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INTUITIONISTIC FUZZY TRANSLATION AND MULTIPLICATION OF PMS-SUBALGEBRAS OF A PMS-ALGEBRA

B. L. Derseh

Abstract. In this paper, we apply the concepts of intuitionistic fuzzy translations and multiplications of intuitionistic fuzzy sets to the PMS-subalgebras of a PMS algebra and investigate several related results. The relationships between intuitionistic fuzzy PMS-subalgebras of a PMS-algebra and intuitionistic fuzzy ω-translations, and intuitionistic fuzzy ζ-multiplications of the intuitionistic fuzzy PMS-subalgebras are discussed. Finally, we discuss the ideas of intuitionistic fuzzy magnified $\zeta\omega$-translations of intuitionistic fuzzy PMS-subalgebras of a PMS-algebra and investigate associated results.

1. Introduction

In 1978, Iseki and Tanaka [8] introduced the concept of BCK-algebra. Iseki [7] also developed the notion of BCI-algebra as the generalization of BCK-algebra in 1980. In 2016, Sithar Selvam and Nagalakshmi [19] introduced a PMS-algebra as a generalization of BCK\BCI\-algebras, TM-algebra, KUS-algebra and PS-algebra, and investigated various related results.

In 1965, Zadeh [21] introduced the idea of fuzzy sets to characterize uncertainty in the universe at large. The study of fuzzy subsets and their applications to various mathematical contexts has given rise to the study of fuzzy mathematics. Fuzzy set theories have been developed in many directions by many researchers and have evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, group, ring, near ring, and vector spaces. Sithar Selvam and Nagalakshmi [18] also introduced the concept of a fuzzy PMS-subalgebra and a fuzzy PMS-ideal of a PMS-algebra and established various properties in detail. Derseh et al. [5, 20] extended the idea of fuzzy PMS-subalgebra and fuzzy PMS-ideal to intuitionistic fuzzy PMS-subalgebra and intuitionistic fuzzy PMS-ideal and investigated various properties.

Fuzzy sets theories were extended to fuzzy translations and fuzzy multiplication, as well as fuzzy magnification, by several researchers. These ideas

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were then applied to various algebraic structures by different researchers. In 2019, Kaviyarasu and Indhira [9] presented the ideas of fuzzy translations, extensions, and fuzzy multiplications of fuzzy INK-ideals in INK-algebras and examined a few related properties. Moreover, these authors investigated the connections between translations, fuzzy extensions, and fuzzy multiplications of fuzzy INK ideals. In 2021, Alshehri [2] introduced the concepts of fuzzy translation and fuzzy multiplication on a BRK algebra and investigated several related properties. In 2022, Almuhaimeed[1] introduced the concepts of fuzzy translation, fuzzy multiplication, fuzzy magnified translation, and fuzzy extension of a KU-algebra and investigated various fundamental properties regarding these concepts. Moreover, Almuhaimeed provided relationships between fuzzy extension, fuzzy translation, and fuzzy multiplication of KU-subalgebras and KU-ideals in KU-algebras

Several researchers have explored the generalization of the notion of fuzzy subsets. For example, Atanassov [3, 4] introduced the idea of an intuitionistic fuzzy set as a generalization of fuzzy sets. Recently, many researchers have also extended the concepts of fuzzy translations, fuzzy multiplications, fuzzy extensions, and fuzzy magnifications to intuitionistic fuzzy translations, intuitionistic fuzzy multiplications, intuitionistic fuzzy extensions, and intuitionistic fuzzy magnifications and applied them to various algebraic structures. For example, Senapati [14] introduced the concept of intuitionistic fuzzy translation to intuitionistic fuzzy subalgebras and ideals in B-algebra and investigated several related properties. The notions of intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of intuitionistic fuzzy H-ideals with some related properties were also studied by Senapati et al.[15] in 2013. In 2019, Khalid et al.[12] introduced intuitionistic fuzzy translation, intuitionistic fuzzy extension, and intuitionistic fuzzy multiplication of intuitionistic fuzzy subalgebra and intuitionistic fuzzy ideal in G-algebra. In 2020, M. Kaviyarasu et al.[10] studied intuitionistic fuzzy translations, extensions, and intuitionistic fuzzy multiplications of intuitionistic fuzzy INK-subalgebra in INK-algebras and investigated associated results. In 2020, Ramesh et al.[13] proposed the intuitionistic fuzzy extensions and multiplications of intuitionistic fuzzy subalgebras in BF-algebras and investigated several related algebraic properties. In 2021, Khalid et al. [11] studied intuitionistic fuzzy translations, intuitionistic fuzzy multiplications, and intuitionistic fuzzy magnified translations in PS-algebras and provided a new line of thought to apply PS-algebras.

In 2021, Hameed et al.[6] discussed the magnified translation of intuitionistic fuzzy AT-ideals, intuitionistic fuzzy extensions, and intuitionistic fuzzy magnified translation of fuzzy AT-ideals on AT-algebras. In 2023, Senapati et al.[17] introduced the notion of intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of intuitionistic fuzzy subalgebras in BG-algebras and investigated several related properties. The notions of intuitionistic fuzzy translations and intuitionistic fuzzy multiplications, as well as intuitionistic fuzzy magnified translations, have been studied in several algebraic structures (see [6, 10, 12, 11, 13, 15, 16, 14, 17]). However, as far as we know, such studies have not been conducted on intuitionistic fuzzy PMS-subalgebras of a PMS-algebra. The studies of intuitionistic fuzzy translation, intuitionistic fuzzy multiplication, intuitionistic fuzzy extension, and intuitionistic fuzzy magnified translation theories in various algebraic structures motivated us to a large extent to extend our present study to PMS-subalgebras of a PMS-algebra. This article aims to study the notion of intuitionistic fuzzy translations and intuitionistic fuzzy multiplication of intuitionistic fuzzy PMS-subalgebras of a PMS-algebra and investigate several related properties.

In this paper, we discuss the relationships between intuitionistic fuzzy PMS-subalgebras of a PMS-algebra and intuitionistic fuzzy ω-translations A^T_ω, and intuitionistic fuzzy ζ-multiplications A^M_ζ of intuitionistic fuzzy PMS-subalgebras in PMS-algebras. We also investigate relationships among intuitionistic fuzzy translation, intuitionistic fuzzy extensions, and intuitionistic fuzzy multiplications of intuitionistic fuzzy PMS-subalgebras. Finally, we introduce the ideas of intuitionistic fuzzy magnified $\zeta\omega$-translations of intuitionistic fuzzy PMS-subalgebras of a PMS-algebra and explore some associated results.

2. PRELIMINARIES

In this section, we consider some basic definitions and results that are necessary for the study of this paper.

Definition 2.1. [19] A nonempty set X with a constant element "0" and a binary operation " $*$ " is called a PMS-algebra if it satisfies the following axioms:

i. $0 * x = x$
ii. $(y * x) * (z * x) = z * y$, for all $x, y, z \in X$.

Definition 2.2. [19] A nonempty subset S of a PMS-algebra X is called a PMS-subalgebra of X if $x * y \in S$, for all $x, y \in S$.

Proposition 2.3. [19] *In any PMS-algebra $(X, *, 0)$ the following properties hold for all $x, y, z \in X$,*

i. $x * x = 0$
ii. $(y * x) * x = y$
iii. $x * (y * x) = y * 0$
iv. $(y * x) * z = (z * x) * y$

Definition 2.4. [21] A fuzzy set A in a nonempty set X is a function $\eta_A : X \to [0, 1]$, where $\eta_A(x)$ defines the degree of membership of the element x in X to set A.

