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ON THE COFINITENESS OF LOCAL COHOMOLOGY MODULES

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Abstract. Let $\mathfrak{a}$ be an ideal of a Noetherian ring R such that the R-modules $H^1_{\mathfrak{a}}(M)$ and $H^3_{\mathfrak{a}}(M)$ are $\mathfrak{a}$-cofinite, for all finitely generated R-modules M. In this paper, it is shown that the R-modules $H^i_{\mathfrak{a}}(M)$ are $\mathfrak{a}$-cofinite, for all finitely generated R-modules M and all integers $i \in \mathbb{N}_0$.

1. Introduction

Throughout this paper, let R denote a commutative Noetherian ring (with identity) and let $\mathfrak{a}$ be an ideal of R. In this paper, we will denote $\operatorname{Supp} R/\mathfrak{a} = \{\mathfrak{p} \in \operatorname{Spec} R : \mathfrak{p} \supseteq \mathfrak{a}\}$ by $V(\mathfrak{a})$. In addition, the symbol $\mathbb{N}$ (respectively $\mathbb{N}_0$) will denote the set of positive (respectively non-negative) integers. The ith local cohomology module of an R-module M with support in $V(\mathfrak{a})$ is defined as:

$$H^i_{\mathfrak{a}}(M) = \varinjlim_{n \in \mathbb{N}} \operatorname{Ext}^i_R(R/\mathfrak{a}^n, M).$$

For further information on the concept of local cohomology, the reader may consult [7] and [12].

It is a well known result that if $(R, \mathfrak{m}, k)$ is a Noetherian local ring, then for each finitely generated R-module M and each $i \in \mathbb{N}_0$, the R-module $H^i_{\mathfrak{m}}(M)$ is Artinian, hence the R-module $\operatorname{Hom}_R(k, H^i_{\mathfrak{m}}(M))$ is finitely generated. Taking this fact, Grothendieck in his algebraic geometry seminar of 1962, (see [11, Exposé XIII, Conjecture 1.1]) conjectured the following:

Conjecture: *For each ideal $\mathfrak{a}$ of a Noetherian ring R and each finitely generated R-module M, the R-modules $\operatorname{Hom}_R(R/\mathfrak{a}, H^i_{\mathfrak{a}}(M))$ are finitely generated for all $i \in \mathbb{N}_0$.*

Two years later, Hartshorne provided a counterexample in [13, Section 3], to show that this question does not have an affirmative answer in general. Furthermore, in the same paper he defined an R-module M to be $\mathfrak{a}$-cofinite if

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the support of M is contained in $V(\mathfrak{a})$ and $\operatorname{Ext}^i_R(R/\mathfrak{a}, M)$ is finitely generated for each $i \in \mathbb{N}_0$ and posed the following question:

Question 1: *For which Noetherian rings R and ideals $\mathfrak{a}$ of R, are the modules $H^i_{\mathfrak{a}}(M)$ $\mathfrak{a}$-cofinite, for all finitely generated R-modules M and all $i \in \mathbb{N}_0$?*

In the sequel, the notation $\mathscr{C}(R, \mathfrak{a})_{cof}$ denotes the category of all $\mathfrak{a}$-cofinite R-modules and $\mathscr{C}^1(R, \mathfrak{a})_{cof}$ denotes the category of all R-modules $M \in \mathscr{C}(R, \mathfrak{a})_{cof}$ such that $\dim M \leq 1$. Also, throughout this paper, let $\mathscr{I}(R)$ be the class of all ideals $\mathfrak{a}$ of R such that $H^i_{\mathfrak{a}}(U) \in \mathscr{C}(R, \mathfrak{a})_{cof}$, for all finitely generated R-modules U and all $i \in \mathbb{N}_0$.

