

ON THE METABOLICITY INDEX OF ORTHOGONAL INVOLUTIONS IN CHARACTERISTIC TWO

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ABSTRACT. The metabolicity index of orthogonal involutions in characteristic two is explicitly computed. As an application, the behavior of this index over separable extensions is studied. The metabolicity index of orthogonal sum of hermitian forms is also studied.

1. INTRODUCTION

Metabolic involutions appear as natural generalizations of metabolic bilinear forms to the theory of algebras with involution. This notion was defined first in [1] and studied in more detail in [4]. While the metabolicity and hyperbolicity of involutions in characteristic different from two are equivalent, metabolic involutions have far more structure than hyperbolic involutions in characteristic two. Hence, they should be studied in more detail in this case.

In [8], the notion of metabolicity index of an involution in characteristic two was defined. It was shown that this index has a nice behavior over certain field extensions. For example, it is preserved over odd degree extensions. Also, it is preserved over certain separable extensions, subject to some additional conditions (see [8, (4.2) and (4.11)]). Moreover, the metabolicity index was used to study Pfister metabolic involutions (see [8, (7.1)]). These results show that the definition of the metabolicity index is very natural and has similar applications to similar notions for symmetric bilinear forms.

In view of [8, (3.5)], the metabolicity index of a hyperbolic involution depends only on the co-index of the underlying algebra. Since every metabolic symplectic involution is hyperbolic, the metabolicity index would be most interesting for orthogonal involutions. The aim of this work is to obtain a formula for the metabolicity index of orthogonal involutions. The main source of inspiration for us is the intuitive idea that hyperbolic involutions are ‘very metabolic’, so symplectic involutions that appear inside metabolic

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involutions will contribute a lot to the metabolicity index. In other words, metabolic algebras with involution with ‘large’ symplectic parts will have large metabolicity indices. Since every algebra with involution decomposes as an orthogonal sum of its symplectic and totally orthogonal parts, the larger the symplectic part is, the smaller the orthogonal part shall be. Considering this, we explicitly compute the metabolicity index, in terms of the co-indices of the underlying algebra and its totally orthogonal part (see 4.5).

In section 5, the behavior of the metabolicity index over separable extensions is studied. For a separable extension K/F and an algebra with involution (A, σ) , we first obtain several sufficient conditions for the equality $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$ in 5.1. Next, in 5.4, a necessary and sufficient condition for the equality $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$ is given. As already mentioned, the metabolicity index over field extensions was studied in [8, §4] in certain special cases. However, our results notably improve that of [8].

In section 6, we use the natural correspondence between hermitian forms and their adjoint involutions to define the metabolicity index of a hermitian form. We then study the metabolicity index of hermitian forms under orthogonal sum. This gives another application of our main result, which is interesting in our opinion (see Propositions 6.6).

2. PRELIMINARIES

Throughout this work, all fields are supposed to be of characteristic two. Also, all algebras considered in this work are supposed to be finite dimensional over their base fields.

Let A be a central simple algebra over a field F . The *degree* of A is defined as $\deg_F A = \sqrt{\dim_F A}$, where $\dim_F A$ is the dimension of A over F . By [5, §9, Theorem 1], there exists a division algebra D over F and a positive integer n such that $A \simeq M_n(D)$, where $M_n(D)$ is the full matrix algebra over D . The integers $\deg_F D$ and n are called the *index* and the *co-index* of A and denoted by $\text{ind}(A)$ and $\mathbf{i}(A)$, respectively. Also, the order of A in the Brauer group of F is called the *exponent* of A .

By an *involution* on A , we mean an antiautomorphism of A of order two. If σ is an involution on A , the pair (A, σ) is simply called an *algebra with involution*. The involution σ is said to be of *the first kind* if it restricts to the identity map on F . Otherwise, it is said to be of *the second kind*. Every involution of the first kind is either *symplectic* or *orthogonal*, depending on whether it becomes adjoint to an alternating or a non-alternating symmetric bilinear form over a splitting field (see [7, (2.5)]).

An involution σ is called *isotropic* if there exists a nonzero element $x \in A$ such that $\sigma(x)x = 0$. Otherwise, it is called *anisotropic*. An involution σ is called *hyperbolic* if there exists an idempotent $e \in A$ satisfying $\sigma(e) = 1 - e$. The involution σ is called *metabolic* if there exists an idempotent $e \in A$ such that $\sigma(e)e = 0$ and $\dim_F eA = \frac{1}{2} \dim_F A$. Every hyperbolic involution is metabolic, but the converse is not true in general (see [4, (4.9) and (4.11)]). Set

$$\text{Sym}(A, \sigma) = \{x \in A \mid \sigma(x) = x\} \quad \text{and} \quad \text{Symd}(A, \sigma) = \{x + \sigma(x) \mid x \in A\}.$$

As in [3], we say that σ is *direct* if $\sigma(x)x \in \text{Symd}(A, \sigma)$ implies that $x = 0$ for $x \in A$.

Let (D, θ) be a division F -algebra with involution. By a *hermitian space* over (D, θ) we mean a pair (V, h) , where V is a right vector space over D and $h : V \times V \rightarrow D$ is a hermitian form. Hence, h is a biadditive map satisfying $h(u\alpha, v\beta) = \theta(\alpha)h(u, v)\beta$ and $h(v, u) = \theta(h(u, v))$, for all $u, v \in V$ and $\alpha, \beta \in D$. The form h is called *even* if $h(v, v) \in \text{Symd}(D, \theta)$ for all $v \in V$. For $\alpha_1, \dots, \alpha_n \in \text{Sym}(D, \theta)$, the diagonal hermitian form $h : D^n \times D^n \rightarrow D$ defined by

$$(u, v) \mapsto \sum_{i=1}^n \theta(u_i) \alpha_i v_i,$$

is denoted by $\langle \alpha_1, \dots, \alpha_n \rangle_{(D, \theta)}$. For a positive integer n , we denote by nh the hermitian form $h \perp \dots \perp h$ (n times), where $\perp$ denotes the orthogonal sum.

