

(u, v) -ABSORBING (PRIME) HYPERIDEALS OVER COMMUTATIVE MULTIPLICATIVE HYPERRINGS

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ABSTRACT. In this paper, we consider the class of multiplicative hyperrings and introduce an expansion of the v -absorbing hyperideals called (u, v) -absorbing hyperideals. A proper hyperideal Q of a multiplicative hyperring A is called a (u, v) -absorbing hyperideal if $x_1 \circ \cdots \circ x_u \subseteq Q$ for non-unit elements $x_1, \dots, x_u \in A$, then there are v of the x_i 's whose product is a subset of Q where $u, v \in \mathbb{N}$ with $u > v$. Moreover, we conceptualize and investigate the concept of (u, v) -absorbing prime hyperideals as an extension of the (u, v) -absorbing hyperideals. We give an example of a (u, v) -absorbing prime hyperideal in A that is not a (u, v) -absorbing hyperideal. We provide some characterizations of these new notions.

1. INTRODUCTION

The study of the ideal structure of rings has been enhanced by the inclusion of absorbing ideals beyond prime ideals. The notion of n -absorbing ideals, where if a product of $n+1$ elements is in the ideal then a product of n of them is also in the ideal, was introduced by Anderson and Badawi in [7]. The idea of (m, n) -absorbing ideals in a commutative ring as a generalization of the n -absorbing ideals was proposed in [20]. A proper ideal Q in a commutative ring R is said to be an (m, n) -absorbing ideal if whenever $x_1 \cdots x_m \in Q$ for some non-units $x_1, \dots, x_m \in R$, then there are n of the x_i 's whose product is in Q . Recently, Badawi et al. have generalized this notion to (m, n) -absorbing prime ideals in [8].

Marty's pioneering work on hyperstructures generalized many concepts in modern algebra by extending their underlying structures to hyperstructures [22]. This new algebraic framework has since been extensively studied and developed by numerous scholars [11, 12, 9, 14, 13, 23, 25, 29]. A hyperoperation " $\circ$ " on non-empty set X is a mapping from $X \times X$ into $P^*(X)$ such that $P^*(X)$ is the family of all non-empty subsets of X . In this case, $(X, \circ)$ is called hypergroupoid [17]. Let X_1, X_2 be two subsets of X and $x \in X$, then $X_1 \circ X_2 = \cup_{x_1 \in X_1, x_2 \in X_2} x_1 \circ x_2$, and $X_1 \circ x = X_1 \circ \{x\}$. This means that the hyperoperation " $\circ$ " on X can be extended to subsets of X . A hypergroupoid

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$(X, \circ)$ is called a semihypergroup if $\cup_{a \in y \circ z} x \circ a = \cup_{b \in x \circ y} b \circ z$ for all $x, y, z \in X$ which means $\circ$ is associative. A semihypergroup X is called a hypergroup if $x \circ X = X = X \circ x$ for each $x \in X$ [17]. A non-empty subset Y of a semihypergroup $(X, \circ)$ refers to a subhypergroup if $x \circ Y = Y = Y \circ x$ for each $x \in Y$ [17]. Multiplicative hyperrings are an important class of algebraic hyperstructures that generalize classical rings. They were introduced by Rota in 1982 [27]. This class of hyperstructures has been widely reviewed in [4, 3, 6, 21, 19, 18, 26, 28]. A triple $(A, +, \circ)$ is called a commutative multiplicative hyperring if **1.** $(A, +)$ is a commutative group, **2.** $(A, \circ)$ is a semihypergroup; **3.** $x \circ (y + z) \subseteq x \circ y + x \circ z$ and $(y + z) \circ x \subseteq y \circ x + z \circ x$ for every $x, y, z \in A$, **4.** $x \circ (-y) = (-x) \circ y = -(x \circ y)$ for every $x, y \in A$, **5.** $x \circ y = y \circ x$ for every $x, y \in A$ [17]. If in (3), the equality holds then the multiplicative hyperring A is called strongly distributive. Consider the ring of integers $(\mathbb{Z}, +, \cdot)$. For each subset $T \in P^*(\mathbb{Z})$ with $|T| \geq 2$, there exists a multiplicative hyperring $(\mathbb{Z}_T, +, \circ)$ where $\mathbb{Z}_T = \mathbb{Z}$ and $a \circ b = \{a.x.b \mid x \in T\}$ for all $a, b \in \mathbb{Z}_T$ [16]. An element $e \in A$ is considered as an identity element if $a \in a \circ e$ for all $a \in A$ [5]. An element $x \in A$ is called unit, if there exists $y \in A$ such that $1 \in x \circ y$. Denote the set of all unit elements in A by $U(A)$ [5]. Moreover, a hyperring A refers to a hyperfield if each non-zero element of A is unit. A non-empty subset B of A is a hyperideal if **i.** $x - y \in B$ for all $x, y \in B$, **ii.** $r \circ x \subseteq B$ for all $x \in B$ and $r \in A$ [17]. A proper hyperideal B in A refers to a prime hyperideal if $x \circ y \subseteq B$ for $x, y \in A$, then $x \in B$ or $y \in B$ [16]. The intersection of all prime hyperideals of A containing a hyperideal B is said to be the prime radical of B , denoted by $rad(B)$. If the multiplicative hyperring A has no prime hyperideal containing B , we define $rad(B) = A$ [16]. Assume that $\mathcal{C}$ is the class of all finite products of elements of A that is $\mathcal{C} = \{a_1 \circ a_2 \circ \dots \circ a_n \mid a_i \in A, n \in \mathbb{N}\} \subseteq P^*(A)$ and B is a hyperideal of A . B refers to a $\mathcal{C}$ -hyperideal of A if for each $C \in \mathcal{C}$ and $B \cap C \neq \emptyset$ imply $C \subseteq B$. Notice that $\{a \in A \mid a^n \subseteq B \text{ for some } n \in \mathbb{N}\} \subseteq rad(A)$. The equality holds if B is a $\mathcal{C}$ -hyperideal of A (see Proposition 3.2 in [16]). Moreover, a hyperideal B of A refers to a strong $\mathcal{C}$ -hyperideal if for each $D \in \mathfrak{U}$ and $D \cap B \neq \emptyset$ imply $D \subseteq B$ such that $\mathfrak{U} = \{\sum_{i=1}^n C_i \mid C_i \in \mathcal{C}, n \in \mathbb{N}\}$ and $\mathcal{C} = \{a_1 \circ a_2 \circ \dots \circ a_n \mid a_i \in A, n \in \mathbb{N}\}$ (for more details see [15]). A proper hyperideal B of A is maximal in A if for each hyperideal M of A with $B \subset M \subseteq A$, then $M = A$ [5]. The intersection of all maximal hyperideals of A is denoted by $J(A)$. Also, A refers to a local multiplicative hyperring if it has just one maximal hyperideal [5]. Assume that B_1 and B_2 are hyperideals of A . We define $(B_2 : B_1) = \{x \in A \mid x \circ B_1 \subseteq B_2\}$ [5]. A proper hyperideal

B of a commutative multiplicative hyperring A is called a v -absorbing hyperideal for $v \in \mathbb{N}$ if whenever $x_1 \circ \cdots \circ x_{v+1} \subseteq B$ for $x_1, \dots, x_{v+1} \in A$, then there are v of the x_i 's whose product is a subset of B [6].