Concerning Question 1, there are several remarkable results in the literature; see e.g. [2, 5, 8, 9, 10, 14, 15, 16, 18, 19]. In fact, in these articles many of authors have found several classes of ideals of a Noetherian ring R satisfying the condition of Question 1. In [4], Bahmanpour solved Question 1, by obtaining an explicit description of the set $\mathscr{I}(R)$ for any Noetherian ring R. More precisely, in [4, Theorem 4.10], he proved that for each ideal $\mathfrak{a}$ of a Noetherian ring R, $\mathfrak{a} \in \mathscr{I}(R)$ if and only if $H^i_{\mathfrak{a}}(R) \in \mathscr{C}^1(R, \mathfrak{a})_{cof}$ for each integer $i \geq 2$. Moreover, in the same paper he proved that in the case that $(R, \mathfrak{m})$ is a local ring, the condition $\mathfrak{a} \in \mathscr{I}(R)$ is equivalent to the condition that for each minimal prime ideal $\mathfrak{P}$ of $\hat{R}$ (the $\mathfrak{m}$-adic completion of R), $\dim \hat{R}/(\mathfrak{a}\hat{R} + \mathfrak{P}) \leq 1$ or $\operatorname{cd}(\mathfrak{a}\hat{R}, \hat{R}/\mathfrak{P}) \leq 1$.

In this paper, we take a step in this direction. Precisely, we show that for a given ideal $\mathfrak{a}$ of a Noetherian ring R, $\mathfrak{a} \in \mathscr{I}(R)$ if and only if the R-modules $H^1_{\mathfrak{a}}(M)$ and $H^3_{\mathfrak{a}}(M)$ are $\mathfrak{a}$-cofinite for all finitely generated R-modules M.

Throughout this paper, for each ideal $\mathfrak{b}$ of R and each R-module M, $\Gamma_{\mathfrak{b}}(M)$ denotes the submodule $\bigcup_{n\in\mathbb{N}}(0 :_M \mathfrak{b}^n)$ of M. We refer the reader to [7, 17] for basic results, notations, and terminology not given in this paper.

2. Main results

The main purpose of this section is to prove Theorem 2.8, which asserts that for a given ideal $\mathfrak{a}$ of a Noetherian ring R, if the R-modules $H^1_{\mathfrak{a}}(M)$ and $H^3_{\mathfrak{a}}(M)$ are $\mathfrak{a}$-cofinite, for all finitely generated R-modules M, then the R-modules $H^i_{\mathfrak{a}}(M)$ are $\mathfrak{a}$-cofinite, for all integers i and all finitely generated R-modules M.

Let us start this section with some useful lemmas.

Lemma 2.1. (See [1, Lemma 2.3]) *Let $\mathfrak{a}$ be an ideal of a Noetherian ring R and $\mathscr{M}$ be a Serre subcategory of the category of R-modules. Let $n \in \mathbb{N}_0$ and M be an R-module such that $\operatorname{Ext}^j_R(R/\mathfrak{a}, H^i_{\mathfrak{a}}(M)) \in \mathscr{M}$, for all $0 \leq i < n$ and* *all* $j \in \mathbb{N}_0$. *If the* R*-modules* $\operatorname{Ext}_R^n(R/\mathfrak{a}, M)$ *and* $\operatorname{Ext}_R^{n+1}(R/\mathfrak{a}, M)$ *are in* $\mathscr{M}$, *then the* R*-modules* $\operatorname{Hom}_R(R/\mathfrak{a}, H_{\mathfrak{a}}^n(M))$ *and* $\operatorname{Ext}_R^1(R/\mathfrak{a}, H_{\mathfrak{a}}^n(M))$ *are in* $\mathscr{M}$.

Lemma 2.2. (See [6, Proposition 2.6]) *Let* $\mathfrak{a}$ *be an ideal of a Noetherian ring* R *and* M *be an* R*-module such that* $\dim M \leq 1$ *and* $\operatorname{Supp} M \subseteq V(\mathfrak{a})$. *Then the following statements are equivalent:*

(1) M *is* $\mathfrak{a}$*-cofinite.*
(2) *The* R*-modules* $\operatorname{Hom}_R(R/\mathfrak{a}, M)$ *and* $\operatorname{Ext}_R^1(R/\mathfrak{a}, M)$ *are finitely generated.*

Lemma 2.3. (See [6, Therem 2.7]) *Let* $\mathfrak{a}$ *be an ideal of a Noetherian ring* R. *Then* $\mathscr{C}^1(R, \mathfrak{a})_{cof}$ *is an Abelian category.*

Lemma 2.4. (See [3, Lemma 2.1]) *For an ideal* $\mathfrak{a}$ *of a Noetherian ring* R, *the following statements are equivalent:*

(1) $\mathfrak{a} \in \mathscr{I}(R)$.
(2) $H_{\mathfrak{a}}^i(R) \in \mathscr{C}^1(R, \mathfrak{a})_{cof}$, *for all integers* $i \geq 2$.
(3) *For each finitely generated* R*-module* M, $H_{\mathfrak{a}}^i(M) \in \mathscr{C}^1(R, \mathfrak{a})_{cof}$, *for all integers* $i \geq 2$.