All hermitian spaces (V, h) considered in this work are supposed to be *nondegenerate*, meaning that there is no nonzero vector $x \in V$ such that $h(x, y) = 0$ for all $y \in V$. Let (V, h) and (V', h') be hermitian spaces over (D, θ) . An *isometry* between (V, h) and (V', h') is an isomorphism of right vector spaces $f : V \rightarrow V'$ such that

$$h'(f(u), f(v)) = h(u, v),$$

for all $u, v \in V$. A hermitian form h is called *isotropic* if $h(v, v) = 0$ for some nonzero vector $v \in V$ and *anisotropic* otherwise. We say that h is *metabolic* if there exists a subspace W of V such that $\dim_D W = \frac{1}{2} \dim_D V$ and $h|_{W \times W} = 0$. The *hyperbolic plane* over (D, θ) is the unique, up to isometry, two-dimensional hermitian form h with a basis $\{u, v\}$ satisfying $h(u, u) = h(v, v) = 0$. The hyperbolic plane over (D, θ) is denoted by $\mathbb{H}_{(D, \theta)}$, or simply $\mathbb{H}$ if the pair (D, θ) is clear from the context. A hermitian form h is called *hyperbolic* if it is isometric to $n\mathbb{H}$ for some positive integer n . According to [6, Ch. I, (3.7.1)], every even metabolic hermitian form is hyperbolic. As

in [3], we say that a hermitian form h is *direct* if there is no nonzero vector $v \in V$ such that $h(v, v) \in \text{Symd}(D, \theta)$.

Let (D, θ) be a division algebra with involution over a field F and let (V, h) be a hermitian space over (D, θ) . Then there exists a decomposition

$$h \simeq h_{\text{dir}} \perp 2\tilde{h} \perp h_{\text{ev}} \perp k\mathbb{H},$$

where $h_{\text{dir}} \perp \tilde{h}$ is direct and h_{ev} is even. Moreover, the form $h_{\text{dir}} \perp 2\tilde{h}$ is unique, up to isomorphism (see [3, (6.5)]). As in [3], we call the form $h_{\text{dir}} \perp 2\tilde{h}$ the *quasi direct part* of h and denote it by h_{qd} . Note that a hermitian form h is even if and only if h_{qd} is trivial.

Let (V, h) be a hermitian space over some division F -algebra with involution of the first kind (D, θ) . The *adjoint involution* of $\text{End}_D(V)$ with respect to h is denoted by $\text{ad}(h)$. By [7, (4.2)], the involution $\text{ad}(h)$ is symplectic if and only if h is even. It is also known that a hermitian form is isotropic (resp. metabolic, hyperbolic, direct) if and only if $\text{ad}(h)$ is isotropic (resp. metabolic, hyperbolic, direct); see [4, (4.8)], [7, (6.7)] and [3, (7.3)]. In particular, every metabolic symplectic involution is hyperbolic.

3. PARTS OF ALGEBRAS WITH INVOLUTION

In this section, we review and complement some results of [3], concerning parts of algebras with involution.

Let (A, σ) be an algebra with involution over a field F . An algebra with involution (B, τ) is called a *part* of (A, σ) if there exists a nonzero idempotent $e \in \text{Sym}(A, \sigma)$ and an F -algebra isomorphism $\phi : B \rightarrow eAe$ such that $\phi \circ \tau = \sigma \circ \phi$. In this case, we say that the idempotent e *defines* (B, τ) . Note that the F -algebra B is Brauer-equivalent to A (see [1, p. 309]).

Let (A_1, σ_1) be a part of (A, σ) defined by $e \in \text{Sym}(A, \sigma)$. The part of (A, σ) defined by $1 - e$ is called a *counterpart* of (A_1, σ_1) . Following [2], we say that (A, σ) is an *orthogonal sum* of (A_1, σ_1) and (A_2, σ_2) , denoted by

$$(A, \sigma) \simeq (A_1, \sigma_1) \boxplus (A_2, \sigma_2),$$

if (A_1, σ_1) is isomorphic to a part of (A, σ) and (A_2, σ_2) is isomorphic to its counterpart.

Let (A, σ) be an algebra with involution of the first kind over a field F . By [3, (4.5)], if σ is not metabolic then there exists a unique, up to isomorphism, F -algebra with anisotropic involution (B, τ) , which is a part of (A, σ) , such that every counterpart of (B, τ) is metabolic. As in [3], we call (B, τ) the *anisotropic part* of (A, σ) and denote it by $(A, \sigma)_{\text{an}}$.

An algebra with involution (A, σ) is called *totally orthogonal* if there is no nonzero idempotent $e \in \text{Symd}(A, \sigma)$. Every algebra with involution of the first kind (A, σ) has a decomposition $(A, \sigma) \simeq (A_1, \sigma_1) \boxplus (A_2, \sigma_2)$, where (A_1, σ_1) is totally orthogonal and (A_2, σ_2) is symplectic. Furthermore, the pairs (A_1, σ_1) and (A_2, σ_2) are unique, up to isomorphism (see [3, (8.3)]). As in [3], we call (A_1, σ_1) and (A_2, σ_2) the *totally orthogonal part* and the *symplectic part* of (A, σ) and denote them by $(A, \sigma)_{\text{to}}$ and $(A, \sigma)_{\text{symp}}$, respectively. Moreover, we denote the underlying algebras A_1 and A_2 by A_{to} and A_{symp} , respectively.