In this paper, we aim to introduce the notions (u, v) -absorbing hyperideals and (u, v) -absorbing prime hyperideals where $u, v \in \mathbb{Z}$ with $u > v$. Indeed, the (u, v) -absorbing hyperideals are an extension of the v -absorbing hyperideals and the (u, v) -absorbing prime hyperideals are a generalization of the (u, v) -absorbing hyperideals. Among many results in this paper, we show that every (u, v) -absorbing prime hyperideals may not be a (u, v) -absorbing hyperideal in Example 3.2. In Proposition 2.6, we conclude that the radical of a (u, v) -absorbing $\mathcal{C}$ -hyperideal Q in a commutative multiplicative hyperring A is the set $\{x \in A \mid x^v \subseteq Q\}$ and this hyperideal is a (k, v) -absorbing hyperideal where $k = \max\{\lceil \frac{u}{v} \rceil, v + 1\}$. By Theorem 2.11, it is shown that if the (u, v) -absorbing hyperideal Q of A has r minimal prime hyperideals, then there are at least $\frac{r!}{(r-k)!k!}$ k -absorbing hyperideals of A that are minimal over Q where $k, r \in \mathbb{N}$ with $2 \leq k \leq r \leq u - 1$. We obtain that a strong $\mathcal{C}$ -hyperideal Q of A is a $(v + 1, v)$ -absorbing prime hyperideal of A if and only if Q is a prime hyperideal or A is local such that $M^v \subseteq Q$ for the only maximal hyperideal M of A in Proposition 3.10. In Theorem 3.15, we get the result that a proper hyperideal Q of A is a (u, v) -absorbing prime hyperideal if and only if for proper hyperideals $Q_1, \dots, Q_u$ of A , $Q_1 \circ \cdots \circ Q_u \subseteq Q$ implies that $Q_1 \circ \cdots \circ Q_v \subseteq Q$ or $Q_{v+1} \circ \cdots \circ Q_u \subseteq Q$. Finally, we analyze the stability of (u, v) -absorbing prime hyperideals in various hyperring-theoretic constructions. Throughout this paper, A denotes a commutative multiplicative hyperring with identity element 1.

2. (u, v) -ABSORBING HYPERIDEALS

In this section, we aim to introduce the class of the (u, v) -absorbing hyperideals as an extension of the v -absorbing hyperideals and present some of their properties.

Definition 2.1. Let $u, v \in \mathbb{N}$ with $u > v$. A proper hyperideal Q of A is called a (u, v) -absorbing hyperideal if whenever $x_1 \circ \cdots \circ x_u \subseteq Q$ for $x_1, \dots, x_u \in A \setminus U(A)$, then there are v of the x_i 's whose product is a subset of Q .

Example 2.2. Consider the multiplicative hyperring $(\mathbb{Z}_T, +, \circ)$.

- (i) Let $T = \{7, 11\}$. Then $Q = \langle 30 \rangle$ is a $(5, 3)$ -absorbing hyperideal of the multiplicative hyperring $\mathbb{Z}_T$. But it is not prime. Because $5 \circ 6 = \{210, 330\} \subseteq Q$ and $5, 6 \notin Q$.
- (ii) Let $T = \{2, 4\}$. Then $Q = \langle 150 \rangle$ is a $(4, 3)$ -absorbing hyperideal of the multiplicative hyperring $\mathbb{Z}_T$. However, it is not $(3, 2)$ -absorbing as the fact that $3 \circ 5^2 = \{300, 600, 1200\} \subseteq Q$ but $3 \circ 5 = \{30, 60\} \not\subseteq Q$ and $5^2 = \{50, 100\} \not\subseteq Q$.

Remark 2.3. Assume that Q is a proper $\mathcal{C}$ -hyperideal of A and $k, r, u, v \in \mathbb{N}$.

- (i) Let Q be a (u, v) -absorbing hyperideal of A . Then:
 - (1) Q is a (k, r) -absorbing hyperideal for every $k \geq u$ and $r \geq v$.
 - (2) Q is a $(u - 1)$ -absorbing hyperideal of A .
- (ii) Q is a (u, v) -absorbing hyperideal of A if and only if $x_1 \circ \cdots \circ x_k \subseteq Q$ for $x_1, \dots, x_k \in A \setminus U(A)$ and $k \geq u$ imply that there are v of the x_i 's whose product is a subset of Q .

Proposition 2.4. Let Q_i be a proper hyperideal of A and $u_i, v_i \in \mathbb{N}$ for every $i \in \{1, \dots, k\}$. If Q_i is a (u_i, v_i) -absorbing hyperideal of A for every $i \in \{1, \dots, k\}$, then $Q = \cap_{i=1}^k Q_i$ is a (u, v) -absorbing hyperideal of A where $v = v_1 + \cdots + v_k$ and $u = \max\{u_1, \dots, u_k, v + 1\}$.

Proof. Assume that Q_i is a (u_i, v_i) -absorbing hyperideal of A for every $i \in \{1, \dots, k\}$. By Remark 2.3 (i), we conclude that Q_i is a (u, v_i) -absorbing hyperideal of A where $v = v_1 + \cdots + v_k$ and $u = \max\{u_1, \dots, u_k, v + 1\}$. Let $x_1 \circ \cdots \circ x_u \subseteq Q$ for $x_1, \dots, x_u \in A \setminus U(A)$. Since Q_i is a (u, v_i) -absorbing hyperideal of A for every $i \in \{1, \dots, k\}$ and $x_1 \circ \cdots \circ x_u \subseteq Q_i$, we get the result that $\Pi_{\alpha \in I_i} x_\alpha \subseteq Q_i$ for each $i \in \{1, \dots, k\}$ such that $I_i \subseteq \{1, \dots, u\}$ and $|I_i| = v_i$. Put $I = \cup_{i=1}^k I_i$. Since $\Pi_{\alpha \in I} x_\alpha \subseteq Q$ and $|I| \leq v$, we conclude that $Q = \cap_{i=1}^k Q_i$ is a (u, v) -absorbing hyperideal of A . $\square$

Example 2.5. In Example 2.2 (ii), the hyperideal $\langle 1155 \rangle = \langle 105 \rangle \cap \langle 77 \rangle$ is $(6, 5)$ -absorbing since the hyperideals $\langle 105 \rangle$ and $\langle 77 \rangle$ are $(4, 3)$ -absorbing and $(3, 2)$ -absorbing hyperideals, respectively.