Lemma 2.5. (See [10, Proposition 2]) *Let* $\mathfrak{a}$, $\mathfrak{b}$ *be two ideals of a Noetherian ring* R *and* M *be an* R*-module with* $\mathfrak{b} M = 0$ *and* $\operatorname{Supp} M \subseteq V(\mathfrak{a})$. *Then* M *is* $\mathfrak{a}$*-cofinite* (*as an* R*-module*) *if and only if* M *is* $(\mathfrak{a} + \mathfrak{b})/\mathfrak{b}$*-cofinite* (*as an* $R/\mathfrak{b}$*-module*).

Lemma 2.6. (See [18, Corollary 3.4]) *Let* $\mathfrak{a}$ *be an ideal of a Noetherian ring* R *and* $x \in \mathfrak{a}$. *Let* M *be an* R*-module with* $\operatorname{Supp} M \subseteq V(\mathfrak{a})$ *such that the* R*-module* $(0 :_M x)$ *and* M/xM *are* $\mathfrak{a}$*-cofinite. Then* M *is* $\mathfrak{a}$*-cofinite.*

The following lemma plays a key role in the proof of Theorem 2.8.

Lemma 2.7. *Let* $(R, \mathfrak{m})$ *be a Noetherian local ring and* $\mathfrak{a}$ *be an ideal of* R. *Then the following statements hold:*

(1) $\mathfrak{a} \in \mathscr{I}(R)$.
(2) *The* R*-modules* $H_{\mathfrak{a}}^1(M)$, $H_{\mathfrak{a}}^3(R)$ *are* $\mathfrak{a}$*-cofinite, for all finitely generated* R*-modules* M.
(3) *For each finitely generated* R*-module* M, $H_{\mathfrak{a}}^i(M) \in \mathscr{C}^1(R, \mathfrak{a})_{cof}$, *for all integers* $i \geq 2$.

Proof. (1)$\Longrightarrow$(2) The assertion is trivial.

(1)$\Longleftrightarrow$(3) The assertion holds by Lemma 2.4.

(2)$\Longrightarrow$(1) We argue by induction on $d = \dim R$. For $d = 0$, the assertion follows from *Grothendieck's Vanishing Theorem* (see [7, Theorem 6.1.2]). Suppose, inductively, that $d > 0$ and the result has been proved for smaller values of d. Assume that M is a finitely generated R-module. Since $\Gamma_{\mathfrak{a}}(R)M \subseteq \Gamma_{\mathfrak{a}}(M)$, so $M/\Gamma_{\mathfrak{a}}(M)$ has an $R/\Gamma_{\mathfrak{a}}(R)$-module structure by means of the natural map $\pi : R \longrightarrow R/\Gamma_{\mathfrak{a}}(R)$. For each $i \in \mathbb{N}$, we have

$$H^i_{\mathfrak{a}}(M) \simeq H^i_{\mathfrak{a}}(M/\Gamma_{\mathfrak{a}}(M)) \simeq H^i_{\mathfrak{a}+\Gamma_{\mathfrak{a}}(R)/\Gamma_{\mathfrak{a}}(R)}(M/\Gamma_{\mathfrak{a}}(M)).$$

In addition, by *Independence Theorem* (see [7, Theorem 4.2.1]) and Lemma 2.5, one sees that $\bigoplus_{i\geq 2} H^i_{\mathfrak{a}}(M) \in \mathscr{C}(R,\mathfrak{a})_{cof}$ if and only if

$$\bigoplus\nolimits_{i\geq 2} H^i_{\mathfrak{a}+\Gamma_{\mathfrak{a}}(R)/\Gamma_{\mathfrak{a}}(R)}(M/\Gamma_{\mathfrak{a}}(M)) \in \mathscr{C}(R/\Gamma_{\mathfrak{a}}(R), \mathfrak{a}+\Gamma_{\mathfrak{a}}(R)/\Gamma_{\mathfrak{a}}(R))_{cof}.$$