Let $(A, \sigma) \simeq \text{ad}(h)$ for some hermitian form h over a division F -algebra with involution of the first kind (D, θ) . Then $(A, \sigma)_{\text{to}} \simeq \text{ad}(h_{\text{qd}})$ by [3, (8.4)]. As already mentioned, a hermitian form is even if and only if its quasi direct part is trivial. Hence, (A, σ) is symplectic if and only if $(A, \sigma)_{\text{to}}$ is trivial.

Lemma 3.1. *Let K/F be a field extension and let (A, σ) be an algebra with involution of the first kind over F . Then there exists an embedding $((A, \sigma)_K)_{\text{to}} \hookrightarrow ((A, \sigma)_{\text{to}})_K$. In particular, we have $\deg_K(A_K)_{\text{to}} \leq \deg_F A_{\text{to}}$.*

Proof. Using the decomposition $(A, \sigma) \simeq (A, \sigma)_{\text{to}} \boxplus (A, \sigma)_{\text{symp}}$, one has

$$(A, \sigma)_K \simeq ((A, \sigma)_{\text{to}})_K \boxplus ((A, \sigma)_{\text{symp}})_K.$$

Clearly, $((A, \sigma)_{\text{symp}})_K$ is symplectic. The uniqueness of $((A, \sigma)_K)_{\text{to}}$ implies that

$$((A, \sigma)_K)_{\text{to}} \simeq (((A, \sigma)_{\text{to}})_K)_{\text{to}} \hookrightarrow ((A, \sigma)_{\text{to}})_K. \quad \square$$

The following result strengthens [3, (9.10)].

Proposition 3.2. *Let (A, σ) be an algebra with involution of the first kind over a field F . If K/F is a separable field extension, then*

$$((A, \sigma)_K)_{\text{to}} \simeq ((A, \sigma)_{\text{to}})_K.$$

Proof. Let L/K be a separable field extension which splits A_K . Then

$$\deg_L(A_L)_{\text{to}} \leq \deg_K(A_K)_{\text{to}} \leq \deg_F A_{\text{to}},$$

by 3.1. On the other hand, $((A, \sigma)_L)_{\text{to}} \simeq ((A, \sigma)_{\text{to}})_L$ by [3, (9.10)]. Hence, $\deg_L(A_L)_{\text{to}} = \deg_L(A_{\text{to}})_L = \deg_F A_{\text{to}}$, which implies that

$$\deg_K(A_K)_{\text{to}} = \deg_F A_{\text{to}} = \deg_K(A_{\text{to}})_K.$$

The result now follows from 3.1. $\square$

Corollary 3.3. *Let K/F be a separable field extension and let (A, σ) be an algebra with orthogonal involution over F . Then $\frac{i(A_K)}{i((A_K)_{\text{to}})} = \frac{i(A)}{i(A_{\text{to}})}$.*

Proof. Since A is Brauer-equivalent to A_{to} , we have

$$\frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} = \frac{\deg(A)}{\deg(A_{\text{to}})}.$$

Similarly, we have $\frac{\mathbf{i}(A_K)}{\mathbf{i}((A_K)_{\text{to}})} = \frac{\deg(A_K)}{\deg((A_K)_{\text{to}})}$. On the other hand,

$$((A, \sigma)_K)_{\text{to}} \simeq ((A, \sigma)_{\text{to}})_K$$

by 3.2. Hence,

$$\frac{\mathbf{i}(A_K)}{\mathbf{i}((A_K)_{\text{to}})} = \frac{\deg(A_K)}{\deg((A_{\text{to}})_K)} = \frac{\deg A}{\deg A_{\text{to}}} = \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})}. \quad \square$$

The next result generalizes [3, (9.9)].

Proposition 3.4. *Let K/F be a separable field extension and let (A, σ) be an algebra with involution over F . If (A, σ) is totally orthogonal then so is $(A, \sigma)_K$.*

Proof. Let τ be a symplectic involution on A . By [3, (4.7)], there exists an algebra with involution (B, ρ) , which is isomorphic to an orthogonal sum of (A, σ) and (A, τ) . Hence, $(B, \rho)_{\text{to}} \simeq (A, \sigma)$. By 3.2, we have $((B, \rho)_K)_{\text{to}} \simeq (A, \sigma)_K$, which implies, in particular, that $(A, \sigma)_K$ is totally orthogonal. $\square$

4. THE METABOLICITY INDEX

In this section, we compute the metabolicity index of an algebra with involution of the first kind. We first recall some constructions from [8]. Let (A, σ) be an algebra with involution over a field F . We say that an F -algebra with involution (B, τ) *divides* (A, σ) if there exists an F -algebra with involution (C, ρ) such that $(A, \sigma) \simeq (B, \tau) \otimes (C, \rho)$. The *metabolicity index* of (A, σ) , denoted by $\mathbf{i}(A, \sigma)$, is defined as the largest integer r for which $\bigotimes_{i=1}^r (M_2(F), \rho_i)$ divides (A, σ) , where each ρ_i is either the canonical involution or the transpose involution on $M_2(F)$. Recall that the *canonical* involution γ on a quaternion algebra Q (i.e., a central simple algebra of degree two) is defined as $\gamma(x) = \text{Trd}_Q(x) - x$, where $\text{Trd}_Q(x)$ is the reduced trace of x . Clearly, we have $\mathbf{i}(A, \sigma) \leq \nu(\mathbf{i}(A))$, where $\nu(\mathbf{i}(A))$ is the largest integer m for which 2^m divides $\mathbf{i}(A)$. Note that σ is metabolic if and only if $\mathbf{i}(A, \sigma) \neq 0$ (see [8, (3.2)]). Also, in view of [8, (3.6)], $\mathbf{i}(A, \sigma)$ is the largest integer r for which there exists a hermitian form h such that $(A, \sigma) \simeq \text{ad}(2^r h)$, except for the case where A splits and σ is hyperbolic. This exceptional case can be easily

settled by the following result concerning the metabolicity index of algebras with hyperbolic involution.