Let $u, v \in \mathbb{N}$. Then the smallest integer not less than $\frac{u}{v}$ is denoted by $\lceil \frac{u}{v} \rceil$. In the following proposition we investigate the radical of a (u, v) -absorbing $\mathcal{C}$ -hyperideal of A .

Proposition 2.6. Let Q be a (u, v) -absorbing $\mathcal{C}$ -hyperideal of A for $u, v \in \mathbb{N}$ with $u > v$. Then:

- (i) $\text{rad}(Q) = \{x \in A \mid x^v \subseteq Q\}$.
- (ii) $\text{rad}(Q)$ is a (k, v) -absorbing hyperideal where $k = \max\{\lceil \frac{u}{v} \rceil, v + 1\}$.

Proof. (i) Assume that Q is a (u, v) -absorbing hyperideal of A . Take any $x \in \text{rad}(Q)$. Then there exists a smallest element $n \in \mathbb{N}$ such that $x^n \subseteq Q$. Let $n > v$. Remark 2.3(i)(2) shows that Q is a $(u - 1)$ -absorbing hyperideal of A . Then we conclude that $n \leq u - 1$ by Theorem 3.6 in [6]. This means $x^u \subseteq Q$ which means $x^v \subseteq Q$ as Q is a (u, v) -absorbing hyperideal of A . This is contradicts the fact that n is a smallest positive integer such that $x^n \subseteq Q$. Therefore $n \leq v$ and $x^v \subseteq Q$.

(ii) Suppose that $x_1 \circ \cdots \circ x_k \subseteq \text{rad}(Q)$ such that $k = \max\{\lceil \frac{u}{v} \rceil, v + 1\}$ and $x_1, \dots, x_k \in A \setminus U(A)$. Then for any $a \in x_1 \circ \cdots \circ x_k$, $a^v \subseteq Q$. Again, $a^v \subseteq (x_1 \circ \cdots \circ x_k)^v \subseteq x_1^v \circ \cdots \circ x_k^v \subseteq Q$. So, $(x_1^v \circ \cdots \circ x_k^v) \cap Q \neq \emptyset$ and then $x_1^v \circ \cdots \circ x_k^v \subseteq Q$ as Q is a $\mathcal{C}$ -hyperideal. The hyperideal Q is a (kv, v) -absorbing hyperideal of A because $k \geq \lceil \frac{u}{v} \rceil$. Then we conclude that

$$x_{i_1}^{r_1} \circ \cdots \circ x_{i_v}^{r_v} \subseteq Q$$

for some $\{i_1, \dots, i_v\} \subseteq \{1, \dots, k\}$ such that $0 \leq i_1, \dots, i_v \leq v$ and $r_1 + \cdots + r_v = v$. This implies that $x_{i_1}^v \circ \cdots \circ x_{i_v}^v \subseteq Q$ which means $x_{i_1} \circ \cdots \circ x_{i_v} \subseteq \text{rad}(Q)$. Consequently, $\text{rad}(Q)$ is a (k, v) -absorbing hyperideal of A . $\square$

Let $(A_1, +_1, \circ_1)$ and $(A_2, +_2, \circ_2)$ be two multiplicative hyperrings with nonzero identity. The set $A_1 \times A_2$ with the operation $+$ and the hyperoperation $\circ$ defined as

$$(x_1, x_2) + (y_1, y_2) = (x_1 +_1 y_1, x_2 +_2 y_2)$$

$$(x_1, x_2) \circ (y_1, y_2) = \{(x, y) \in A_1 \times A_2 \mid x \in x_1 \circ_1 y_1, y \in x_2 \circ_2 y_2\}$$

is a multiplicative hyperring [28]. The following example verifies that if Q_1 and Q_2 are two (u, v) -absorbing hyperideal of A_1 and A_2 , respectively, $Q_1 \times Q_2$ may not be a (u, v) -absorbing hyperideal of $A_1 \times A_2$

Example 2.7. Consider the multiplicative hyperring $(\mathbb{Z}_T, +, \circ)$ where $T = \{2, 5\}$. Then $Q_1 = \langle 33 \rangle$ and $Q_2 = \langle 91 \rangle$ are two $(4, 2)$ -absorbing hyperideal of the multiplicative hyperring $(\mathbb{Z}_T, +, \circ)$. But, $Q_1 \times Q_2$ is not a $(4, 2)$ -absorbing hyperideal of $\mathbb{Z}$ since

$$(3, 1) \circ (11, 1) \circ (1, 7) \circ (1, 13) = \{(a, b) \mid a \in \{264, 660, 1650, 2475\}, \\ b \in \{728, 1820, 4550, 11375\}\} \subseteq Q_1 \times Q_2 \text{ and}$$

$$(3, 1) \circ (11, 1), (3, 1) \circ (1, 7), (3, 1) \circ (1, 13), (11, 1) \circ (1, 7), \\ (11, 1) \circ (1, 13), (1, 7) \circ (1, 13) \not\subseteq Q_1 \times Q_2.$$

Theorem 2.8. Assume that $A = A_1 \times A_2$ such that A_1, A_2 are commutative multiplicative hyperrings, $Q = Q_1 \times A_2$ such that Q_1 is a proper hyperideal of

A_1 and $u, v \in \mathbb{N}$ with $u > v$. Then Q is a (u, v) -absorbing hyperideal of A if and only if $x_1 \circ_1 \cdots \circ_1 x_u \subseteq Q_1$ for $x_1, \dots, x_u \in A_1$ implies that there are v of the x_i 's whose product is a subset of Q_1 .

Proof. ($\implies$) Suppose on the contrary, $x_1 \circ_1 \cdots \circ_1 x_u \subseteq Q_1$ for some $x_1, \dots, x_u \in A_1$ and none of product of the terms v of the x_i 's is not a subset of Q_1 . Since Q is a (u, v) -absorbing hyperideal of A , $(x_1, 0) \circ \cdots \circ (x_u, 0) \subseteq Q$ and none of the $(x_i, 0)$'s are unit, there exist v of the $(x_i, 0)$'s whose product is a subset of Q which implies there exist v of the x_i 's whose is a subset of Q_1 which is a contradiction.

($\impliedby$) Let $(x_1, y_1) \circ \cdots \circ (x_u, y_u) \subseteq Q$ for $(x_1, y_1), \dots, (x_u, y_u) \in A \setminus U(A)$. This means that $x_1 \circ_1 \cdots \circ_1 x_u \subseteq Q_1$. By the hypothesis, there exist v of the x_i 's, say $x_{i_1}, \dots, x_{i_v}$, whose product is a subset of Q_1 . It follows that $(x_{i_1}, y_{i_1}) \circ \cdots \circ (x_{i_v}, y_{i_v}) \subseteq Q$ and so Q is a (u, v) -absorbing hyperideal of A . $\square$

Theorem 2.9. Assume that Q is a hyperideal of A and $u, v \in \mathbb{N}$ with $u > v$. Then there exists a (u, v) -absorbing hyperideal of A which is minimal in the set of all (u, v) -absorbing hyperideals of A containing Q .