Definition 2.5. [18] A fuzzy set A in a PMS-algebra X is called a fuzzy PMS-subalgebra of X if it $\eta_A(x * y) \geq \min\{\eta_A(x), \eta_A(y)\}$, for all $x, y \in X$.

Definition 2.6. [9, 15] Let A be a fuzzy subset of a nonempty set X and $\omega \in [0,\ 1 - \sup\{\eta_A(x) \mid x \in X\}]$. A mapping $(\eta_A)^T_\omega : X \to [0, 1]$ is said to be a fuzzy ω-translation of η_A if it satisfies $(\eta_A)^T_\omega (x) = \eta_A(x) + \omega$, for all $x \in X$.

Definition 2.7. [9, 15] Let A be a fuzzy subset of a nonempty set X and $\zeta \in [0, 1]$. A mapping $(\eta_A)^M_\zeta : X \to [0, 1]$ is said to be a fuzzy ζ-multiplication of η_A if it satisfies $(\eta_A)^M_\zeta (x) = \zeta.\eta_A(x)$, for all $x \in X$.

Definition 2.8. [15] Let A be a fuzzy subset of a nonempty X, and

$$\omega \in [0,\ 1 - \sup\{\eta_A(x) \mid x \in \text{ X }\}],$$

and $\zeta \in [0, 1]$. A mapping $(\eta_A)^{\text{MT}}_{\zeta\omega} : X \to [0, 1]$ is said to be a fuzzy magnified $\zeta\omega$-translation of A if it satisfies $(\eta_A)^{\text{MT}}_{\zeta\omega} (x) = \zeta.\eta_A(x) + \omega$, for all $x \in X$.

Definition 2.9. [3, 4] An intuitionistic fuzzy set (IFS) A in a nonempty set X is an object having the form

$$A = \{\langle x, \eta_A(x), \lambda_A(x)\rangle | x \in X\},$$

where the functions $\eta_A : X \to [0, 1]$ and $\lambda_A : X \to [0, 1]$ define the degree of membership and the degree of nonmembership respectively, with the condition $0 \leq \eta_A(x) + \lambda_A(x) \leq 1$, for all $x \in X$.

Definition 2.10. [3, 4] Let A and B be two intuitionistic fuzzy subsets of the set X, where $A = \{\langle x, \eta_A(x), \lambda_A(x)\rangle | x \in X\}$ and

$$B = \{\langle x, \eta_B(x), \lambda_B(x)\rangle | x \in X\},$$

then

i. $A \subseteq B$ if and only if $\eta_A(x) \leq \eta_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$, for all $x \in X$.
ii. $A \cap B = \{\langle x, \min\{\eta_A(x), \eta_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\}\rangle \, | x \in X\}$

iii. $A \cup B = \{\langle x, \max\{\eta_A(x), \eta_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\}\rangle \,|x \in X\}$

Definition 2.11. [5] An IFS $A = (\eta_A, \lambda_A)$ in a PMS-algebra X is called an intuitionistic fuzzy PMS-subalgebra (IF PMS-SA) of X if it satisfies the following conditions for all $x, y, z \in X$.

i. $\eta_A(x * y) \geq \min\{\eta_A(x), \eta_A(y)\}$

ii. $\lambda_A(y * x) \leq \max\{\lambda_A(x), \lambda_A(y)\}$

Definition 2.12. [16] Let $A = (\eta_A, \lambda_A)$ and $A = (\eta_B, \lambda_B)$ be two intuitionistic fuzzy subsets of a nonempty set X. Then B is said to be an intuitionistic fuzzy extension of A if $A \subseteq B$ *i.e*, $\eta_A(x) \leq \eta_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$, for all $x \in X$.

3. INTUITIONISTIC FUZZY TRANSLATION OF PMS-ALGEBRAS

In this section, we discuss the notion of intuitionistic fuzzy ω-translation in a PMS-algebra and investigate some interesting results. Throughout this and the subsequent sections X refers to a PMS-algebra and

$$\Upsilon = 1 - sup\{\eta_A(x) : x \in X\},$$

unless otherwise stated .

Definition 3.1. Let $A = (\eta_A, \lambda_A)$ be an IFS in X and $\omega \in [0, \Upsilon]$. An object of the form $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ is called an intuitionistic fuzzy ω-translation (IFOT) of A if it satisfies the condition

$$(\eta_A)^T_\omega(x) = \eta_A(x) + \omega \text{ and } (\lambda_A)^T_\omega(x) = \lambda_A(x) - \omega,$$

for all $x \in X$.

Example 3.2. Let $\mathbb{Z}$ be the set of integers and "$*$" is a binary operation in $\mathbb{Z}$ defined by $x * y = \text{y} - x,\ \ \forall x,\ y\ \in \mathbb{Z}$, where "$-$" is the usual subtraction of integers. Then $(\mathbb{Z}, *, 0)$ is a PMS-algebra. Let $A = \{\langle x, \eta_A(x), \lambda_A(x)\rangle | x \in \mathbb{Z}\}$ be an IFS defined by

$$\eta_A(x) = \begin{cases} 0.5 & if\ x \in \langle 2\rangle \\ 0.3 & otherwise \end{cases} \text{ and } \lambda_A(x) = \begin{cases} 0.4 & if\ x \in \langle 2\rangle \\ 0.6 & otherwise \end{cases}$$

Now, $\Upsilon = 1 - sup\{\eta_A(x) :\ x \in X\ \} = 1 - 0.5 = 0.5$.

Choose $\omega = 0.4 \in [0, \Upsilon]$. Then an IFOT $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ of an IFS $A = (\eta_A, \lambda_A)$ in $\mathbb{Z}$ is given by

$$(\eta_A)^T_{0.4}(x) = \begin{cases} 0.5 + 0.4 = 0.9 & if\ x \in \langle 2\rangle \\ 0.3 + 0.4 = 0.7 & otherwise \end{cases}$$

and

$$(\lambda_A)^T_{0.4}(x) = \begin{cases} 0.4 - 0.4 = 0 & if\ x \in \langle 2 \rangle \\ 0.6 - 0.4 = 0.2 & otherwise \end{cases}$$

Theorem 3.3. *Let $A = (\eta_A, \lambda_A)$ be an IF PMS-SA of X and $\omega \in [0, \Upsilon]$. Then the IFOT $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ of A is an IF PMS-SA of X.*

Proof. Let $x,\ y \in X$ and $\omega \in [0, \Upsilon]$. Then

$$\begin{aligned}(\eta_A)^T_\omega(x * y) = \eta_A(x * y) + \omega &\geq min\{\eta_A(x),\ \eta_A(y)\} + \omega \\ &= \min\{\eta_A(x) + \omega,\ \eta_A(y) + \omega\} \\ &= \min\{(\eta_A)^T_\omega(x), (\eta_A)^T_\omega(y)\}\end{aligned}$$

and

$$\begin{aligned}(\lambda_A)^T_\omega(x * y) = \lambda_A(x * y) - \omega &\leq \max\{\lambda_A(x),\ \lambda_A(y)\} - \omega \\ &= \max\{\lambda_A(x) - \omega,\ \lambda_A(y) - \omega\} \\ &= max\{(\lambda_A)^T_\omega(x), (\lambda_A)^T_\omega(y)\}\end{aligned}$$

