the following result.

Proposition 2.26. *The following statements are equivalent for a commutative ring R.*

1. *Every Artinian R-module is Noetherian.*
2. *Every quotient finite dimensional m-short module M is both Artinian and Noetherian for all integers $m \geq -1$.*
3. *Every quotient finite dimensional α-short module M is both Artinian and Noetherian for all ordinal α.*
4. *Every quotient finite dimensional m-almost Noetherian module M is both Artinian and Noetherian for all integers $m \geq -1$.*

(5) *Every quotient finite dimensional α-almost Noetherian module M is both Artinian and Noetherian for all integers $m \geq -1$.*
(6) *Every quotient finite dimensional m-short type module M is both Artinian and Noetherian for all integers $m \geq -1$.*
(7) *Every quotient finite dimensional α-short type module M is both Artinian and Noetherian for all ordinal α.*
(8) *Every quotient finite dimensional m-almost Noetherian type module M is both Artinian and Noetherian for all integers $m \geq -1$.*
(9) *Every quotient finite dimensional α-almost Noetherian type module M is both Artinian and Noetherian for all ordinal α.*
(10) *No homomorphic image of R can be isomorphic to a dense subring of a complete local domain of Krull dimension 1.*

3. Properties of α-short type modules and α-almost Noetherian type modules

In this section some properties of α-short type modules over an arbitrary ring R are investigated.

Remark 3.1. Let M be a quotient finite dimensional module and N be a submodule of M such that $ucn\text{-}\dim N = \alpha$ and $ucn\text{-}\dim \frac{M}{N} = \beta$. If $\sup\{ucn\text{-}\dim N, ucn\text{-}\dim \frac{M}{N}\} = \gamma$, then $\gamma \leq ucn\text{-}\dim M \leq \omega_1 + \gamma$, where ω_1 is the first uncountable ordinal number.

Proof. We know that $n\text{-}\dim N \leq \omega_1 + \alpha$ and $n\text{-}\dim \frac{M}{N} \leq \omega_1 + \beta$, see [6, Theorem 3.11]. Therefore $n\text{-}\dim M = \sup\{n\text{-}\dim N, n\text{-}\dim \frac{M}{N}\} \leq \omega_1 + \gamma$. But by [6, Remark 3.2], we get $ucn\text{-}\dim M \leq n\text{-}\dim M$. In view of [6, Corollary 3.6], we get $\gamma \leq ucn\text{-}\dim M$. This implies that $\gamma \leq ucn\text{-}\dim M \leq \omega_1 + \gamma$ and we are done. □

In the following two propositions, we investigate the connection between α-short modules and α-short type modules.

Proposition 3.2. *Let M be an α-short R-module. Then M is a β-short type module and $\alpha \leq \omega_1 + \beta + 1$.*

Proof. Let N be any uncountably generated submodule of M, then $ucn\text{-}\dim N \leq n\text{-}\dim N \leq \alpha$ or $ucn\text{-}\dim \frac{M}{N} \leq n\text{-}\dim \frac{M}{N} \leq \alpha$, see [6, Remark 3.2]. This implies that M is β-short type for some $\beta \leq \alpha$. If M is β-short type, then $ucn\text{-}\dim M = \beta$ or $ucn\text{-}\dim M = \beta + 1$. Hence

$$\beta \leq n\text{-}\dim M \leq \omega_1 + \beta + 1,$$

see [6, Theorem 3.11]. In other hand by [8, Proposition 1.12], we get $\alpha \leq n\text{-dim}\, M \leq \alpha + 1$. Therefore $\alpha \leq \omega_1 + \beta + 1$. □

Proposition 3.3. *Let M be a quotient finite dimensional module. If M be a β-short type R-module, then M is an α-short R-module and $\alpha \leq \omega_1 + \beta + 1$.*

Proof. By Proposition 2.15, $ucn\text{-dim}\, M = \beta$ or $ucn\text{-dim}\, M = \beta + 1$. This implies that M has Noetherian dimension and $\beta \leq n\text{-dim}\, M \leq \omega_1 + \beta + 1$, see [6, Theorem 3.11]. Thus M is α-short for some ordinal number $\alpha \leq \omega_1 + \beta + 1$, see [8, Remark 1.2.]. □

We note that the $\mathbb{Z}$-module $\mathbb{Z} \oplus \mathbb{Z}_{p^\infty}$ is -1-short type.