Proof. Suppose that

$$\Gamma = \{P \mid P \text{ is a } (u, v)\text{-absorbing hyperideal of } A \text{ and } Q \subseteq P\}.$$

Let M be a maximal hyperideal of A containing Q . Then M is a (u, v) -absorbing hyperideal for all $u, v \in \mathbb{N}$ with $u > v$ and so $M \in \Gamma$ which means $\Gamma \neq \emptyset$. Order Γ by $\leq$ such that $P_1 \leq P_2$ if and only if $P_1 \supseteq P_2$ for every $P_1, P_2 \in \Gamma$. Let $\{P_i\}_{i \in \Delta}$ be a non-empty chain of hyperideals in Γ . Put $I = \bigcap_{i \in \Delta} P_i$. Assume that $x_1 \circ \cdots \circ x_u \subseteq I$ for $x_1, \dots, x_u \in A \setminus U(A)$ such that none of the product of the terms v of x_i 's is a subset of I . Then that there exists P_j in the chain such that none of the product of the terms v of x_i 's is a subset of P_j which is impossible. Hence there are v of the x_i 's whose product is a subset of I . Now, by Zorn's lemma, we conclude that $(\Gamma, \leq)$ has a maximal element which means Γ has a minimal (u, v) -absorbing hyperideal of A containing Q . $\square$

As an immediate consequence of the previous theorem, we have the following result.

Corollary 2.10. Suppose that Q is a hyperideal of A and $v \in \mathbb{N}$. Then there exists a minimal v -absorbing hyperideal P of A such that $Q \subseteq P$.

Theorem 2.11. Assume that Q is a (u, v) -absorbing hyperideal of A and $k, r \in \mathbb{N}$ with $2 \leq k \leq r \leq u - 1$. If Q has r minimal prime hyperideals, then there are at least $\frac{r!}{(r-k)!k!}$ k -absorbing hyperideals of A that are minimal over Q .

Proof. Suppose that Q has r minimal prime hyperideals and $k = 2$. Then there exist the distinct prime hyperideals $Q_1, \dots, Q_r$ of A that are minimal over Q . Hence $Q_i \cap Q_j$ is a 2-absorbing hyperideal of A for each $i, j \in \{1, \dots, r\}$ and $i \neq j$ by Theorem 3.4 in [6]. Hence we have $Q \subseteq Q_{ij} \subseteq Q_i \cap Q_j$ for some minimal 2-absorbing hyperideal Q_{ij} of A by Corollary 2.10. Therefore we conclude that Q_i and Q_j are only minimal prime hyperideals over Q_{ij} by Theorem 3.8 in [6]. Assume that Q_{ij} 's are not distinct such that $Q_{ij} = Q_{st}$ for some $i, j, s, t \in \{1, \dots, r\}$ and $\{i, j\} \neq \{s, t\}$. Then we conclude that Q_i, Q_j, Q_s and Q_t are distinct minimal prime hyperideals over Q_{ij} which contradicts the fact that there are at most two prime hyperideals of A that are minimal over the 2-absorbing hyperideal Q_{ij} . Hence the Q_{ij} 's are distinct. Thus we get the result that Q has at least $\frac{r!}{(r-k)!k!}$ minimal 2-absorbing hyperideals. If $k > 2$, one can easily complete the proof by using a similar argument. $\square$

Lemma 2.12. Let Q be a (u, v) -absorbing hyperideals of A , $Q_1, \dots, Q_v$ be incomparable prime $\mathcal{C}$ -hyperideals of A such that $Q \subseteq \cap_{i=1}^v Q_i$ and $k_1, \dots, k_v \in \mathbb{N}$. If $x_1^{k_1} \circ \dots \circ x_v^{k_v} \subseteq Q$ for $x_i \in Q_i \setminus (\cup_{j \neq i} Q_j)$, then

$$x_1 \circ \dots \circ x_v \subseteq Q.$$

Proof. Suppose that $k_1, \dots, k_v \in \mathbb{N}$. Put $k = k_1 + \dots + k_v$. Let $k \geq u$. So Q is a (k, v) -absorbing hyperideal of A by Remark 2.3(i). Then there exist integers $r_1, \dots, r_v$ such that $x_1^{r_1} \circ \dots \circ x_v^{r_v} \subseteq Q$, $0 \leq r_i \leq k_i$ and $r_1 + \dots + r_v = v$. Assume that one of the r_i 's is zero. Then we have $x_1^{r_1} \circ \dots \circ x_{i-1}^{r_{i-1}} \circ x_{i+1}^{r_{i+1}} \circ \dots \circ x_v^{r_v} \subseteq Q \subseteq Q_i$. Then there exists $j \in \{1, \dots, i-1, i+1, \dots, v\}$ such that $x_j \in Q_i$ which is impossible. Hence $x_1 \circ \dots \circ x_v \subseteq Q$. $\square$

Theorem 2.13. Assume that Q is a (u, v) -absorbing strong $\mathcal{C}$ -hyperideal of A for $u, v \in \mathbb{N}$ with $2 \leq v < u$ and $Q_1, \dots, Q_v$ are the only minimal prime $\mathcal{C}$ -hyperideals over Q . If $z_i \in Q_i \setminus (\cup_{j \neq i} Q_j)$, then $Q_j \circ \prod_{i \neq j} z_i \subseteq Q$.

Proof. Assume that $x \in P_j$. If $x \notin \cup_{j \neq i} Q_j$, then we have

$$x \circ \prod_{i \neq j} z_i \subseteq \cap_{i=1}^v Q_i = \text{rad}(Q)$$

as the Q_i 's are the only minimal prime hyperideals over Q . Therefore we conclude that $x \circ \prod_{i \neq j} z_i \subseteq Q$ by Proposition 2.6 and Lemma 2.12. Let

$x \in Q_j \cap (\cup_{j \neq i} Q_j)$. Then there exist $y \in A$ and $z \in Q_j \setminus (\cup_{j \neq i} Q_j)$ such that $w + x \in Q_j \setminus (\cup_{j \neq i} Q_j)$ for every $w \in y \circ z$ by Corollary 3.12 in [6]. It follows that $(w + x) \circ \Pi_{i \neq j} z_i \subseteq Q$ and so $w \circ \Pi_{i \neq j} z_i + x \circ \Pi_{i \neq j} z_i \subseteq Q$ as Q is a strong $\mathcal{C}$ -hyperideal of A . Since $w \circ \Pi_{i \neq j} z_i \subseteq Q$, we obtain $x \circ \Pi_{i \neq j} z_i \subseteq Q$ which means $Q_j \circ \Pi_{i \neq j} z_i \subseteq Q$. $\square$