Proposition 4.1. *Let (A, σ) be an algebra with involution of the first kind over a field F . If σ is hyperbolic, then $\mathbf{i}(A, \sigma) = \nu(\mathbf{i}(A))$.*

Proof. See [8, (3.5)]. □

Proposition 4.2. *Let (A, σ) be an algebra with involution of the first kind over a field F and let r be a nonnegative integer. If σ is metabolic, then $\mathbf{i}(A, \sigma) > r$ if and only if $\nu(\mathbf{i}(A)) > r$ and $\mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^r} \mathbf{i}(A)$.*

Proof. If σ is symplectic, then it is hyperbolic and A_{to} is trivial. Hence, the result follows from 4.1. Suppose that σ is orthogonal. Write $(A, \sigma) \simeq \text{ad}(h)$, where h is a hermitian form over some division F -algebra with involution (D, θ) . The metabolicity of σ implies that h is also metabolic. By [3, (6.4)], one can write $h \simeq 2\eta \perp k\mathbb{H}$, where η is a direct hermitian form over (D, θ) and k is a nonnegative integer. Set $s = \dim_D \eta$. Then $\mathbf{i}(A) = \dim_D h = 2s + 2k$. Also, $(A, \sigma)_{\text{to}} \simeq \text{ad}(2\eta)$ and $\mathbf{i}(A_{\text{to}}) = 2s$.

Suppose first that $\mathbf{i}(A, \sigma) > r$. Then $\nu(\mathbf{i}(A)) \geq \mathbf{i}(A, \sigma) > r$. We have to prove that $\mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^r} \mathbf{i}(A)$. Since $\mathbf{i}(A, \sigma) > r$, there exists a hermitian form ξ over (D, θ) such that $h \simeq 2^{r+1}\xi$. Moreover, by [8, (2.8)], there exists an integer k' such that $2^{r+1}\xi \simeq 2\xi \perp k'\mathbb{H}$. Thus,

$$h \simeq 2\eta \perp k\mathbb{H} \simeq 2\xi \perp k'\mathbb{H}.$$

Since 2ξ is metabolic, by [3, (6.4)] we have $2\xi \simeq 2\xi' \perp k''\mathbb{H}$, where ξ' is a direct hermitian form and k'' is a nonnegative integer. Hence,

$$h \simeq 2\eta \perp k\mathbb{H} \simeq 2\xi' \perp k'\mathbb{H} \perp k''\mathbb{H}.$$

Using [3, (6.5)], one concludes that $2\eta \simeq 2\xi'$. Thus, $k = k' + k''$, which implies, in particular, that $k \geq k'$. Set $s' = \dim_D \xi$. The relations

$$\mathbf{i}(A) = \dim_D h = 2s + 2k = 2s' + 2k',$$

imply that $s' \geq s$. Hence, $2^{r+1}s' \geq 2^{r+1}s$. Also, since $\mathbf{i}(A) = 2s + 2k = 2^{r+1}s'$, we have $2s + 2k \geq 2^{r+1}s$. Dividing by 2^r , one concludes that

$$\mathbf{i}(A_{\text{to}}) = 2s \leq \frac{1}{2^r}(2s + 2k) = \frac{1}{2^r} \mathbf{i}(A).$$

Conversely, suppose that $\nu(\mathbf{i}(A)) > r$ and $\mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^r} \mathbf{i}(A)$. Then 2^{r+1} divides $\mathbf{i}(A)$, which implies that 2^r divides $s + k$. Since

$$2s = \mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^r} \mathbf{i}(A) = \frac{1}{2^r}(2s + 2k),$$

one has $2^r s \leq s + k$. Hence, $k - (2^r - 1)s \geq 0$. We also have

$$k + s - (k - (2^r - 1)s) = 2^r s,$$

and $2^r \mid k + s$, which implies that 2^r divides $k - (2^r - 1)s$. Hence,

$$k - (2^r - 1)s = 2^r m$$

for some nonnegative integer m . We now consider two cases.

Case 1. Suppose that either $D \neq F$ or m is even. In both cases, there exists an even hermitian form η_1 of dimension m over (D, θ) . Set $\eta' = \eta \perp \eta_1$. The form $2^{r+1}\eta_1$ is metabolic and even, hence it is hyperbolic, i.e., $2^{r+1}\eta_1 \simeq 2^r m \mathbb{H}$. Since $\dim_D \eta = s$, by [8, (2.8)] we have $2^{r+1}\eta \simeq 2\eta \perp (2^r - 1)s\mathbb{H}$. Hence,

$$\begin{aligned} 2^{r+1}\eta' &\simeq 2^{r+1}\eta \perp 2^{r+1}\eta_1 \\ &\simeq 2\eta \perp (2^r - 1)s\mathbb{H} \perp 2^r m\mathbb{H} \\ &\simeq 2\eta \perp ((2^r - 1)s + 2^r m)\mathbb{H}. \end{aligned}$$

Since $(2^r - 1)s + 2^r m = k$, one has $2^{r+1}\eta' \simeq 2\eta \perp k\mathbb{H} \simeq h$. Hence, $\mathbf{i}(A, \sigma) > r$, proving the result.