Also, from the hypothesis (ii), *Independence Theorem* and Lemma 2.5, we can deduce that the $R/\Gamma_{\mathfrak{a}}(R)$-modules

$$H^1_{\mathfrak{a}+\Gamma_{\mathfrak{a}}(R)/\Gamma_{\mathfrak{a}}(R)}(N),\ H^3_{\mathfrak{a}+\Gamma_{\mathfrak{a}}(R)/\Gamma_{\mathfrak{a}}(R)}(N),$$

are $\mathfrak{a}+\Gamma_{\mathfrak{a}}(R)/\Gamma_{\mathfrak{a}}(R)$-cofinite, for all finitely generated $R/\Gamma_{\mathfrak{a}}(R)$-modules N. So, by replacing R, $\mathfrak{a}$ and M with $R/\Gamma_{\mathfrak{a}}(R)$, $\mathfrak{a}+\Gamma_{\mathfrak{a}}(R)/\Gamma_{\mathfrak{a}}(R)$ and $M/\Gamma_{\mathfrak{a}}(M)$, we can make the additional assumption that $\Gamma_{\mathfrak{a}}(R) = 0$ and $\Gamma_{\mathfrak{a}}(M) = 0$. Then, by *Prime Avoidance Theorem*, we have

$$\mathfrak{a} \nsubseteq \left(\bigcup_{\mathfrak{p}\in \operatorname{Ass}_R R} \mathfrak{p}\right) \bigcup \left(\bigcup_{\mathfrak{q}\in \operatorname{Ass}_R M} \mathfrak{q}\right).$$

Thus, we can find an element $x \in \mathfrak{a}$ with $x \notin \bigcup_{\mathfrak{p}\in \operatorname{Ass}_R R} \mathfrak{p}$ and $x \notin \bigcup_{\mathfrak{q}\in \operatorname{Ass}_R M} \mathfrak{q}$. Then, it is clear that $\dim R/xR = d - 1$. Also, from the hypothesis (ii), *Independence Theorem* and Lemma 2.5, it follows that the R/xR-modules $H^1_{\mathfrak{a}+xR/xR}(L)$, $H^3_{\mathfrak{a}+xR/xR}(L)$ are $\mathfrak{a}+xR/xR$-cofinite, for every finitely generated R/xR-module L. So, by inductive hypothesis we have

$$\mathfrak{a}+xR/xR \in \mathscr{I}(R/xR).$$

Therefore, Lemma 2.3 implies that

$$H^i_{\mathfrak{a}+xR/xR}(M/xM) \in \mathscr{C}^1(R/xR, \mathfrak{a}+xR/xR)_{cof},$$

for all integers $i \geq 2$. Hence, by *Independence Theorem* and Lemma 2.5, we can deduce that $H^i_{\mathfrak{a}}(M/xM) \in \mathscr{C}^1(R,\mathfrak{a})_{cof}$, for all integers $i \geq 2$. As $x \in \mathfrak{a}$, for each $i \in \mathbb{N}_0$, we see that $\dim H^i_{\mathfrak{a}}(M) \leq 1$ if and only if $\dim\,(0 :_{H^i_{\mathfrak{a}}(M)} x) \leq 1$. Moreover, the exact sequence

$$0 \longrightarrow M \xrightarrow{x} M \longrightarrow M/xM \longrightarrow 0$$

induces a long exact sequence

$$\cdots \longrightarrow H^j_{\mathfrak{a}}(M) \xrightarrow{x} H^j_{\mathfrak{a}}(M) \longrightarrow H^j_{\mathfrak{a}}(M/xM)$$
$$\longrightarrow H^{j+1}_{\mathfrak{a}}(M) \xrightarrow{x} H^{j+1}_{\mathfrak{a}}(M) \longrightarrow \cdots .$$