Let Q be a u -absorbing hyperideal of A for $u \in \mathbb{N}$. We define

$$Abs(Q) = \min\{v \mid Q \text{ is a } v\text{-absorbing hyperideal of } A\},$$

otherwise $Abs(Q) = \infty$ [6]. Moreover, we define

$$abs(Q) = \min\{v \mid Q \text{ is an } (Abs(Q) + 1, v)\text{-absorbing hyperideal of } A\}.$$

It is clear that $abs(Q) \leq Abs(Q)$. Thus, in Proposition 2.4, we get

$$abs(\cap_{i=1}^k Q_i) \leq abs(Q_1) + \cdots + abs(Q_k)$$

and

$$Abs(\cap_{i=1}^k Q_i) \leq \max\{Abs(Q_1), \dots, Abs(Q_k), abs(Q_1) + \cdots + abs(Q_k)\}.$$

Also, in Proposition 2.6, we have $abs(rad(Q)) \leq abs(Q)$ and

$$Abs(rad(Q)) \leq \max\{\lceil \frac{Abs(Q) + 1}{abs(Q)} \rceil - 1, abs(Q)\}.$$

Recall from [5] that the hyperideals I and J are coprime if $I + J = A$.

Theorem 2.14. *Assume that Q is a (u, v) -absorbing strong $\mathcal{C}$ -hyperideal of A for $u, v \in \mathbb{N}$ with $2 \leq v < u$ and $Q_1, \dots, Q_v$ are the only minimal prime $\mathcal{C}$ -hyperideals over Q . Then*

- (i) $Q_1 \circ \cdots \circ Q_v \subseteq Q$ and $abs(Q) = v$.
- (ii) If $Q_1, \dots, Q_v$ are pairwise coprime, then $Q_1 \circ \cdots \circ Q_v = Q$ and $abs(Q) = v$.

Proof. (i) Let $Q_1, \dots, Q_v$ be the only minimal prime $\mathcal{C}$ -hyperideals over Q . Take any $x_i \in Q_i$ for each $1 \leq i \leq v$ and $z_i \in Q_i \setminus (Q_1 \cup (\cup_{j \neq i} Q_j))$ for each $2 \leq i \leq v$. Therefore $x_1 \circ \Pi_{2 \leq i \leq v} z_i \subseteq Q$ by Theorem 2.13. Suppose that $x_1 \circ \cdots \circ x_k \circ \Pi_{k+1 \leq i \leq v} z_i \subseteq Q$ for $1 \leq k \leq v - 1$. By Corollary 3.12 in [6], there exist $a_{k+1} \in Q_{k+1} \setminus (\cup_{i \neq k+1} Q_i)$ and $b_{k+1} \in A$ such that

$$c_{k+1} + x_{k+1} \in Q_{k+1} \setminus (\cup_{i \neq k+1} Q_i)$$

for each $c_{k+1} \in a_{k+1} \circ b_{k+1}$ as $Q_1, \dots, Q_v$ are incomparable prime hyperideals of A . This implies that

$$\begin{aligned} & (x_1 \circ \dots \circ x_k) \circ (c_{k+1} + x_{k+1}) \circ \prod_{k+2 \leq i \leq v} z_i \\ & \subseteq (x_1 \circ \dots \circ x_k \circ c_{k+1} \circ \prod_{k+2 \leq i \leq v} z_i) + (x_1 \circ \dots \circ x_k \circ x_{k+1} \circ \prod_{k+2 \leq i \leq v} z_i). \end{aligned}$$

Put $z_{k+1} = c_{k+1} + x_{k+1}$. Since Q is a strong $\mathcal{C}$ -hyperideal of A and

$$x_1 \circ \dots \circ x_k \circ \prod_{k+1 \leq i \leq v} z_i \subseteq Q,$$

we get $(x_1 \circ \dots \circ x_k \circ c_{k+1} \circ \prod_{k+2 \leq i \leq v} z_i) + (x_1 \circ \dots \circ x_{k+1} \circ \prod_{k+2 \leq i \leq v} z_i) \subseteq Q$ which means $x_1 \circ \dots \circ x_{k+1} \circ \prod_{k+2 \leq i \leq v} z_i \subseteq Q$. If $k = v - 1$, then we have $(x_1 \circ \dots \circ x_{v-1}) \circ (c_v + x_v) \subseteq Q$ for $c_v \in a_v \circ b_v$ which means $x_1 \circ \dots \circ x_v \subseteq Q$. Thus $Q_1 \circ \dots \circ Q_v \subseteq Q$ and $\text{abs}(Q) = v$.

(ii) Assume that $Q_1, \dots, Q_v$ are prime hyperideals of A that are pairwise coprime. Therefore $\cap_{i=1}^v Q_i = \prod_{1 \leq i \leq v} Q_i \subseteq Q$ by (i). As $Q_1, \dots, Q_v$ are the only minimal prime $\mathcal{C}$ -hyperideals over Q , we conclude that $Q_1 \circ \dots \circ Q_v = Q$ and so $\text{abs}(Q) = v$. □

Theorem 2.15. *Assume that Q is a radical $\mathcal{C}$ -hyperideal of A . Then Q is a (u, v) -absorbing hyperideal of A if and only if Q is a v -absorbing hyperideal of A .*

Proof. ($\implies$) Suppose that Q is a (u, v) -absorbing hyperideal of A and $x_1 \circ \dots \circ x_{u-1} \subseteq Q$ for $x_1, \dots, x_{u-1} \in A \setminus U(A)$. Since

$$x_1 \circ \dots \circ x_{k-1} \circ x_k^2 \circ x_{k+1} \circ \dots \circ x_{u-1} \subseteq Q,$$

we conclude that either $x_{i_1} \circ \dots \circ x_{i_v} \subseteq Q$ for some

$$i_j \in \{1, \dots, k-1, k, k+1, \dots, u-1\}$$

which implies Q is a $(u-1, v)$ -absorbing hyperideal of A or

$$x_k^2 \circ x_{i_1} \circ \dots \circ x_{i_{v-2}} \subseteq Q$$

for some $i_j \in \{1, \dots, k-1, k+1, \dots, u-1\}$ which means

$$x_k \circ x_{i_1} \circ \dots \circ x_{i_{v-2}} \subseteq \text{rad}(Q).$$

Since Q is a radical hyperideal, we get $x_k \circ x_{i_1} \circ \dots \circ x_{i_{v-2}} \subseteq Q$ which implies Q is a $(u-1, v)$ -absorbing hyperideal of A . By continuing this process, we conclude Q is a $(v+1, v)$ -absorbing hyperideal and so it is a v -absorbing hyperideal of A by Remark 2.3(i).