Hence, the IFOT A^T_ω is an IF PMS-SA of X. □

Theorem 3.4. *Let $A = (\eta_A, \lambda_A)$ be an IFS in X such that the IFOT $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ of A is an IF PMS-SA of X, for some $\omega \in [0, \Upsilon]$. Then $A = (\eta_A, \lambda_A)$ is an IF PMS-SA of X.*

Proof. Let A^T_ω is an IF PMS-SA of X for some $\omega \in [0, \Upsilon]$ and $x, y \in X$. Then

$$\begin{aligned}eta_A(x * y) + \omega = (\eta_A)^T_\omega(x * y) &\geq\ min\{(\eta_A)^T_\omega(x), (\eta_A)^T_\omega(y)\} \\ &= \min\{\eta_A(x) + \omega,\ \eta_A(y) + \omega\} \\ &= \min\{\eta_A(x),\ \eta_A(y)\} + \omega \\ \implies \eta_A(x * y) &\geq \min\{\eta_A(x),\ \eta_A(y)\}\end{aligned}$$

and

$$\begin{aligned}\lambda_A(x * y) - \omega = (\lambda_A)^T_\omega(x * y) &\leq max\{(\lambda_A)^T_\omega(x), (\lambda_A)^T_\omega(y)\} \\ &= \max\{\lambda_A(x) - \omega,\ \lambda_A(y) - \omega\} \\ &= \max\{\lambda_A(x),\ \lambda_A(y)\} - \omega \\ \implies \lambda_A(x * y) &\leq \max\{\lambda_A(x),\ \lambda_A(y)\}\end{aligned}$$

Hence, $A = (\eta_A, \lambda_A)$ is an IF PMS-SA of X. □

Theorem 3.5. *Let $A = (\eta_A, \lambda_A)$ and $B = (\eta_B, \lambda_B)$ be any two IF PMS-SAs of X. Then $(A \cap B)^T_\omega$ is also an IF PMS-SA of X.*

Proof. Let A and B be two IF PMS-SAs of X. Then for each $x, y \in X$ and $\omega \in [0, \Upsilon]$, we have

$$\begin{aligned}
(\eta_{A\cap B})^T_\omega (x*y) &= \eta_{A\cap B}(x*y) + \omega \\
&= \min\{\eta_A(x*y),\ \eta_B(x*y)\} + \omega \\
&\geq \min\{\ min\{\eta_A(x),\ \eta_A(y)\},\ \min\{\eta_B(x),\ \eta_B(y)\} + \omega \\
&= \min\{\ min\{\eta_A(x),\ \eta_B(x)\},\ \min\{\eta_A(y),\ \eta_B(y)\}\} + \omega \\
&= \min\{\eta_{A\cap B}(x),\ \eta_{A\cap B}(y)\} + \omega \\
&= \min\{\eta_{A\cap B}(x) + \omega,\ \eta_{A\cap B}(y) + \omega\} \\
&= \min\{(\eta_{A\cap B})^T_\omega (x), (\eta_{A\cap B})^T_\omega (y)
\end{aligned}$$

and

$$\begin{aligned}
(\lambda_{A\cap B})^T_\omega (x*y) &= \lambda_{A\cap B}(x*y) - \omega \\
&= \max\{\lambda_A(x*y),\ \lambda_B(x*y)\} - \omega \\
&\leq \max\{\max\{\lambda_A(x),\ \lambda_A(y)\},\ \ \max\{\lambda_B(x),\ \lambda_B(y)\}\} - \omega \\
&= \max\{\{\ max\{\lambda_A(x),\ \lambda_B(x)\},\ \max\{\lambda_A(y),\ \lambda_B(y)\}\} - \omega \\
&= \max\{\lambda_{A\cap B}(x),\ \lambda_{A\cap B}(y)\} - \omega \\
&= \max\{\lambda_{A\cap B}(x) - \omega,\ \lambda_{A\cap B}(y) - \omega\} \\
&= \max\{\lambda_{A\cap B})^T_\omega(x), (\lambda_{A\cap B})^T_\omega (y)
\end{aligned}$$

Therefore, $(A \cap B)^T_\omega$ is also an IF PMS-SA of X. □

Lemma 3.6. *If $A = (\eta_A, \lambda_A)$is an IFS of X such that the IFOT $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ of A is an IF PMS-SA of X for some $\omega \in [0, \Upsilon]$. Then $(\eta_A)^T_\omega(0) \geq (\eta_A)^T_\omega (x)$ and $(\lambda_A)^T_\omega(0) \leq (\lambda_A)^T_\omega (x)$, for all $x \in X$.*

Theorem 3.7. *Let the IFOT $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ of A be an IF PMS-SA of X for $\omega \in [0, \Upsilon]$. If $x \leq y$, then $(\eta_A)^T_\omega (x) = (\eta_A)^T_\omega (y)$ and*

$$(\lambda_A)^T_\omega (x) = (\lambda_A)^T_\omega (y).$$

Proof. Let $x, y \in X$ such that $x \leq y$. Then $x * y = 0$. Now we have

$$\begin{aligned}
(\eta_A)^T_\omega (x) = (\eta_A)^T_\omega (0*x) &= (\eta_A)^T_\omega ((y*y)*x) \\
&= (\eta_A)^T_\omega ((x*y)*y) \\
&= (\eta_A)^T_\omega (0*y) = (\eta_A)^T_\omega (y)
\end{aligned}$$

Hence $(\eta_A)^T_\omega(x) = (\eta_A)^T_\omega(y)$. Similarly, we can also show that

$$(\lambda_A)^T_\omega(x) = (\lambda_A)^T_\omega(y)$$

□

Theorem 3.8. *If $A = (\eta_A, \lambda_A)$is an IFS of X such that the IFOT $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ of A is an IF PMS-SA of X for some $\omega \in [0, \Upsilon]$. Then the sets $T_{\eta_A} = \{x \in X| \, (\eta_A)^T_\omega(x) = (\eta_A)^T_\omega(0)\}$ and*

$$T_{\lambda_A} = \{\, x \in X| \, (\lambda_A)^T_\omega(x) = (\lambda_A)^T_\omega(0)\}$$

are PMS-SA of X.