Consequently, for all integers $j \geq 0$, there is a short exact sequence,

$$0 \longrightarrow H^j_{\mathfrak{a}}(M)/xH^j_{\mathfrak{a}}(M) \longrightarrow H^j_{\mathfrak{a}}(M/xM) \longrightarrow (0 :_{H^{j+1}_{\mathfrak{a}}(M)} x) \longrightarrow 0,$$

which implies that $\dim (0 :_{H^i_{\mathfrak{a}}(M)} x) \leq 1$, for all integers $i \geq 3$. Therefore, $\dim H^i_{\mathfrak{a}}(M) \leq 1$, for all integers $i \geq 3$. Hence, by Lemma 2.3, the R-module $(0 :_{H^3_{\mathfrak{a}}(M)} x)$ is $\mathfrak{a}$-cofinite since $H^3_{\mathfrak{a}}(M) \in \mathscr{C}^1(R, \mathfrak{a})_{cof}$. Therefore, by the exact sequence

$$0 \longrightarrow H^2_{\mathfrak{a}}(M)/xH^2_{\mathfrak{a}}(M) \longrightarrow H^2_{\mathfrak{a}}(M/xM) \longrightarrow (0 :_{H^3_{\mathfrak{a}}(M)} x) \longrightarrow 0,$$

it is concluded that $H^2_{\mathfrak{a}}(M)/xH^2_{\mathfrak{a}}(M)$ is $\mathfrak{a}$-cofinite. By the exact sequence

$$\Gamma_{\mathfrak{a}}(M/xM) \longrightarrow (0 :_{H^1_{\mathfrak{a}}(M)} x) \longrightarrow 0,$$

one deduces that $(0 :_{H^1_{\mathfrak{a}}(M)} x)$ is a finitely generated R-module. Therefore, the exact sequences

$$0 \longrightarrow (0 :_{H^1_{\mathfrak{a}}(M)} x) \longrightarrow H^1_{\mathfrak{a}}(M) \longrightarrow xH^1_{\mathfrak{a}}(M) \longrightarrow 0,$$

and

$$0 \longrightarrow xH^1_{\mathfrak{a}}(M) \longrightarrow H^1_{\mathfrak{a}}(M) \longrightarrow H^1_{\mathfrak{a}}(M)/xH^1_{\mathfrak{a}}(M) \longrightarrow 0,$$

show that $H^1_{\mathfrak{a}}(M)/xH^1_{\mathfrak{a}}(M)$ is $\mathfrak{a}$-cofinite. Therefore, the exact sequence

$$0 \longrightarrow H^1_{\mathfrak{a}}(M)/xH^1_{\mathfrak{a}}(M) \longrightarrow H^1_{\mathfrak{a}}(M/xM) \longrightarrow (0 :_{H^2_{\mathfrak{a}}(M)} x) \longrightarrow 0,$$

implies that $(0 :_{H^2_{\mathfrak{a}}(M)} x)$ is $\mathfrak{a}$-cofinite. Since $x \in \mathfrak{a}$ and both of the R-modules $(0 :_{H^2_{\mathfrak{a}}(M)} x)$, $H^2_{\mathfrak{a}}(M)/xH^2_{\mathfrak{a}}(M)$ are $\mathfrak{a}$-cofinite, so by Lemma 2.6, the R-module $H^2_{\mathfrak{a}}(M)$ is $\mathfrak{a}$-cofinite.

Now, using induction on i, we prove that the R-modules

$$\Gamma_{\mathfrak{a}}(M),\ H^1_{\mathfrak{a}}(M),\ H^2_{\mathfrak{a}}(M),\ H^3_{\mathfrak{a}}(M),\ \ldots,\ H^i_{\mathfrak{a}}(M),$$

are $\mathfrak{a}$-cofinite for all integers $i \geq 3$. For $i = 3$, there is nothing to prove. Suppose, inductively, that $i > 3$ and the result has been proved for $i - 1$. Then, by inductive hypothesis, the R-modules

$$\Gamma_{\mathfrak{a}}(M),\ H^1_{\mathfrak{a}}(M),\ H^2_{\mathfrak{a}}(M),\ H^3_{\mathfrak{a}}(M),\ \ldots,\ H^{i-1}_{\mathfrak{a}}(M),$$