Case 2. Suppose that $D = F$ and $m = 2l + 1$ for some nonnegative integer l . In this case, h and η are symmetric bilinear forms over F . Since η is anisotropic, there exists $a_1, \dots, a_s \in F^\times$ such that $\eta = \langle a_1, \dots, a_s \rangle_{(F, \text{id})}$. Note that $s > 0$, as σ is orthogonal. Write $\eta = \eta_1 \perp \eta_2$, where $\eta_1 = \langle a_1, \dots, a_{s-1} \rangle_{(F, \text{id})}$ and $\eta_2 = \langle a_s \rangle_{(F, \text{id})}$. Set $\mathbf{b} = \eta \perp \eta_2 \perp l\mathbb{H}$. By repeated use of [8, (2.8)] we have

$$\begin{aligned} 2^{r+1}\mathbf{b} &\simeq 2\eta \perp (2^r - 1)s\mathbb{H} \perp 2\eta_2 \perp (2^r - 1)\mathbb{H} \perp 2^{r+1}l\mathbb{H} \\ &\simeq 2\eta_1 \perp 4\eta_2 \perp ((2^r - 1)s + 2^{r+1}l + 2^r - 1)\mathbb{H} \\ &\simeq 2\eta_1 \perp 2\eta_2 \perp \mathbb{H} \perp ((2^r - 1)s + 2^r(2l + 1) - 1)\mathbb{H} \\ &\simeq 2\eta \perp ((2^r - 1)s + 2^r m)\mathbb{H} \\ &\simeq 2\eta \perp k\mathbb{H} \simeq h. \end{aligned}$$

It follows that $\mathbf{i}(A, \sigma) > r$, completing the proof. $\square$

Proposition 4.3. *Let (A, σ) be an algebra with orthogonal involution over a field F and set $m = \max\{s \in \mathbb{Z} \mid \mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^s} \mathbf{i}(A)\}$. If σ is metabolic then $\mathbf{i}(A, \sigma) = \min\{\nu(\mathbf{i}(A)), m + 1\}$.*

Proof. Set $r = \nu(\mathbf{i}(A))$. We consider two cases.

Case 1. $m + 1 \leq r$. Then $m \leq r - 1 < \nu(\mathbf{i}(A))$. We also have $\mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^m} \mathbf{i}(A)$.

Hence, $\mathbf{i}(A, \sigma) > m$ by 4.2. Now, if $\mathbf{i}(A, \sigma) > m + 1$, then $\mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^{m+1}}\mathbf{i}(A)$ by 4.2, contradicting the maximality of m . Hence,

$$\mathbf{i}(A, \sigma) = m + 1 = \min\{\nu(\mathbf{i}(A)), m + 1\}.$$

Case 2. $r < m + 1$. Then $\mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^{r-1}}\mathbf{i}(A)$. Since $\nu(\mathbf{i}(A)) = r > r - 1$, 4.2 implies that $\mathbf{i}(A, \sigma) > r - 1$, or equivalently, $\mathbf{i}(A, \sigma) \geq r$. On the other hand, $r = \nu(\mathbf{i}(A)) \geq \mathbf{i}(A, \sigma)$. Hence,

$$\mathbf{i}(A, \sigma) = r = \min\{\nu(\mathbf{i}(A)), m + 1\},$$

proving the result. $\square$

4.3 can be generalized to an arbitrary involution of the first kind, replacing the maximum by supremum, as follows.

Theorem 4.4. *Let (A, σ) be an algebra with involution of the first kind over a field F and set $m = \sup\{s \in \mathbb{Z} \mid \mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^s}\mathbf{i}(A)\}$. If σ is metabolic then $\mathbf{i}(A, \sigma) = \min\{\nu(\mathbf{i}(A)), m + 1\}$.*

Proof. If σ is orthogonal, then A_{to} is nontrivial and

$$m = \max\{s \in \mathbb{Z} \mid \mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^s}\mathbf{i}(A)\}.$$

Hence, the result follows from 4.3.

Suppose that σ is symplectic, or equivalently, A_{to} is trivial. Then $\mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^s}\mathbf{i}(A)$ for every nonnegative integer s , i.e., $m = \infty$. Hence,

$$\min\{\nu(\mathbf{i}(A)), m + 1\} = \nu(\mathbf{i}(A)).$$

On the other hand, since σ is symplectic and metabolic, it is hyperbolic and $\mathbf{i}(A, \sigma) = \nu(\mathbf{i}(A))$ by 4.1. This completes the proof. $\square$

According to 4.1, the metabolicity index of an algebra with symplectic involution (A, σ) can be easily computed. Indeed, if σ is not metabolic, then $\mathbf{i}(A, \sigma) = 0$. Otherwise, σ is hyperbolic and $\mathbf{i}(A, \sigma) = \nu(\mathbf{i}(A))$. The next result gives an explicit formula for the metabolicity index of orthogonal involutions. For a real number x , we denote by $\lfloor x \rfloor$ the greatest integer less than or equal to x .

Theorem 4.5. *Let (A, σ) be an algebra with orthogonal involution over a field F . If σ is metabolic then*

$$\mathbf{i}(A, \sigma) = \min \left\{ \nu(\mathbf{i}(A)), \left\lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \right\rfloor + 1 \right\}.$$

Proof. Observe first that since σ is orthogonal, A_{to} is nontrivial, i.e., $\mathbf{i}(A_{\text{to}}) \neq 0$. Set

$$m = \max\{s \in \mathbb{Z} \mid \mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^s} \mathbf{i}(A)\}.$$

We have $\mathbf{i}(A_{\text{to}}) \leq \frac{1}{2^m} \mathbf{i}(A)$, which implies that $2^m \leq \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})}$. Hence, $m \leq \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})}$. The maximality of m implies that $m = \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor$. The result now follows from 4.3. $\square$

Remark 4.6. 4.5 cannot be applied for symplectic involutions, as A_{to} is trivial and the quotient $\frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})}$ is undefined in this case. Also, the result is not necessarily true for nonmetabolic involutions. To construct an example, let (Q, γ) be a quaternion algebra with the canonical involution over a field F . Suppose that there exists $\alpha \in Q$ such that the form $\langle 1, \alpha \rangle_{(Q, \gamma)}$ is not isotropic (i.e., $\gamma(x)x \neq \alpha$ for all $x \in Q$). Set

$$(A, \sigma) = \text{ad}(\langle 1, \alpha \rangle_{(Q, \gamma)}) \simeq \text{ad}(\langle 1, \alpha \rangle) \otimes (Q, \gamma).$$

Then $\mathbf{i}(A) = 2$, i.e., $\nu(\mathbf{i}(A)) = 1$. Since $\mathbf{i}(A) \geq \mathbf{i}(A_{\text{to}})$, we have $\lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor \geq 0$. It follows that $\min\{\nu(\mathbf{i}(A)), \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor + 1\} = 1$. However, $\mathbf{i}(A, \sigma) = 0$, because σ is not metabolic.