Proof. Suppose $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ is an IF PMS-SA of X for some $\omega \in [0, \Upsilon]$. Let $x, y \in T_{\eta_A}$. Then $(\eta_A)^T_\omega(x) = (\eta_A)^T_\omega(0) = (\eta_A)^T_\omega(y)$. Since $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ is an IF PMS-SA of X, we have

$$(\eta_A)^T_\omega(x * y) \geq \min\left\{(\eta_A)^T_\omega(x), (\eta_A)^T_\omega(y)\right\} = (\eta_A)^T_\omega(0).$$

This implies $(\eta_A)^T_\omega(x * y) \geq (\eta_A)^T_\omega(0)$. By Lemma 3.6, it follows that $(\eta_A)^T_\omega(x * y) = (\eta_A)^T_\omega(0)$. So that $x * y \in T_{\eta_A}$. Therefore, T_{η_A} is a PMS-SA of X.
Also, Let $x, y \in T_{\lambda_A}$. Then$(\lambda_A)^T_\omega(x) = (\lambda_A)^T_\omega(0) = (\lambda_A)^T_\omega(y)$. Since $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ is an IF PMS-SA of X, we have

$$(\lambda_A)^T_\omega(x * y) \leq \max\left\{(\lambda_A)^T_\omega(x), (\lambda_A)^T_\omega(y)\right\} = (\lambda_A)^T_\omega(0).$$

This implies $(\lambda_A)^T_\omega(x * y) \leq (\lambda_A)^T_\omega(0)$. Again, by Lemma 3.6, it follows that $(\lambda_A)^T_\omega(x * y) = (\lambda_A)^T_\omega(0)$. So that $x * y \in T_{\lambda_A}$. Therefore, T_{λ_A} is a PMS-SA of X. □

Definition 3.9. For an IFS $A = (\eta_A, \lambda_A)$ of X, $\omega \in [0, \Upsilon]$ and $t, \; s \in [0, 1]$ with $t \geq \omega$, The sets $U_\omega(\eta_A, t) = \{x \in X| \; (\eta_A)^T_\omega \geq t\}$ and

$$L_\omega(\lambda_A, s) = \{x \in X| \, (\lambda_A)^T_\omega \leq s\}$$

are upper level and lower level subsets of an intuitionistic fuzzy translation A^T_ω of A.

Theorem 3.10. *Let $A = (\eta_A, \lambda_A)$ be an IFS of X and $\omega \in [0, \Upsilon]$. Then the IFOT A^T_ω of A is an IF PMS-SA of X if and only if $U_\omega(\eta_A, t)$ and $L_\omega(\lambda_A, s)$ are PMS-SAs of X for all $t \in Im(\eta_A)$ and $s \in Im(\lambda_A)$ with $t \geq \omega$.*

Proof. Suppose that $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ is an IF PMS-SA of X. Let $x, y \in X$ such that $x, y \in U_\omega(\eta_A, t)$ for all $t \in \mathrm{Im}(\eta_A)$ with $t \geq \omega$. Then $(\eta_A)^T_\omega \geq t$ and $(\eta_A)^T_\omega \geq t$. Since A^T_ω is an IF PMS-SA of X, we have

$$(\eta_A)^T_\omega (x * y) \geq min\left\{(\eta_A)^T_\omega (x), (\eta_A)^T_\omega (y)\right\} \geq min\{t, t\} = t$$
$$\Longrightarrow (\eta_A)^T_\omega (x * y) \geq t.$$

Hence, $x * y \in U_\omega(\eta_A, t)$. Therefore, $U_\omega(\eta_A, t)$ is a PMS-SA of X.
Again, let $x, y \in X$ such that $x, y \in L_\omega(\lambda_A, s)$, for all $s \in \mathrm{Im}(\lambda_A)$.
Then $(\lambda_A)^T_\omega (x) \leq s$ and $(\lambda_A)^T_\omega (y) \leq s$. Since A^T_ω is an IF PMS-SA of X, we have

$$(\lambda_A)^T_\omega (x * y) \leq max\left\{(\lambda_A)^T_\omega (x), (\lambda_A)^T_\omega (y)\right\} \leq max\{s, s\} = s$$
$$\Longrightarrow (\lambda_A)^T_\omega (x * y) \leq s.$$

Hence, $x * y \in L_\omega(\lambda_A, s)$. Therefore, $L_\omega(\lambda_A, s)$ is a PMS-SA of X.

Conversely, suppose that $U_\omega(\eta_A, t)$ and $L_\omega(\lambda_A, s)$ are PMS-SAs of X, for all $t \in \mathrm{Im}(\eta_A)$ and $s \in \mathrm{Im}(\lambda_A)$ with $t \geq \omega$. Assume that there exist $x, y \in X$ such that

$$(\eta_A)^T_\omega (x * y) < \gamma \leq \min\{(\eta_A)^T_\omega (x),\ (\eta_A)^T_\omega (y)\}.$$

Then $(\eta_A)^T_\omega (x) \geq \gamma$ and $(\eta_A)^T_\omega (y) \geq \gamma$ but $(\eta_A)^T_\omega (x * y) < \gamma$.
Hence, $x, y \in U_\omega(\eta_A, t)$ but $x * y \notin U_\omega(\eta_A, t)$, which is a contradiction.

Therefore $(\eta_A)^T_\omega (x * y) \geq \min\{(\eta_A)^T_\omega (x),\ (\eta_A)^T_\omega (y)\}$, for all $x.y \in X$.

Again, assume that there exist $a, b \in X$ such that

$$(\lambda_A)^T_\omega (a * b) > \beta \geq \max\{(\lambda_A)^T_\omega (a),\ (\lambda_A)^T_\omega (b)\}.$$

Then $(\lambda_A)^T_\omega (a) \leq \beta$ and $(\lambda_A)^T_\omega (b) \leq \beta$ but $(\lambda_A)^T_\omega (a * b) > \beta$. Hence, $a, b \in L_\omega(\lambda_A, s)$ but $a * b \notin L_\omega(\lambda_A, s)$, which is a contradiction.

Therefore, $(\lambda_A)^T_\omega (x * y) \leq \max\{(\lambda_A)^T_\omega (x),\ (\lambda_A)^T_\omega (y)\}$ for all $x.y \in X$.
Consequently, $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ is an IF PMS-SA of X. □

Definition 3.11. Let $A = (\eta_A, \lambda_A)$ and $B = (\eta_B, \lambda_B)$ be intuitionistic fuzzy subsets of X. Then B is called an IF PMS-SA extension of A if the following conditions hold

(i) B is an intuitionistic fuzzy extension of A.
(ii) If A is an IF PMS-SA of X, then so is B.

Theorem 3.12. *Let $A = (\eta_A, \lambda_A)$ be an IFS of X and $\omega \in [0, \Upsilon]$. Then the IFOT $A_\omega^T = \left((\eta_A)_\omega^T, (\lambda_A)_\omega^T\right)$ of A is an IF PMS-SA extension of A.*

Proof. By the definition of $A_\omega^T = \left((\eta_A)_\omega^T, (\lambda_A)_\omega^T\right)$, we have

$$(\eta_A)_\omega^T(x) = \eta_A(x) + \omega$$

and $(\lambda_A)_\omega^T = \lambda_A(x) - \omega$ for all $x \in X$. So, $\eta_A(x) \leq \eta_A(x) + \omega = (\eta_A)_\omega^T(x)$ and $\lambda_A(x) \geq \lambda_A(x) - \omega = (\lambda_A)_\omega^T$, for all $x \in X$. Hence, A_ω^T is an intuitionistic fuzzy extension of A. Also, by Theorem 3.3, if A is an IF PMS-SA of X, then so is A_ω^T. Therefore, by Definition 3.11, A_ω^T is an IF PMS-SA extension of A. □

The converse of Theorem 3.12 need not be true in general, as justified in the following example.