($\impliedby$) Let Q is a v -absorbing hyperideal of A and $x_1 \circ \dots \circ x_u \subseteq Q$ for $x_1, \dots, x_{u-1} \in A \setminus U(A)$. Take any $c \in (x_{i_1} \circ \dots \circ x_{i_{u-v}}) \setminus Q$ such that

$i_j \in \{1, \dots, u\}$. Then a product of v of the x_i 's other than the x_{i_j} with c is a subset of Q and so a product of v of the x_i 's other than the x_{i_j} is a subset of Q as Q is a v -absorbing hyperideal of A . Thus Q is a (u, v) -absorbing hyperideal of A . $\square$

Let us give the following theorem without proof as a result of Theorem 2.15.

Corollary 2.16. *Let Q be a (u, v) -absorbing radical $\mathcal{C}$ -hyperideal of A . Then $abs(Q) = Abs(Q)$.*

Recall from [16] that a proper hyperideal Q of A is primary if for any $x, y \in A$, $x \circ y \subseteq Q$ implies that $x \in Q$ or $y^t \subseteq Q$ for some $t \in \mathbb{N}$. Since the radical of a primary $\mathcal{C}$ -hyperideal Q is a prime hyperideal of A , say P , Q is referred to as a P -primary $\mathcal{C}$ -hyperideal of A . Now, we consider the relationship between (u, v) -absorbing hyperideals and primary hyperideals.

Theorem 2.17. *Assume that $u, v \in \mathbb{N}$ and Q is a P -primary $\mathcal{C}$ -hyperideal of A where P is a prime hyperideal of A with $P^v \subseteq Q$. Then Q is a (u, v) -absorbing hyperideal of A for any $u \geq v + 1$. Furthermore, if P^v is a P -primary $\mathcal{C}$ -hyperideal of A , then P^v is a (u, v) -absorbing hyperideal of A for any $u \geq v + 1$ and $abs(P^v) = v$.*

Proof. The proof follows from Theorem 4.1 in [6]. $\square$

Theorem 2.18. *Assume that Q is a (u, v) -absorbing strong $\mathcal{C}$ -hyperideal of A such that $Q \subset rad(Q)$ and $u, v \in \mathbb{N}$ with $u > v \geq 2$. If $a^n \subseteq Q$ for $a \in rad(Q) \setminus Q$ and the least positive integer $n \geq 2$, then $(Q : a^{n-1})$ is a $(u - n + 1, v)$ -absorbing hyperideal of A such that $r = \min\{u - n, v\}$.*

Proof. Let Q be a (u, v) -absorbing hyperideal of A . By Remark 2.3, Q is a $(u - 1)$ -absorbing of A . Put $E = (Q : a^{n-1})$ for $a \in rad(Q) \setminus Q$ and the least positive integer $n \geq 2$. Therefore we conclude that E is a $(u - n)$ -absorbing hyperideal of A by Theorem 4.3 in [6]. This means that E is a $(u - n + 1, u - n)$ -absorbing hyperideal of A . Assume that $x_1 \circ \dots \circ x_{u-n+1} \subseteq E$ for $x_1, \dots, x_{u-n+1} \in A \setminus U(A)$ and $v \leq u - n$. Since Q be a (u, v) -absorbing hyperideal of A and $a^{n-1} \circ x_1 \circ \dots \circ x_{u-n+1} \subseteq E$, there are k of the x_i 's, say $x_1, \dots, x_k$ such that $a^{v-k} \circ x_1 \circ \dots \circ x_k \subseteq Q$ which implies $x_1 \circ \dots \circ x_k \subseteq E$. Since $k \leq v$, we get $x_1 \circ \dots \circ x_v \subseteq E$. Consequently, E is $(u - n + 1, v)$ -absorbing hyperideal of A . $\square$

Let us give the following corollary without proof as a result of Theorem 2.18.

Corollary 2.19. *Let $u, v \in \mathbb{N}$ with $u \geq 4$ and Q be a (u, v) -absorbing strong $\mathcal{C}$ -hyperideal of A such that $Q \subset \text{rad}(Q)$. If $a^{u-2} \subseteq Q$ for $a \in \text{rad}(Q) \setminus Q$ and the least positive integer $u - 2$, then $(Q : a^{u-3})$ is a 2-absorbing hyperideal of A .*

Theorem 2.20. *Let Q be a proper strong $\mathcal{C}$ -hyperideal of A such that $a + 1 \notin U(A)$ for some $a \in Q$ and $u, v \in \mathbb{N}$ with $u > v$. Then Q is a (u, v) -absorbing hyperideal if and only if $x_1 \circ \cdots \circ x_u \subseteq Q$ for $x_1, \dots, x_u \in A$ implies that there are v of the x_i 's whose product is a subset of Q .*

Proof. ($\implies$) Assume that $x_1 \circ \cdots \circ x_u \subseteq Q$ for $x_1, \dots, x_u \in A$. Let $x_1, \dots, x_{s-1}$ be non-unit elements in A and $x_s, \dots, x_u$ be unit elements in A for some $s \geq v + 2$. Since $x_1 \circ \cdots \circ x_u \subseteq Q$ and $x_s, \dots, x_u$ are units in A , we get $x_1 \circ \cdots \circ x_s \subseteq Q$. Put $y_i = 1 + a$ for each $i \in \{1, \dots, s\}$. Then the y_i 's are non-unit. Since Q is a (u, v) -absorbing hyperideal and $x_1 \circ \cdots \circ x_{s-1} \circ y_s \circ \cdots \circ y_u \subseteq Q$, we conclude that there exist v elements of the x_i 's and the y_j 's whose product is a subset of Q . Take any $n \in \mathbb{N}$. Assume that $(a + 1)^n \subseteq Q$. This implies that $(\sum_{k=0}^n \binom{n}{k} a^{n-k} \circ 1^k) \cap Q \neq \emptyset$. Since Q is a strong $\mathcal{C}$ -hyperideal of A , we have $(\sum_{k=0}^n \binom{n}{k} a^{n-k} \circ 1^k) \subseteq Q$ and so $1 \in Q$, a contradiction. Then $(a + 1)^n \not\subseteq Q$ for all $n \geq 1$. Thus $x_{i_1} \circ \cdots \circ x_{i_h} \circ (a + 1)^{v-h} \subseteq Q$ for some $h \in \{1, \dots, v\}$ and $i_j \in \{1, \dots, s-1\}$. Clearly, $\sum_{k=0}^{v-h} \binom{v-h}{k} a^{v-h-k} \circ 1^k = X + 1$ where $X \subseteq Q$. Since $x_{i_1} \circ \cdots \circ x_{i_h} \circ (X + 1) \subseteq Q \cap ((x_{i_1} \circ \cdots \circ x_{i_h} \circ X) + (x_{i_1} \circ \cdots \circ x_{i_h} \circ 1))$, we obtain $(x_{i_1} \circ \cdots \circ x_{i_h} \circ X) + (x_{i_1} \circ \cdots \circ x_{i_h} \circ 1) \subseteq Q$. Hence we get the result that $x_{i_1} \circ \cdots \circ x_{i_h} \subseteq Q$, as required.