5. SEPARABLE EXTENSIONS

In this section, we study conditions under which the metabolicity index of an algebra with involution is preserved under separable extensions. As observed in [8, (4.1)], if (A, σ) is an algebra with symplectic metabolic involution over a field F , then $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$ if and only if $\mathbf{i}(A_K) = \mathbf{i}(A)$. We therefore consider orthogonal involutions in the sequel. We start with some sufficient conditions for the equality $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$.

Proposition 5.1. *Let K/F be a separable field extension and let (A, σ) be an algebra with orthogonal involution over F . Suppose that σ is metabolic. If any of the following conditions occur, then $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$.*

- (i) $\mathbf{i}(A_K) = \mathbf{i}(A)$;
- (ii) $\mathbf{i}((A_K)_{\text{to}}) = \mathbf{i}(A_{\text{to}})$;
- (iii) $\nu(\mathbf{i}(A)) > \mathbf{i}(A, \sigma)$;
- (iv) $\mathbf{i}(A, \sigma) = \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor + 1$.
- (v) $\nu(\mathbf{i}(A)) > \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor$.

Proof. Observe first that conditions (i) and (ii) are equivalent by 3.3. Also, since σ is metabolic, using 4.5 one concludes that (iv) and (v) are equivalent; and (iii) implies (iv). Hence, it suffices to prove the result for (i) and (iv).

If $\mathbf{i}(A_K) = \mathbf{i}(A)$ then $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$ by 3.3 and 4.5.

Suppose that $\mathbf{i}(A, \sigma) = \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor + 1$. Then

$$\lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor + 1 \leq \nu(\mathbf{i}(A)).$$

In view of 3.3, we have

$$\lfloor \log_2 \frac{\mathbf{i}(A_K)}{\mathbf{i}((A_K)_{\text{to}})} \rfloor + 1 = \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor + 1 \leq \nu(\mathbf{i}(A)) \leq \nu(\mathbf{i}(A_K)).$$

It follows from 4.5 that

$$\mathbf{i}((A, \sigma)_K) = \lfloor \log_2 \frac{\mathbf{i}(A_K)}{\mathbf{i}((A_K)_{\text{to}})} \rfloor + 1 = \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor + 1 = \mathbf{i}(A, \sigma). \quad \square$$

Remark 5.2. (1) Condition (iii) in 5.1 was already stated in [8, (4.9)] with a different proof.

(2) 5.1 can be used to give an immediate proof of [8, (4.2)], in the case where σ is metabolic. It should be noted that the remaining case where σ is not metabolic can be easily proved, as in [8, (4.2)].

Remark 5.3. Condition (iv) in 5.1 automatically implies that σ is metabolic. However, for other conditions, the result is no longer valid without the hypothesis that σ is metabolic. This can be checked by Example 4.5 in [8]. Indeed, this example shows that if σ is not metabolic, then the condition $\mathbf{i}(A_K) = \mathbf{i}(A)$ does not guarantee that $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$. As already observed, conditions (i) and (ii) in 5.1 are equivalent. Hence, for these conditions, 5.1 is not true in the case where σ is not metabolic. Conditions (iii) and (v) can also be treated by this example. Before that, it is worth noting that the equivalence of (iv) and (v), stated in the proof of 5.1, is based on 4.5, which is no longer valid if σ is not metabolic.

We now check conditions (iii) and (v). Let $K = F(x, y, z, w)$, where F is a field and x, y, z, w are independent variables. Let ϕ be the anisotropic part of the quadratic form $\langle\langle z, y, x \rangle\rangle \perp [1, w]$ and set $L = K(\phi)$ be the function field of ϕ over K . Let Q be the quaternion algebra over L whose norm form is $\langle\langle y, x \rangle\rangle$. Set $(A, \sigma) = \text{Ad}(\langle\langle 1, z \rangle\rangle_{(Q, \gamma)})$, where γ is the canonical involution on Q . Consider the L -algebra with involution

$$(Q \otimes_L M_2(L), \rho \otimes t),$$

where ρ is any orthogonal involution on Q . Note that A is Brauer-equivalent to $Q \otimes_L M_2(L)$. Also, the involutions σ and $\rho \otimes t$ are of the first kind. Hence, by [3, (4.7)], there exists an L -algebra with involution (B, τ) , which is isomorphic to an orthogonal sum of (A, σ) and $(Q \otimes_L M_2(L), \rho \otimes t)$. Note that $\mathbf{i}(B) = 4$ and τ is not metabolic, i.e., $\mathbf{i}(B, \tau) = 0$. Hence, $\nu(\mathbf{i}(B)) > \mathbf{i}(B, \tau)$. We also have $B_{\text{to}} \simeq (M_2(L), t) \otimes (Q, \rho)$, which implies that $\mathbf{i}(B_{\text{to}}) = 2$. It follows that $\nu(\mathbf{i}(B)) > \lfloor \log_2 \frac{\mathbf{i}(B)}{\mathbf{i}(B_{\text{to}})} \rfloor$. Hence, conditions (iii) and (v) are both satisfied for (B, τ) . However, as proved in [8, (4.5)], (B, τ) is metabolic over the quadratic separable extension $L(\wp^{-1}(w))/L$. Hence, $\mathbf{i}((B, \tau)_{L(\wp^{-1}(w))}) \neq \mathbf{i}(B, \tau)$.