Example 3.13. Consider a PMS-algebra $(\mathbb{Z}, *, 0)$ is a PMS-algebra as defined in Example 3.2. Let $A = (\eta_A, \lambda_A)$ and $B = (\eta_B, \lambda_B)$ be intuitionistic fuzzy subsets of X, respectively, defined by

$$\eta_A(x) = \begin{cases} 0.6 & if\ x\ is\ an\ odd\ integer \\ 0.4 & if\ x\ is\ an\ even\ integer \end{cases}$$

$$\lambda_A(x) = \begin{cases} 0.3 & if\ x\ is\ an\ odd\ integer \\ 0.5 & if\ x\ is\ an\ even\ integer \end{cases}$$

and

$$\eta_B(x) = \begin{cases} 0.8 & if\ x\ is\ an\ odd\ integer \\ 0.5 & if\ x\ is\ an\ even\ integer \end{cases}$$

$$\lambda_B(x) = \begin{cases} 0.2 & if\ x\ is\ an\ odd\ integer \\ 0.4 & if\ x\ is\ an\ even\ integer \end{cases}$$

Clearly, A and B are IF PMS-SAs of X, B is an IF PMS-SA extension of A. But it is not the intuitionistic fuzzy ω-translation $A_\omega^T = \left((\eta_A)_\omega^T, (\lambda_A)_\omega^T\right)$ of A for all $\omega \in [0, \Upsilon]$.

Theorem 3.14. *Let $A = (\eta_A, \lambda_A)$ be an IF PMS-SA of X and let $\omega, \gamma \in [0, \Upsilon]$. If $\omega \geq \gamma$, then the intuitionistic fuzzy ω-translation $A_\omega^T = \left((\eta_A)_\omega^T, (\lambda_A)_\omega^T\right)$ of A is an IF PMS-SA extension of the intuitionistic fuzzy γ-translation $A_\gamma^T = \left((\eta_A)_\gamma^T, (\lambda_A)_\gamma^T\right)$ of A.*

Proof. Since $A = (\eta_A, \lambda_A)$ is an IF PMS-SA of X, by Theorem 3.3, A^T_ω and A^T_γ are IF PMS-SAs of X, for $\omega, \gamma \in [0, \Upsilon]$. Now,

$$(\eta_A)^T_\omega(x) = \eta_A(x) + \omega \geq \eta_A(x) + \gamma = (\eta_A)^T_\gamma(x)$$

and $(\lambda_A)^T_\omega(x) = \lambda_A(x) - \omega \leq \lambda_A(x) - \gamma = (\lambda_A)^T_\gamma(x)$ as $\omega \geq \gamma$. This implies $(\eta_A)^T_\gamma(x) \leq (\eta_A)^T_\omega(x)$ and $(\lambda_A)^T_\gamma(x) \geq (\lambda_A)^T_\omega(x)$, for all $x \in X$. Therefore, $A^T_\omega = \left((\eta_A)^T_\omega, (\lambda_A)^T_\omega\right)$ is an IF PMS-SA extension of $A^T_\gamma = \left((\eta_A)^T_\gamma, (\lambda_A)^T_\gamma\right)$. □

4. INTUITIONISTIC FUZZY MULTIPLICATION OF PMS-ALGEBRAS

In this section, the notion of intuitionistic fuzzy ζ-multiplication in a PMS-algebra is discussed, and some interesting results are investigated.

Definition 4.1. Let $A = (\eta_A, \lambda_A)$ be an IFS in X and $\zeta \in [0, 1]$. An object of the form $A^M_\zeta = \left((\eta_A)^M_\zeta, (\lambda_A)^M_\zeta\right)$ is called an intuitionistic fuzzy ζ-multiplication (IFZM) of A if it satisfies the condition $(\eta_A)^M_\zeta(x) = \zeta.\eta_A(x)$ and $(\lambda_A)^M_\zeta(x) = \zeta.\lambda_A(x)$, for all $x \in X$.

Example 4.2. Let $\mathbb{Z}$ be the set of integers and $(\mathbb{Z}, *, 0)$ is a PMS-algebra as defined in Example 3.2. Let $A = \{\langle x, \eta_A(x), \lambda_A(x)\rangle | x \in \mathbb{Z}\}$ be an IFS defined by

$$\eta_A(x) = \begin{cases} 0.8 & if\ x \in \langle 3\rangle \\ 0.4 & otherwise \end{cases} \quad \text{and} \quad \lambda_A(x) = \begin{cases} 0.2 & if\ x \in \langle 3\rangle \\ 0.5 & otherwise \end{cases}$$

Choose $\zeta = 0.5 \in [0, 1]$. Then an IFZM $A^M_\zeta = \left((\eta_A)^T_\zeta, (\lambda_A)^T_\zeta\right)$ of an IFS $A = (\eta_A, \lambda_A)$ in $\mathbb{Z}$ is given by

$$(\eta_A)^M_\zeta(x) = \begin{cases} 0.5(0.8) = 0.16 & if\ x \in \langle 3\rangle \\ 0.5(0.4) = 0.20 & otherwise \end{cases}$$

and

$$(\lambda_A)^M_\zeta(x) = \begin{cases} 0.5(0.2) = 0.10 & if\ x \in \langle 3\rangle \\ 0.5(0.5) = 0.25 & otherwise \end{cases}$$

Theorem 4.3. *Let $A = (\eta_A, \lambda_A)$ be an IF PMS-SA of X and $\zeta \in [0, 1]$. Then the IFZM $A^M_\zeta = \left((\eta_A)^M_\zeta, (\lambda_A)^M_\zeta\right)$ of A is an IF PMS-SA of X.*

Proof. Let $x,\ y \in X$ and $\zeta \in [0,1]$. Then

$$\begin{aligned}(\eta_A)_\zeta^M(x*y) = \zeta.\eta_A(x*y) &\geq \zeta.min\{\eta_A(x),\ \eta_A(y)\}\\ &= \min\{\zeta.\eta_A(x),\ \zeta.\eta_A(y)\}\\ &= min\{(\eta_A)_\zeta^M(x), (\eta_A)_\zeta^M(y)\end{aligned}$$

and

$$\begin{aligned}(\lambda_A)_\zeta^M(x*y) = \zeta.\lambda_A(x*y) &\leq \zeta.max\{\lambda_A(x),\ \lambda_A(y)\}\\ &= \max\{\zeta.\lambda_A(x),\ \zeta.\lambda_A(y)\}\\ &= max\{(\lambda_A)_\zeta^M(x), (\lambda_A)_\zeta^M(y)\}.\end{aligned}$$

Hence, the IFZM A_ζ^M is an IF PMS-SA of X. □

Theorem 4.4. *Let $A = (\eta_A, \lambda_A)$ be an IFS in X such that the IFZM $A_\zeta^M = \left((\eta_A)_\zeta^M, (\lambda_A)_\zeta^M\right)$ of A is an IF PMS-SA of X for each $\zeta \in (0,1]$. Then $A = (\eta_A, \lambda_A)$ is an IF PMS-SA of X.*

Proof. Let A_ζ^M is an IF PMS-SA of X for each $\zeta \in (0,1]$ and $x, y \in X$.

$$\begin{aligned}\text{Then. } \zeta.\eta_A(x*y) = (\eta_A)_\zeta^M(x*y) &\geq\ min\{(\eta_A)_\zeta^M(x), (\eta_A)_\zeta^M(y)\}\\ &= \min\{\zeta.\eta_A(x),\ \zeta.\eta_A(y)\}\\ &= \zeta.min\{\eta_A(x),\ \eta_A(y)\}\\ \Longrightarrow \eta_A(x*y) &\geq\ min\{\eta_A(x),\ \eta_A(y)\},\ \text{since } \zeta \neq 0\end{aligned}$$

and

$$\begin{aligned}\zeta.\lambda_A(x*y) = (\lambda_A)_\zeta^M(x*y) &\leq max\{(\lambda_A)_\zeta^M(x), (\lambda_A)_\zeta^M(y)\}\\ &= \max\{\zeta.\lambda_A(x),\ \zeta.\lambda_A(y)\}\\ &= \zeta.max\{\lambda_A(x),\ \lambda_A(y)\}\\ \Longrightarrow \lambda_A(x*y) &\leq \max\{\lambda_A(x),\ \lambda_A(y)\},\ \text{since } \zeta \neq 0.\end{aligned}$$