Proposition 3.4. *Let R be a ring and M be a nonzero α-short type module, which is not a atomic type module. Then M contains an uncountably generated submodule L such that $ucn\text{-dim}\, \frac{M}{L} \leq \alpha$.*

Proof. Since M is not atomic type, we infer that there exists an uncountably generated submodule $L \subset M$, such that $ucn\text{-dim}\, L = ucn\text{-dim}\, M$. We know that $ucn\text{-dim}\, M = \alpha$ or $ucn\text{-dim}\, M = \alpha + 1$, by Proposition 2.15. If $ucn\text{-dim}\, M = \alpha$ it is clear that $ucn\text{-dim}\, \frac{M}{L} \leq \alpha$. Hence we may suppose that $ucn\text{-dim}\, L = ucn\text{-dim}\, M = \alpha + 1$. If $ucn\text{-dim}\, \frac{M}{L} = \alpha + 1$, then M is γ-short type module for some $\gamma \geq \alpha + 1$, which is a contradiction. Consequently, $ucn\text{-dim}\, \frac{M}{L} \leq \alpha$ and we are done. □

Theorem 3.5. *Let α be an ordinal number and M be an R-module. If every proper uncountably generated submodule of M is γ-short type for some ordinal number $\gamma \leq \alpha$, then $ucn\text{-dim}\, M \leq \alpha + 2$, in particular, M is μ-short for some ordinal $\mu \leq \alpha + 2$.*

Proof. Let $N \subset M$ be any uncountably generated submodule. Since N is γ-short type for some ordinal number $\gamma \leq \alpha$, we infer that

$$ucn\text{-dim}\, N \leq \gamma + 1 \leq \alpha + 1,$$

by Proposition 2.15. This immediately implies that $ucn\text{-dim}\, M \leq \alpha + 2$, see [6, Lemma 3.7]. The final part is now evident. □

The next result is the dual of Theorem 3.5.

Theorem 3.6. *Let M be a nonzero R-module and α be an ordinal number. Let for each proper uncountably generated submodule N of M, $\frac{M}{N}$ be γ-short type for some ordinal number $\gamma \leq \alpha$. Then $ucn\text{-dim}\, M \leq \alpha + 2$, in particular, M is μ-short for some ordinal $\mu \leq \alpha + 2$.*

Proof. Let N be any proper uncountably generated submodule of M. Then $\frac{M}{N}$ is γ-short type for some ordinal number $\gamma \leq \alpha$. In view of Proposition 2.15, we infer that $ucn\text{-}\dim \frac{M}{N} \leq \gamma + 1 \leq \alpha + 1$. Therefore

$$\begin{aligned} & ucn\text{-}\dim M \\ & \leq \sup\{ucn\text{-}\dim \frac{M}{N} : \text{N is uncountably generated submodule of M}\} + 1 \\ & \leq \alpha + 2, \end{aligned}$$

see [6, Lemma 3.8]. The final part is now evident. □

The next immediate result is the counterparts of Theorems 3.5, 3.6, for α-almost Noetherian type modules.

Proposition 3.7. *Let M be an R-module and α be an ordinal number. If each proper uncountably generated submodule N of M (resp., for each proper uncountably generated submodule N of M, $\frac{M}{N}$) is γ-almost Noetherian type with $\gamma \leq \alpha$, then $ucn\text{-}\dim M \leq \alpha + 1$ and M is an μ-almost Noetherian type module with $\mu \leq \alpha + 2$ (resp., $ucn\text{-}\dim M \leq \alpha + 1$ and M is an μ-almost Noetherian type module with $\mu \leq \alpha + 2$).*

The followig result is evident.

Proposition 3.8. *If M has finite Goldie dimension, then $ucn\text{-}\dim M \leq \sup\{ucn\text{-}\dim \frac{M}{E} + 1 : E \subset_e M \text{and } E \text{ is uncountably generated}\}$ if either side exists.*

Proof. Let

$$\alpha = \sup\{ucn\text{-}\dim \tfrac{M}{E} + 1 : \text{E is essential and uncountably generated}\},$$

then it sufficient to show that $ucn\text{-}\dim M$ exists and $ucn\text{-}\dim M \leq \alpha$. Now, let $N_1 \subset N_2 \subset \cdots \subset N_i \subset \ldots$ be an infinite ascending chain of uncountably generated submodule of M, then by our assumption there exists some integer k such that N_i is essential in N_{i+1} for all $i \geq k$ (note, M has finite Goldie dimension). This means that there exists a submodule P of M such that $N_i \oplus P$ is essential in M for all $i \geq k$. It is clear that for each i, $N_i \oplus P$ is an uncountably generated submodule of M (note, if $N_i \oplus P$ is countably generated, then N_i is countably generated which is a contradiction). But $\frac{N_{i+1}}{N_i} \simeq \frac{N_{i+1} \oplus P}{N_i \oplus P}$ for all $i \geq k$. In view of [6, Lemma 3.5], we infer that

$$ucn\text{-}\dim \tfrac{N_{i+1}}{N_i} = ucn\text{-}\dim \tfrac{N_{i+1} \oplus P}{N_i \oplus P} \leq ucn\text{-}\dim \tfrac{M}{N_i \oplus P} < \alpha$$

for each $i \geq k$ and hence $ucn\text{-}\dim M \leq \alpha$. □

Proposition 3.9. *Let R be a semiprime ring. If the right R-module R is quotient finite dimensional and it is also α-short type, then ucn-$\dim R = \alpha$ or ucn-$\dim \frac{R}{E} \leq \alpha$ for each essential uncountably generated right ideal E of R.*

Proof. Suppose that there exists an essential uncountably generated right ideal E' of R such that ucn-$\dim \frac{R}{E'} \nleq \alpha$. Since R is α-short type, we infer that ucn-$\dim E' \leq \alpha$. In view of Corollary 2.17, R has Noetherian dimension. Therefore R is a right Goldie ring, see [10, Corollary 3.4]. Hence there exists a regular element c in E', which implies that

$$ucn\text{-}\dim R = ucn\text{-}\dim cR \leq ucn\text{-}\dim E'_R \leq \alpha.$$

□

Acknowledgments

The author would like to thank the referees for carefully reading the paper, detailed report and giving very helpful comments.