We are now ready to give the following necessary and sufficient condition for the equality $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$.

Theorem 5.4. *Let K/F be a separable field extension and let (A, σ) be an algebra with orthogonal involution over F . If σ is metabolic, then $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$ if and only if $\mathbf{i}(A_K) = \mathbf{i}(A)$ or $\nu(\mathbf{i}(A)) > \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor$.*

Proof. The ‘if’ implication follows from 5.1. Conversely, suppose that $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$. If $\nu(\mathbf{i}(A)) \leq \lfloor \log_2 \frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} \rfloor$, then

$$\mathbf{i}(A, \sigma) = \nu(\mathbf{i}(A)), \quad (5.1)$$

by 4.5. The assumption $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$ and the equality $\frac{\mathbf{i}(A)}{\mathbf{i}(A_{\text{to}})} = \frac{\mathbf{i}(A_K)}{\mathbf{i}((A_K)_{\text{to}})}$ in 3.3 imply that

$$\mathbf{i}((A, \sigma)_K) \leq \lfloor \log_2 \frac{\mathbf{i}(A_K)}{\mathbf{i}((A_{\text{to}})_K)} \rfloor.$$

Hence, $\mathbf{i}((A, \sigma)_K) = \nu(\mathbf{i}(A_K))$ by 4.5. It now follows from (5.1) and the equality $\mathbf{i}((A, \sigma)_K) = \mathbf{i}(A, \sigma)$ that $\nu(\mathbf{i}(A_K)) = \nu(\mathbf{i}(A))$. Note that A has exponent two, as it carries an involution of the first kind. In view of [5, Ch. 9, Thm. 11], one concludes that $\text{ind}(A)$ is a power of two. Hence, the equality $\nu(\mathbf{i}(A_K)) = \nu(\mathbf{i}(A))$ implies that $\mathbf{i}(A_K) = \mathbf{i}(A)$, proving the result. $\square$

6. THE METABOLICITY INDEX OF HERMITIAN FORMS

Throughout this section, (D, θ) is a division algebra with involution of the first kind over a field F .

Let (V, h) be a hermitian space over (D, θ) . Then $\text{ad}(h)$ is an involution of the first kind. We define the *metabolicity index* of h as $\mathbf{i}(h) = \mathbf{i}(\text{ad}(h))$. In view of [8, (3.6)], $\mathbf{i}(h)$ is the largest integer r for which there exists a hermitian form h' over h such that $h \simeq 2^r h'$, except in the case where D splits and h

is hyperbolic. In this exceptional case, using 4.1, we have $\mathbf{i}(h) = \nu(\dim_D h)$. Equivalently, if $h \simeq k\mathbb{H}$, then $\mathbf{i}(h) = \nu(k) + 1$.

The next result follows from 4.5 and 4.1.

Theorem 6.1. *Let h be a metabolic hermitian form over (D, θ) . If h is even then $\mathbf{i}(h) = \nu(\dim_D h)$. Otherwise,*

$$\mathbf{i}(h) = \min \left\{ \nu(\dim_D h), \left\lfloor \log_2 \frac{\dim_D h}{\dim_D h_{\text{qd}}} \right\rfloor + 1 \right\}.$$

Lemma 6.2. *Let (V, h) be a hermitian space over (D, θ) . Then*

$$\mathbf{i}(kh) \geq \mathbf{i}(h) + \nu(k)$$

for every positive integer k .

Proof. If h is not metabolic, the result is evident, so let h be metabolic. If h is even, then so is kh . Hence,

$$\mathbf{i}(kh) = \nu(\dim_D kh) = \nu(k \dim_D h) = \nu(k) + \nu(\dim_D h) = \nu(k) + \mathbf{i}(h).$$

Suppose that h is not even and set $r = \mathbf{i}(h)$. Then there exists a hermitian form h' such that $h \simeq 2^r h'$. Write $k = 2^{\nu(k)} s$ for some positive odd integer s . Then $kh \simeq 2^{r+\nu(k)} s h'$, hence $\mathbf{i}(kh) \geq \mathbf{i}(h) + \nu(k)$. $\square$

The inequality in 6.2 can be strict, as the following example shows.

Example 6.3. Let h' be a 2-dimensional direct hermitian form over (D, θ) and set $h = 2h' \perp 2\mathbb{H}$. Then $h_{\text{qd}} = 2h'$, hence $\mathbf{i}(h) = 2$ by 6.1. Now, consider the form $3h \simeq 6h' \perp 6\mathbb{H}$. By [8, (2.8)], we have $3h \simeq 2h' \perp 10\mathbb{H}$. Hence, $(3h)_{\text{qd}} \simeq 2h'$. It follows from 6.1 that $\mathbf{i}(3h) = 3 > \mathbf{i}(h) + \nu(3)$.

The following result shows that if k is a power of 2 and h is metabolic, we have equality in 6.2.