Hence, $A = (\eta_A, \lambda_A)$ is an IF PMS-SA of X. □

Theorem 4.5. *Let A be an intuitionistic fuzzy subset of X such that $\omega \in [0, \Upsilon]$ and $\zeta = 1$. Then every IFOT A_ω^T of A is an IF PMS-SA extension of the IFZM A_ζ^M of A.*

Proof. Let $\omega \in [0, \Upsilon]$ and $\zeta = 1$. Then for every $x \in X$, we have

$$\begin{aligned}&(\eta_A)_\zeta^M(x) = \zeta.\eta_A(x) = \eta_A(x) \leq \eta_A(x) + \omega = (\eta_A)_\omega^T(x)\\ &\Longrightarrow (\eta_A)_\zeta^M(x) \leq (\eta_A)_\omega^T(x)\end{aligned}$$

and

$$(\lambda_A)^M_\zeta(x) = \zeta.\lambda_A(x) = \lambda_A(x) \geq \lambda_A(x) - \omega = (\lambda_A)^T_\omega(x)$$
$$\Longrightarrow (\lambda_A)^M_\zeta(x) \geq (\lambda_A)^T_\omega(x).$$

So, A^T_ω is an intuitionistic fuzzy extension of A^M_ζ.

Assume that A^M_ζ is an IF PMS-SA of X. Then A is an IF PMS-SA of X by Theorem 4.4. It follows from Theorem 3.3 that A^T_ω is also an intuitionistic fuzzy subalgebra of X, for all $\omega \in [0, \Upsilon]$. Hence every IFOT A^T_ω of A is an IF PMS-SA extension of the IFZM A^M_ζ of A. □

Theorem 4.5 is not valid for $\zeta = 0$. This fact is shown by the following example.

Example 4.6. Consider a PMS-algebra $(\mathbb{Z}, *, 0)$ where $\mathbb{Z}$ is the set of all integers and $*$ is a binary operation in $\mathbb{Z}$ defined as in example 3.2. Define an IFS $A = \{\langle x, \eta_A(x), \lambda_A(x)\rangle | x \in \mathbb{Z}\}$ by

$$\eta_A(x) = \begin{cases} 0.4 & if\ x \geq 0 \\ 0.2 & if\ x < 0 \end{cases} \quad \text{and} \quad \lambda_A(x) = \begin{cases} 0.3 & if\ x \geq 0 \\ 0.5 & if\ x < 0 \end{cases}$$

If $\zeta = 0$, we see that $(\eta_A)^M_0(x * y) = 0 = min\{(\eta_A)^M_0(x),\ (\eta_A)^M_0(y)\}$ and $(\lambda_A)^M_0(x * y) = 0 = max\{(\lambda_A)^M_0(x),\ (\lambda_A)^M_0(y)\}$ for all $x, y \in$ Z. So A^M_ζ is an IF PMS-SA of $\mathbb{Z}$. But if we take $x = 3$ and $y = 2$, then

$$\begin{aligned}(\eta_A)^T_\omega(x * y) = (\eta_A)^T_\omega(-1) &= \eta_A(-1) + \omega \\ &= 0.2 + \omega \\ &< 0.4 + \omega \\ &= min\{(\eta_A)^T_\omega(x),\ (\eta_A)^T_\omega(y)\} \\ \Longrightarrow (\eta_A)^T_\omega(x * y) &< min\{(\eta_A)^T_\omega(x),\ (\eta_A)^T_\omega(y)\}\end{aligned}$$

for all $\omega \in [0, 1]$. And also,

$$\begin{aligned}(\lambda_A)^T_\omega(x * y) = (\lambda_A)^T_\omega(-1) &= \lambda_A(-1) - \omega \\ &= 0.5 - \omega \\ &> 0.3 - \omega = max\{(\lambda_A)^T_\omega(x),\ (\lambda_A)^T_\omega(y)\} \\ \Longrightarrow (\lambda_A)^T_\omega(x * y) &> max\{(\lambda_A)^T_\omega(x),\ (\lambda_A)^T_\omega(y)\}\end{aligned}$$

for all $\omega \in [0, 1]$. Hence, A^T_ω is not an IF PMS-SA of X.
Therefore, the IFOT A^T_ω of A is not an IF PMS-SA extension of the IFZM A^M_ζ of A for $\zeta = 0$.

Theorem 4.7. *Let $A = (\eta_A, \lambda_A)$ and $B = (\eta_B, \lambda_B)$ be any two IF PMS-SAs of X. Then $(A \cap B)_\zeta^M$ is also an IF PMS-SA of X.*

Proof. Let A and B be any two IF PMS-SAs of X. Then for each $x, y \in X$ and $\zeta \in [0, 1]$, we have

$$\begin{aligned}
(\eta_{A\cap B})_\zeta^M(x * y) &= \zeta.\eta_{A\cap B}(x * y)\\
&= \zeta.\min\{\eta_A(x * y), \eta_B(x * y)\}\\
&\geq \zeta.\min\{\min\{\eta_A(x),\ \eta_A(y)\},\ \min\{\eta_B(x), \eta_B(y)\}\}\\
&= \zeta \min\{\min\{\eta_A(x), \eta_B(x)\},\ \min\{\eta_A(y),\ \eta_B(y)\}\}\\
&= \zeta.\min\{\eta_{A\cap B}(x),\ \eta_{A\cap B}(y)\}\\
&= \min\{\zeta.\eta_{A\cap B}(x),\ \zeta.\eta_{A\cap B}(y)\}\\
&= \min\{(\eta_{A\cap B})_\zeta^M(x), (\eta_{A\cap B})_\zeta^M(y)
\end{aligned}$$

and

$$\begin{aligned}
(\lambda_{A\cap B})_\zeta^M(x * y) &= \zeta.\lambda_{A\cap B}(x * y)\\
&= \zeta.\max\{\lambda_A(x * y),\ \lambda_B(x * y)\}\\
&\leq \zeta.\max\{\max\{\lambda_A(x), \lambda_A(y)\}, \max\{\lambda_B(x),\ \lambda_B(y)\}\\
&= \zeta.\max\{\ \max\{\lambda_A(x), \lambda_B(x)\},\ \max\{\lambda_A(y), \lambda_B(y)\}\}\\
&= \zeta.\max\{\lambda_{A\cap B}(x), \lambda_{A\cap B}(y)\}\\
&= \max\{\zeta.\lambda_{A\cap B}(x), \zeta.\lambda_{A\cap B}(y)\}\\
&= max\{(\lambda_{A\cap B})_\zeta^M(x), (\lambda_{A\cap B})_\zeta^M(y)
\end{aligned}$$

Therefore, $(A \cap B)_\zeta^M$ is an IF PMS-SA of X. □

5. INTUITIONISTIC FUZZY MAGNIFIED $\zeta\omega-$TRANSLATION OF PMS-ALGEBRAS

In this section, the idea of intuitionistic fuzzy magnified $\zeta\omega$-translation (IFMZOT) of IF PMS-SA is introduced, and some associated results are explored.