are $\mathfrak{a}$-cofinite. Therefore, by Lemma 2.1, the R-modules

$$\mathrm{Hom}_R(R/\mathfrak{a}, H^i_{\mathfrak{a}}(M)),\ \ \mathrm{Ext}^1_R(R/\mathfrak{a}, H^i_{\mathfrak{a}}(M)),$$

are finitely generated. Since $\dim H^i_{\mathfrak{a}}(M) \leq 1$, it follows from Lemma 2.2, that the R-module $H^i_{\mathfrak{a}}(M)$ is $\mathfrak{a}$-cofinite. This completes the inductive hypothesis. So that, $\mathfrak{a} \in \mathscr{I}(R)$. □

Now we are ready to state and prove our main result.

Theorem 2.8. *Let R be a Noetherian ring and $\mathfrak{a}$ be an ideal of R. Then the following statements hold:*

(1) $\mathfrak{a} \in \mathscr{I}(R)$.
(2) $H^1_{\mathfrak{a}}(M)$ *and* $H^3_{\mathfrak{a}}(M)$ *are $\mathfrak{a}$-cofinite for every finitely generated R-module M.*

Proof. (1)$\Longrightarrow$(2) The assertion is obvious.

(2)$\Longrightarrow$(1) We claim that $\dim H^i_{\mathfrak{a}}(R) \leq 1$, for all integers $i \geq 2$. Assume that this is not the case. Then there is an integer $j \geq 2$ such that $\dim H^i_{\mathfrak{a}}(R) \geq 2$. Thus, there exists a maximal prime ideal $\mathfrak{n}$ of R such that $\dim H^j_{\mathfrak{a}R_{\mathfrak{n}}}(R_{\mathfrak{n}}) = \dim (H^j_{\mathfrak{a}}(R))_{\mathfrak{n}} \geq 2$. From the hypothesis (ii), it is concluded that the $R_{\mathfrak{n}}$-modules $H^1_{\mathfrak{a}R_{\mathfrak{n}}}(N)$ and $H^3_{\mathfrak{a}R_{\mathfrak{n}}}(N)$ are $\mathfrak{a}R_{\mathfrak{n}}$-cofinite, for every finitely generated $R_{\mathfrak{n}}$-module N. Hence, by Lemma 2.7, we have $\dim (H^j_{\mathfrak{a}}(R))_{\mathfrak{n}} = \dim H^j_{\mathfrak{a}R_{\mathfrak{n}}}(R_{\mathfrak{n}}) \leq 1$, which is a contradiction. Therefore, $\dim H^i_{\mathfrak{a}}(R) \leq 1$, for all integers $i \geq 2$. Moreover, by the hypothesis, the R-modules $\Gamma_{\mathfrak{a}}(R)$ and $H^1_{\mathfrak{a}}(R)$ are $\mathfrak{a}$-cofinite. Therefore, applying the method used in the proof of Lemma 2.7, one sees that the R-modules $H^i_{\mathfrak{a}}(R)$ are $\mathfrak{a}$-cofinite for all integers $i \in \mathbb{N}_0$. Consequently, $H^i_{\mathfrak{a}}(R) \in \mathscr{C}^1(R, \mathfrak{a})_{cof}$, for all integers $i \geq 2$. Therefore, $\mathfrak{a} \in \mathscr{I}(R)$, by Lemma 2.4. □

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ON THE COFINITENESS OF LOCAL COHOMOLOGY MODULES

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فرض کنیم $\mathfrak{a}$ ایده‌آلی از حلقه نوتری R باشد به‌طوری که برای هر R-مدول متناهی مولد M، R-مدول‌های $H^{1}_{\mathfrak{a}}(M)$ و $H^{3}_{\mathfrak{a}}(M)$، $\mathfrak{a}$-هم‌متناهی باشند. در این مقاله ثابت شده است که برای هر R-مدول متناهی مولد M و به ازای هر $i \in \mathbb{N}_0$، $H^{i}_{\mathfrak{a}}(M)$، $\mathfrak{a}$-هم‌متناهی است.

کلمات کلیدی: مدول هم‌متناهی، کوهمولوژی موضعی، حلقه نوتری، بعد کرول.