Proposition 6.4. *Let (V, h) be a hermitian space over (D, θ) . If h is metabolic then $\mathbf{i}(2^t h) = \mathbf{i}(h) + t$ for every positive integer t .*

Proof. If h is even, then so is $2^t h$ and the result follows from the relation $\mathbf{i}(h) = \nu(\dim_D h)$. Suppose that h is not even and set $r = \mathbf{i}(h)$. Then there exists a hermitian form h' such that $h \simeq 2^r h'$. We already know that $\mathbf{i}(2^t h) \geq r + t$, thanks to 6.2. Suppose that $\mathbf{i}(2^t h) > r + t$. Then there exists a hermitian form h'' such that $2^t h \simeq 2^{r+t+1} h''$. Since $h \simeq 2^r h'$, one concludes that $2^{r+t} h' \simeq 2^{r+t+1} h''$. It follows from [8, (2.9)] that $2h' \simeq 4h''$. Note that $r \geq 1$, because h is metabolic. Hence,

$$h \simeq 2^r h' \simeq 2^{r-1}(2h') \simeq 2^{r-1}(4h'') \simeq 2^{r+1} h'',$$

which contradicts $\mathbf{i}(h) = r$. $\square$

Remark 6.5. 6.4 is not necessarily true if h is not metabolic. For example, suppose that $D \neq F$ and let $h = 2\tilde{h} \perp h_1$, where $\tilde{h}$ is a one-dimensional direct hermitian form and h_1 is a two-dimensional even nonmetabolic hermitian form over (D, θ) . Then $\mathbf{i}(h) = 0$, as h is not metabolic. We claim that $\mathbf{i}(2h) = 3 \neq \mathbf{i}(h) + 1$. Note that $2h \simeq 4\tilde{h} \perp 2h_1$. The form $2h_1$ is even and metabolic, so it is hyperbolic. Hence, $2h_1 \simeq 2\mathbb{H}$ by dimension count. It follows from [8, (2.8)] that $2h \simeq 4\tilde{h} \perp 2\mathbb{H} \simeq 2\tilde{h} \perp 3\mathbb{H}$. Hence, $(2h)_{\text{qd}} \simeq 2\tilde{h}$. Now, 6.1 implies that $\mathbf{i}(2h) = 3$.

Proposition 6.6. *If (V, h) and (V', h') are two hermitian spaces over (D, θ) then $\mathbf{i}(h \perp h') \geq \min\{\mathbf{i}(h), \mathbf{i}(h')\}$.*

Proof. If h or h' is not metabolic, the result is evident. Suppose that h and h' are both metabolic. Set $r = \mathbf{i}(h)$ and $r' = \mathbf{i}(h')$. We consider 3 cases.

Case 1. h and h' are even. Then

$$\begin{aligned} \mathbf{i}(h \perp h') &= \nu(\dim_D h + \dim_D h') \\ &\geq \min\{\nu(\dim_D h), \nu(\dim_D h')\} \\ &= \min\{\mathbf{i}(h), \mathbf{i}(h')\}. \end{aligned}$$

Case 2. Exactly one of h or h' is even, say h . Then there exists a positive odd integer s such that $h \simeq 2^{r-1}s\mathbb{H}$. Also, there exists a hermitian form h_1 over (D, θ) such that $h' \simeq 2^{r'}h_1$. Now, if $r - 1 \geq r'$, then

$$h \perp h' \simeq 2^{r-1}s\mathbb{H} \perp 2^{r'}h_1 \simeq 2^{r'}(2^{r-1-r'}s\mathbb{H} \perp h_1),$$

hence $\mathbf{i}(h \perp h') \geq r' = \min\{\mathbf{i}(h), \mathbf{i}(h')\}$. Otherwise, we have $r - 1 < r'$. Then

$$h \perp h' \simeq 2^{r-1}s\mathbb{H} \perp 2^{r'}h_1 \simeq 2^{r-1}(s\mathbb{H} \perp 2^{r'-r+1}h_1) = 2^{r-1}\eta,$$

where $\eta := s\mathbb{H} \perp 2^{r'-r+1}h_1$. Since $r' - r + 1$ is a positive integer, the form $2^{r'-r+1}h_1$ is metabolic. It follows that the form η is also metabolic, i.e., $\mathbf{i}(\eta) \geq 1$. Hence, Using 6.4, one concludes that

$$\mathbf{i}(h \perp h') = \mathbf{i}(2^{r-1}\eta) = r - 1 + \mathbf{i}(\eta) \geq r - 1 + 1 = r = \min\{\mathbf{i}(h), \mathbf{i}(h')\}.$$

Case 3. h and h' are both noneven. Then there exists hermitian forms h_1 and h'_1 over (D, θ) such that $h \simeq 2^r h_1$ and $h' \simeq 2^{r'} h'_1$. Hence,

$$h \perp h' \simeq 2^r h_1 \perp 2^{r'} h'_1,$$

which implies that $\mathbf{i}(h \perp h') \geq \min\{\mathbf{i}(h), \mathbf{i}(h')\}$. $\square$

Remark 6.7. The inequality in 6.6 can be strict or not. For example, if $h = h'$ is a metabolic form, then 6.4 implies that

$$\mathbf{i}(h \perp h') = \mathbf{i}(2h) = \mathbf{i}(h) + 1 > \min\{\mathbf{i}(h), \mathbf{i}(h')\}.$$

Now, let $\langle \alpha, \beta \rangle$ be a direct hermitian form over (D, θ) and set $h = \langle \alpha, \alpha \rangle$ and $h' = \langle \beta, \beta \rangle$. Then $\mathbf{i}(h) = \mathbf{i}(h') = 1$. We also have $h \perp h' = 2\langle \alpha, \beta \rangle$. Since $\langle \alpha, \beta \rangle$ is direct, we have $(h \perp h')_{\text{qd}} = 2\langle \alpha, \beta \rangle$. Hence,

$$\mathbf{i}(h \perp h') = 1 = \min\{\mathbf{i}(h), \mathbf{i}(h')\}$$

by 6.1.

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ON THE METABOLICITY INDEX OF ORTHOGONAL INVOLUTIONS
IN CHARACTERISTIC TWO

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اندیس متابولیک برگردان‌های متعامد در مشخصه دو

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در این مقاله اندیس متابولیک برگردان‌های متعامد در مشخصه دو را محاسبه می‌نماییم. به‌عنوان یک کاربرد از این محاسبه، رفتار این اندیس تحت توسیع‌های میدانی جدایی‌پذیر را بررسی می‌کنیم. همچنین اندیس متابولیک مجموع فرم‌های هرمیتی را مطالعه می‌نماییم.

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