Definition 5.1. Let $A = (\eta_A, \lambda_A)$ be an IFS in X and $\omega \in [0, \zeta.\Upsilon]$, $\zeta \in (0, 1]$. An object of the form $A_{\zeta\omega}^{\mathrm{MT}} = \left((\eta_A)_{\zeta\omega}^{\mathrm{MT}}, (\lambda_A)_{\zeta\omega}^{\mathrm{MT}}\right)$ is called an intuitionistic fuzzy magnified $\zeta\omega$-translation (IFMZOT) of A if it satisfies the conditions $(\eta_A)_{\zeta\omega}^{\mathrm{MT}}(x) = \zeta.\eta_A(x) + \omega$ and $(\lambda_A)_\zeta^{\mathrm{MT}}(x) = \zeta.\lambda_A(x) - \omega$, for all $x \in X$.

$*$	0	1	2
0	0	1	2
1	2	0	1
2	1	2	0

TABLE 1.

Example 5.2. Let $X = \{0, 1, 2\}$ be a set with the binary operation $*$ as given in Table 1.
Then $(X, *, 0)$ is a PMS-algebra. Define an IFS

$$A = \{\langle x, \eta_A(x), \lambda_A(x)\rangle | x \in X\}$$

in X by

$$\eta_A(x) = \begin{cases} 0.6 & if\ x = 0 \\ 0.4 & if\ x \neq 0 \end{cases} \quad \text{and} \quad \lambda_A(x) = \begin{cases} 0.4 & if\ x = 0 \\ 0.5 & if\ x \neq 0 \end{cases}$$

Now, $\Upsilon = 1 - sup\,\{\eta_A(x) :\ x \in X\ \} = 1 - 0.6 = 0.4$.
Choose $\omega = 0.2 \in [0,\ 0.28]$ and $\zeta = 0.7 \in (0, 1]$. Then an IFMZOT $A^{\mathrm{MT}}_{\zeta\omega} = \left((\eta_A)^{\mathrm{MT}}_{\zeta\omega}, (\lambda_A)^{\mathrm{MT}}_{\zeta\omega}\right)$ of an IFS $A = (\eta_A, \lambda_A)$ in X is given by

$$(\eta_A)^{\mathrm{MT}}_{\zeta\omega}(x) = \begin{cases} 0.7(0.6) + 0.2 = 0.62 & if\ x \in \langle 3\rangle \\ 0.7(0.4) + 0.2 = 0.48 & otherwise \end{cases}$$

and

$$(\lambda_A)^{\mathrm{MT}}_{\zeta\omega}(x) = \begin{cases} 0.7(0.4) - 0.2 = 0.08 & if\ x \in \langle 3\rangle \\ 0.7(0.5) - 0.2 = 0.15 & otherwise \end{cases}$$

Theorem 5.3. *Let $A = (\eta_A, \lambda_A)$ be an IF PMS-SA of X and $\omega \in [0, \zeta.\Upsilon]$, $\zeta \in (0, 1]$. Then the IFMZOT $A^{MT}_{\zeta\omega} = \left((\eta_A)^{MT}_{\zeta\omega}, (\lambda_A)^{MT}_{\zeta\omega}\right)$ of A is an IF PMS-SA of X.*

Proof. Let $x, y \in X$ and $\omega \in [0, \zeta.\Upsilon]$, $\zeta \in [0, 1]$. Then

$$\begin{aligned}(\eta_A)^{\mathrm{MT}}_{\zeta\omega}(x * y) &= \zeta.\eta_A(x * y) + \omega \\ &\geq \zeta.min\{\eta_A(x),\ \eta_A(y)\} + \omega \\ &= \min\{\zeta.\eta_A(x),\ \zeta.\eta_A(y)\} + \omega \\ &= \min\{\zeta.\eta_A(x) + \omega,\ \zeta.\eta_A(y) + \omega\} \\ &= min\{(\eta_A)^{\mathrm{MT}}_{\zeta\omega}(x), (\eta_A)^{\mathrm{MT}}_{\zeta\omega}(y)\end{aligned}$$

and

$$\begin{aligned}(\lambda_A)^{\mathrm{MT}}_{\zeta\omega}(x * y) &= \zeta.\lambda_A(x * y) - \omega \\ &\leq \zeta.max\{\lambda_A(x),\ \lambda_A(y)\} - \omega\end{aligned}$$

$$
\begin{aligned}
&= \max\{\zeta.\lambda_A(x),\ \zeta.\lambda_A(y)\} - \omega \\
&= \max\{\zeta.\lambda_A(x) - \omega,\ \zeta.\lambda_A(y) - \omega\} \\
&= max\{(\lambda_A)^{\mathrm{MT}}_{\zeta\omega}(x),\ (\lambda_A)^{\mathrm{MT}}_{\zeta\omega}(y)\}
\end{aligned}
$$

Hence, the IFMZOT $A^{\mathrm{MT}}_{\zeta\omega}$ is an IF PMS-SA of X. □

Theorem 5.4. *Let $A = (\eta_A, \lambda_A)$ be an IFS in X such that the IFMZOT $A^{MT}_{\zeta\omega} = \left((\eta_A)^{MT}_{\zeta\omega}, (\lambda_A)^{MT}_{\zeta\omega}\right)$ of A is an IF PMS-SA of X for some $\omega \in [0, \Upsilon]$ and $\zeta \in (0, 1]$. Then $A = (\eta_A, \lambda_A)$ is an IF PMS-SA of X.*

Proof. Let A^M_ζ is an IF PMS-SA of X for some $\zeta \in [0, 1]$ and $x, y \in X$. Then

$$
\begin{aligned}
\zeta.\eta_A(x * y) + \omega = (\eta_A)^{\mathrm{MT}}_{\zeta\omega}(x * y) &\geq\ min\{(\eta_A)^{\mathrm{MT}}_{\zeta\omega}(x), (\eta_A)^{\mathrm{MT}}_{\zeta\omega}(y)\} \\
&= \min\{\zeta.\eta_A(x) + \omega,\ \zeta.\eta_A(y) + \omega\} \\
&= \min\{\zeta.\eta_A(x),\ \zeta.\eta_A(y)\} + \omega \\
&= \zeta.min\{\eta_A(x),\ \eta_A(y)\} + \omega \\
\Longrightarrow \eta_A(x * y) &\geq\ \min\{\eta_A(x),\ \eta_A(y)\}
\end{aligned}
$$

and

$$
\begin{aligned}
\zeta.\lambda_A(x * y) - \omega = (\lambda_A)^{\mathrm{MT}}_{\zeta\omega}(x * y) &\leq max\{(\lambda_A)^{\mathrm{MT}}_{\zeta\omega}(x), (\lambda_A)^{\mathrm{MT}}_{\zeta\omega}(y)\} \\
&= \max\{\zeta.\lambda_A(x) - \omega,\ \zeta.\lambda_A(y) - \omega\} \\
&= \max\{\zeta.\lambda_A(x),\ \zeta.\lambda_A(y)\} - \omega \\
&= \zeta.max\{\lambda_A(x),\ \lambda_A(y)\} - \omega \\
\Longrightarrow \lambda_A(x * y) &\leq \max\{\lambda_A(x),\ \lambda_A(y)\}
\end{aligned}
$$