References

1. T. Albu and S. Rizvi, Chain conditions on Quotient finite dimensional modules, *Comm. Algebra*, **29**(5) (2001), 1909–1928.
2. T. Albu and P. F. Smith, Dual Krull dimension and duality, *Rocky Mountain J. Math.*, **29** (1999), 1153–1164.
3. T. Albu and P. Vamos, Global Krull dimension and global dual Krull dimension of valuation rings, In: *Abelian Groups, Module Theory, and Topology*, Marcel Dekker, (1998), 37–54.
4. G. Bilhan and P. F. Smith, Short modules and almost Noetherian modules, *Math. Scand.*, **98** (2006), 12–18.
5. L. Chambless, N-Dimension and N-critical modules, application to Artinian modules, *Comm. Algebra*, **8**(16) (1980), 1561–1592.
6. M. Davoudian, Dimension of uncountably generated submodules, *Int. Electron. J. Algebra*, **35** (2024), 149–159.
7. M. Davoudian and O. A. S. Karamzadeh, Artinian serial modules over commutative (or left Noetherian) rings are at most one step away from being Noetherian, *Comm. Algebra*, **44**(9) (2016), 3907–3917.
8. M. Davoudian, O. A. S. Karamzadeh and N. Shirali, On α-short modules, *Math. Scand.*, **114**(1) (2014), 26–37.
9. R. Gordon, Gabriel and Krull dimension, In: *Ring Theory (Proceeding of the Oklahoma Conference)*, Lecture Notes in Pure and Appl. Math., Vol. 7, Dekker, NewYork, (1974), 241–295.
10. R. Gordon and J. C. Robson, *Krull dimension*, Mem. Amer. Math. Soc., 1973.

11. J. Hashemi, O. A. S. Karamzadeh and N. Shirali, Rings over which the Krull dimension and Noetherian dimension of all modules coincide, *Comm. Algebra*, **37**(2) (2009), 650–662.
12. O. A. S. Karamzadeh, *Noetherian-dimension*, Ph.D. Thesis, Exeter, 1974.
13. O. A. S. Karamzadeh and M. Motamedi, On α-*Dicc* modules, *Comm. Algebra*, **22** (1994), 1933–1944.
14. O. A. S. Karamzadeh and A. R. Sajedinejad, Atomic modules, *Comm. Algebra*, **29**(7) (2001), 2757–2773.
15. O. A. S. Karamzadeh and A. R. Sajedinejad, On the Loewy length and the Noetherian dimension of Artinian modules, *Comm. Algebra*, **30** (2002), 1077–1084.
16. O. A. S. Karamzadeh and N. Shirali, On the Countablity of Noetherian Dimension of Modules, *Comm. Algebra*, **32** (2004), 4073–4083.
17. D. Kirby, Dimension and length for Artinian modules, *Quart. J. Math. Oxford.*, **41** (1990), 419–429.
18. B. Lemonnier, Deviation des ensembless et groupes totalement ordonnes, *Bull. Sci. Math.*, **96** (1972), 289–303.
19. B. Lemonnier, Dimension de Krull et codeviation, Application au theorem d'Eakin, *Comm. Algebra*, **6** (1978), 1647–1665.
20. J. C. McConell and J. C. Robson, *Noncommutative Noetherian Rings*, Wiley-Interscience, New York, 1987.
21. R. N. Roberts, Krull dimension for Artinian modules over quasi local commutative rings, *Quart. J. Math. Oxford.*, **26** (1975), 269–273.

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Journal of Algebraic Systems

ON α–SHORT TYPE MODULES

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دربارهی مدولهای از نوع α–کوتاه

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در این مقاله مفهوم مدولهای از نوع α–کوتاه را معرفی و مطالعه میکنیم. با استفاده از این مفهوم برخی از مفاهیم اصلی مدولهای α–کوتاه را به مدولهای از نوع α–کوتاه، تعمیم میدهیم. نشان میدهیم اگر M یک مدول از نوع α–کوتاه باشد، آنگاه بعد نوتری M کوچکتر یا مساوی $\omega_1 + \alpha + 1$ است، که ω_1 اولین عدد ترتیبی ناشمارا است.

کلمات کلیدی: مدولهای α–کوتاه، مدولهای α– تقریباً نوتری، مدولهای از نوع α–کوتاه، مدولهای از نوع α– تقریباً نوتری، بعد نوتری ناشمارا مولد.