($\impliedby$) It is obvious. □

As a result of the previous theorems, we give the following result.

Corollary 2.21. *Let Q be a proper strong $\mathcal{C}$ -hyperideal of A such that $Q \not\subseteq J(A)$ and $u, v \in \mathbb{N}$ with $u > v$. Then Q is a (u, v) -absorbing hyperideal if and only if $x_1 \circ \cdots \circ x_u \subseteq Q$ for $x_1, \dots, x_u \in A$ implies that there are v of the x_i 's whose product is a subset of Q .*

3. (u, v) -ABSORBING PRIME HYPERIDEALS

In this section, we aim to extend the class of the (u, v) -absorbing hyperideals to (u, v) -absorbing prime hyperideals.

Definition 3.1. Let $u, v \in \mathbb{N}$ with $u > v$. A proper hyperideal Q of A refers to a (u, v) -absorbing prime hyperideal if $x_1 \circ \cdots \circ x_u \subseteq Q$ for $x_1, \dots, x_u \in A \setminus U(A)$ implies either $x_1 \circ \cdots \circ x_v \subseteq Q$ or $x_{v+1} \circ \cdots \circ x_u \subseteq Q$.

Example 3.2. In Example 2.2, the hyperideal $\langle 15 \rangle$ is a $(3, 1)$ -absorbing prime hyperideal. However, since $3^2 \circ 5 = \{180, 360, 720\} \subseteq \langle 15 \rangle$ but $3, 5 \notin \langle 15 \rangle$, then $\langle 15 \rangle$ is not a $(3, 1)$ -absorbing hyperideal.

A proper hyperideal Q of A is 1-absorbing if for $x, y, z \in A \setminus U(A)$, $x \circ y \circ z \subseteq Q$ implies that $x \in Q$ or $y \circ z \subseteq Q$ [18]. Every 1-absorbing prime $\mathcal{C}$ -hyperideal of a multiplicative hyperring A is a (u, v) -absorbing prime hyperideal for all $u \geq 3$ and $v \geq 1$. But the converse of the explanation may not be generally true.

Example 3.3. Let $\mathbb{Z}$ be the ring of integers. Define the hyperoperation $a \circ ab = \{2ab, 3ab\}$ for all $a, b \in \mathbb{Z}$. The $\mathcal{C}$ -hyperideal $Q = \langle 455 \rangle$ is a $(4, 3)$ -absorbing prime hyperideal in the multiplicative hyperring $(\mathbb{Z}, +, \circ)$. However, it is not a 1-absorbing hyperideal of $(\mathbb{Z}, +, \circ)$ as

$$3 \circ 5 \circ 13 = \{1820, 2730, 4095\} \subseteq Q$$

but $5, 7, 13 \notin Q$, $5 \circ 7 = \{70, 105\} \not\subseteq Q$, $7 \circ 13 = \{182, 273\}$ and

$$5 \circ 13 = \{130, 195\} \not\subseteq Q.$$

Remark 3.4. Let $u, v \in \mathbb{N}$ with $u > v$ and Q be a $\mathcal{C}$ -hyperideal of A .

- (i) Q is a (u, v) -absorbing prime hyperideal of A if and only if Q is a $(u, u - v)$ -absorbing prime hyperideal.
- (ii) If Q is a (u, v) -absorbing prime hyperideal of A , then Q is a $(u+1, v+1)$ -absorbing prime hyperideal.

The following example shows that the converse of Remark 3.4 (ii) may not be true, in general.

Example 3.5. In Example 3.3, the hyperideal $\langle 385 \rangle$ is a $(4, 3)$ -absorbing prime hyperideal. However, it is not $(3, 2)$ -absorbing since

$$5 \circ 7 \circ 11 = \{1540, 2310, 3465\} \subseteq \langle 385 \rangle$$

but $5, 7, 11 \notin \langle 385 \rangle$, $5 \circ 7 = \{70, 105\} \not\subseteq \langle 385 \rangle$, $5 \circ 11 = \{110, 165\} \not\subseteq \langle 385 \rangle$ and $7 \circ 11 = \{154, 231\} \not\subseteq \langle 385 \rangle$.

Proposition 3.6. Assume that Q is a strong $\mathcal{C}$ -hyperideal of A such that A is not local and $u, v \in \mathbb{N}$ with $u > v$. Then Q is a (u, v) -absorbing prime hyperideal of A if and only if Q is a $(u - 1, v - 1)$ -absorbing prime hyperideal.

Proof. ($\implies$) Let Q be a (u, v) -absorbing prime hyperideal of A . Suppose on contrary that Q is not a $(u-1, v-1)$ -absorbing prime hyperideal of A . Then there exist $x_1, \dots, x_{u-1} \in A \setminus U(A)$ such that $x_1 \circ \dots \circ x_{u-1} \subseteq Q$ but neither $x_1 \circ \dots \circ x_{v-1} \subseteq Q$ nor $x_v \circ \dots \circ x_{u-1} \subseteq Q$. Take any $y \in A \setminus U(A)$. Since Q is a (u, v) -absorbing prime hyperideal of A ,

$$y \circ x_1 \circ \dots \circ x_{u-1} \subseteq Q$$

and $x_v \circ \dots \circ x_{u-1} \not\subseteq Q$, we have $y \circ x_1 \circ \dots \circ x_{v-1} \subseteq Q$. Take any $z \in U(A)$. If $y + z \notin U(A)$, then we obtain $(y + z) \circ x_1 \circ \dots \circ x_{v-1} \subseteq Q$ as $(y + z) \circ x_1 \circ \dots \circ x_{u-1} \subseteq Q$, $x_v \circ \dots \circ x_{u-1} \not\subseteq Q$ and Q is a (u, v) -absorbing prime hyperideal of A . Since Q is a strong $\mathcal{C}$ -hyperideal of A and $(y + z) \circ x_1 \circ \dots \circ x_{v-1} \subseteq y \circ x_1 \circ \dots \circ x_{v-1} + z \circ x_1 \circ \dots \circ x_{v-1}$, we conclude that $y \circ x_1 \circ \dots \circ x_{v-1} + z \circ x_1 \circ \dots \circ x_{v-1} \subseteq Q$. This implies that $x_1 \circ \dots \circ x_{v-1} \subseteq Q$ because $y \circ x_1 \circ \dots \circ x_{v-1} \subseteq Q$. This is a contradiction. Then we have $y + z \in U(A)$. By Lemma 2.6 in [19], we get the result that A is local which is a contradiction. Thus Q is a $(u-1, v-1)$ -absorbing prime hyperideal of A .