Hence, $A = (\eta_A, \lambda_A)$ is an IF PMS-SA of X. □

6. CONCLUSION

The main objective of this article was to investigate intuitionistic fuzzy structures in PMS-algebras with an emphasis on intuitionistic fuzzy translations, intuitionistic fuzzy multiplications, and intuitionistic fuzzy extensions of intuitionistic fuzzy PMS-subalgebras of PMS-algebras. We also extended these concepts to intuitionistic fuzzy magnified translations of intuitionistic fuzzy PMS-subalgebras in PMS-algebras. We discussed the relationships between intuitionistic fuzzy PMS-subalgebras of PMS-algebras and intuitionistic fuzzy ω-translations A^T_ω and intuitionistic fuzzy ζ-multiplications A^T_ζ of intuitionistic fuzzy PMS-subalgebras in PMS-algebras. In addition, we looked into the relationships between the intuitionistic fuzzy PMS-subalgebras and their intuitionistic fuzzy translations, intuitionistic fuzzy extensions, and intuitionistic fuzzy multiplications. Furthermore, we defined the level subsets of the intuitionistic fuzzy translations of PMS-subalgebras and characterized the intuitionistic fuzzy translations of PMS-subalgebras using their level subsets. Finally, we investigated the intuitionistic fuzzy magnified $\zeta\omega$-translations of intuitionistic fuzzy PMS-subalgebras in PMS-algebras and obtained some intriguing findings. We hope that the findings of this work will provide a new thought to the structures of PMS-algebras based on intuitionistic fuzzy translations, intuitionistic fuzzy multiplications, and intuitionistic fuzzy magnified translations and that the results of this study will serve as the foundations for further studies in the theory of PMS-algebras. In our future works, we will extend the results of this study to the interval-valued intuitionistic fuzzy translations and multiplications, as well as to the interval-valued magnified translations of intuitionistic fuzzy PMS-algebras.

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References

1. A. Almuhaimeed, Applications on fuzzy translation and fuzzy multiplication of KU-algebras, *Int. J. Math. Comput. Sci.*, **17**(2) (2022), 667–677.
2. H. A. Alshehri, Fuzzy translation and fuzzy multiplication in BRK-algebras, *Eur. J. Pure Appl. Math.*, **14**(3) (2021), 737–745.
3. K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **20**(1) (1986), 87–96.
4. K. Atanassov, New operations defined over the intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **61**(2) (1994), 137–142.
5. B. L. Derseh, B. A. Alaba and Y. G. Wondifraw, Intuitionistic fuzzy PMS-subalgebra of a PMS-algebra, *Korean J. Math.*, **29**(3) (2021), 563–576.
6. A. T. Hameed, M. S. Mahdi Zabiba and N. H. Malik, Magnified translation of intuitionistic fuzzy AT-ideals on AT-algebra, *J. Discrete Math. Sci. Cryptogr.*, **24**(6) (2021), 1645–1650.
7. K. Iseki, On BCI-algebras, *Mathematics Seminar Notes*, **8** (1980), 125–130.
8. K. Iseki and S. Tanaka, An introduction to the theory of BCK-algebras, *Mathematica Japonica*, **23** (1978), 1–26.
9. M. Kaviyarasu and K. Indhira, Fuzzy Translation of INK-ideal of INK-algebras, *International Journal of Innovative Technology and Exploring Engineering*, **8**(12) (2019), 4319–4322.
10. M. Kaviyarasu, K. Indhira and V. M. Chandrasekaran, Intuitionistic fuzzy translation on INK-Algebra, *Advances in Mathematics: Scientific Journal*, **9**(1) (2020), 295–303.

11. M. Khalid, Z. Iqbal and R. Iqbal, Intuitionistic Fuzzy Magnified Translation of PS-Algebra, *Journal of New Theory*, **35** (2021), 32–47.
12. M. Khalid, R. Iqbal, S. Zafar and H. Khalid, Intuitionistic fuzzy translation and multiplication of G-algebra, *The Journal of Fuzzy Mathematics*, **27**(3) (2019), 543–559.
13. D. Ramesh, K. K. Rao, R. D. Prasad, N. Srimannarayana and B. Satyanarayana, Translations of intuitionistic fuzzy subalgebras in BF-algebras, *Advances in Mathematics: Scientific Journal*, **9**(10) (2020), 8837–8844.
14. T. Senapati, Translation of Intuitionistic Fuzzy B-Algebras, *Fuzzy Information and Engineering*, **7**(4) (2015), 389–404.
15. T. Senapati, M. Bhowmik and M. Pal, Atanassov's Intuitionistic Fuzzy Translations of Intuitionistic Fuzzy H-Ideals in BCK/BCI-Algebras, *Notes on Intuitionistic Fuzzy Sets*, **19**(1) (2013), 32–47.
16. T. Senapati, M. Bhowmik, M. Pal and B. Davvaz, Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras and ideals in BCK\BCI-algebras, *Eurasian Math. J.*, **6**(1) (2015), 96–114.
17. T. Senapati, A. Iampan and R. Chinram, Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras in BG-algebras, *TWMS Journal of Applied and Engineering Mathematics*, **13**(3) (2023), 911–919.
18. P. M. Sithar Selvam and K. T. Nagalakshmi, Fuzzy PMS ideals in PMS-algebras, *Annals of pure and applied mathematics*, **12** (2016), 153–159.
19. P. M. Sithar Selvam and K. T. Nagalakshmi, On PMS-algebras, *Transylvanian Review*, **24** (2016), 1622–1628.
20. Y. G. Wondifraw, B. A. Alaba and B. L. Derseh, Intuitionistic fuzzy PMS-ideals of a PMS-algebra, *Thai J. Math.*, **20**(4) (2022), 1535–1548.
21. L. A. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), 338–353.

Beza Lamesgin Derseh
Department of Mathematics, Debre Markos University, P.O. Box 79, Debre Markos, Ethiopia.
Email: dbezalem@gmail.com, beza_lamesgin@dmu.edu.et

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INTUITIONISTIC FUZZY TRANSLATION AND MULTIPLICATION OF PMS-SUBALGEBRAS OF A PMS-ALGEBRA

B. L. DERSEH

ترجمه و ضرب فازی شهودی PMS-زیرجبرهای یک PMS-جبر

بیزا لامسگین درسه

گروه ریاضی، دانشگاه دبر مارکوس، دبر مارکوس، اتیوپی

در این مقاله، مفاهیم ترجمه و ضرب فازی شهودی مجموعه‌های فازی شهودی را برای PMS-زیرجبرهای یک PMS-جبر به کار گرفته‌ایم و نتایج مرتبط را بررسی کرده‌ایم. روابط بین PMS-زیرجبرهای فازی شهودی یک PMS-جبر و ω-ترجمه‌های فازی شهودی، و ζ-ضرب‌های فازی شهودی این PMS-زیرجبرهای فازی شهودی بررسی شده‌اند. در آخر، ایده‌های $\zeta\omega$-ترجمه‌های فازی شهودی بزرگ‌نمایی‌شده از زیرحلقه‌های فازی شهودی PMS-زیرجبرهای فازی شهودی یک PMS-جبر مورد مطالعه قرار گرفته و نتایج مربوطه ارائه شده‌اند.

کلمات کلیدی: PMS-جبر، PMS-زیرجبر، PMS-زیرجبر فازی شهودی، ترجمه فازی شهودی، ضرب فازی شهودی، ترجمه فازی شهودی بزرگ‌نمایی‌شده.