($\impliedby$) It is obvious by Remark 3.4 (ii). $\square$

Proposition 3.7. *Assume that $u, v \in \mathbb{N}$ with $u > v$ and Q is a strong $\mathcal{C}$ -hyperideal of A such that A is not local. Then Q is a $(u-v+1, 1)$ -absorbing prime hyperideal of A if and only if Q is a prime hyperideal.*

Proof. ($\implies$) Let Q be a $(u-v+1, 1)$ -absorbing prime hyperideal of A . Then we get the result that Q is a $(u-v+1, u-v)$ -absorbing prime hyperideal of A by Remark 3.4 (i). Hence we conclude that Q is a prime hyperideal by Proposition 3.6.

($\impliedby$) Straightforward. $\square$

For a $\mathcal{C}$ -hyperideal Q of A that the hyperring A is not local, it can be easily proved, by induction, that Q is a $(u-1, v-1)$ -absorbing prime hyperideal for $u, v \in \mathbb{N}$ with $u > v$ if and only if Q is a $(u-v+1, 1)$ -absorbing prime hyperideal. Thus in view of Proposition 3.6 and Proposition 3.7, we have the following result.

Corollary 3.8. *In a hyperring A that is not local, every (u, v) -absorbing prime hyperideal is prime and vice versa.*

Theorem 3.9. *Assume that Q_1 and Q_2 are strong $\mathcal{C}$ -hyperideals of the hyperring A_1 and A_2 , respectively. Let $u, v \in \mathbb{N}$ with $u > v$. Then the following are equivalent:*

- (i) $Q_1 \times Q_2$ is a (u, v) -absorbing prime hyperideal of $A_1 \times A_2$.
- (ii) $Q_1 \times Q_2$ is a prime hyperideal of $A_1 \times A_2$.
- (iii) Q_1 is a prime hyperideal of A_1 and $Q_2 = A_2$ or Q_2 is a prime hyperideal of A_2 and $Q_1 = A_1$.

Proof. It can be easily seen that the idea is true by Corollary 3.8 as $A_1 \times A_2$ is not local. $\square$

Proposition 3.10. *Let $u, v \in \mathbb{N}$ with $u > v$ and Q be a strong $\mathcal{C}$ -hyperideal of A . Then Q is a $(v+1, v)$ -absorbing prime hyperideal of A if and only if Q is a prime hyperideal or A is local such that $M^v \subseteq Q$ for the only maximal hyperideal M of A .*

Proof. ($\implies$) Let Q be a $(v+1, v)$ -absorbing prime hyperideal of A and the hyperring A is not local. Then Q is prime by Corollary 3.8. Suppose that A is local and M is the only maximal hyperideal of A such that $M^v \not\subseteq Q$. If Q is not a $(v, v-1)$ -absorbing prime hyperideal of A , then there exist $x_1, \dots, x_v \in A \setminus U(A)$ such that $x_1 \circ \dots \circ x_v \subseteq Q$ but $x_1 \circ \dots \circ x_{v-1} \not\subseteq Q$ and $x_v \notin Q$. Take any $a \in a_1 \circ \dots \circ a_v$ for every $a_1, \dots, a_v \in M$. Since Q is a $(v+1, v)$ -absorbing prime hyperideal of A , $a \circ x_1 \circ \dots \circ x_v \subseteq Q$ and $x_v \notin Q$, we get $a \circ x_1 \circ \dots \circ x_{v-1} \subseteq Q$. Let $b \in x_1 \circ \dots \circ x_{v-1}$. So $a \circ b \subseteq a \circ x_1 \circ \dots \circ x_{v-1} \subseteq Q$. On the other hand, $a \circ b \subseteq a_1 \circ \dots \circ a_v \circ b$. Then we get $a_1 \circ \dots \circ a_v \circ b \subseteq Q$ as Q is a strong $\mathcal{C}$ -hyperideal of A . If $b \in Q$, then $x_1 \circ \dots \circ x_{v-1} \cap Q \neq \emptyset$ and so $x_1 \circ \dots \circ x_{v-1} \subseteq Q$ which is impossible. Therefore $b \notin Q$. Since Q is a $(v+1, v)$ -absorbing prime hyperideal of A , $a_1 \circ \dots \circ a_v \circ b \subseteq Q$ and $b \notin Q$, we conclude that $a_1 \circ \dots \circ a_v \subseteq Q$ which means $M^v \subseteq Q$, a contradiction. Then we get the result that Q is a $(v, v-1)$ -absorbing prime hyperideal of A . By induction, we conclude that Q is prime as $M^i \not\subseteq Q$ for any $i \leq v$.

($\impliedby$) It is obvious. $\square$

Corollary 3.11. *Assume that $u, v \in \mathbb{N}$ with $u > v$ and Q is a $(v+1, v)$ -absorbing prime strong $\mathcal{C}$ -hyperideal of A that is not prime. If P is a hyperideal of A containing Q , then P is a (u, v) -absorbing prime hyperideal of A .*

Proof. Suppose that Q is a $(v+1, v)$ -absorbing prime strong $\mathcal{C}$ -hyperideal of A that is not prime. By Proposition 3.10, we get the result that A is local and $M^v \subseteq Q$ where M is the only maximal hyperideal of A . Then $M^v \subseteq P$ and so we are done by Proposition 3.10. $\square$

Theorem 3.12. *Let Q be a (u, v) -absorbing prime strong $\mathcal{C}$ -hyperideal of A where $u, v \in \mathbb{N}$ and $u > v$. Then the following statements hold:*

- (i) $\text{rad}(Q)$ is a prime hyperideal of A .

- (ii) If Q is not a prime hyperideal of A , then A is local. In particular, if $u = v + 1$, then $\text{rad}(Q) = M$ where M is the only maximal hyperideal of A .

Proof. (i) Suppose that Q is a (u, v) -absorbing prime strong $\mathcal{C}$ -hyperideal of A . Let $a \circ b \subseteq \text{rad}(Q)$ for $a, b \in A$. We may assume $a, b \in A \setminus U(A)$, without loss of generality. Then we have $a^n \circ b^n \subseteq Q$ for some $n \in \mathbb{N}$. Take any $x \in a^n$ and $y \in b^n$. If x or $y \in U(A)$, then we obtain

$$b^n \subseteq e \circ b^n \subseteq x^{-1} \circ x \circ b^n \subseteq x^{-1} \circ a^n \circ b^n \subseteq Q$$

which means $b \in \text{rad}(Q)$ or

$$a^n \subseteq a^n \circ e \subseteq a^n \circ y \circ y^{-1} \subseteq a^n \circ b^n \circ y^{-1} \subseteq Q$$

which means $a \in \text{rad}(Q)$. Now, we assume that $x, y \notin U(A)$. Since $a^{v-1} \circ x \circ b^{u-v-1} \circ y \subseteq Q$ and Q is a (u, v) -absorbing prime hyperideal of A , we get the result that $a^{v-1} \circ x \subseteq Q$ or $b^{u-v-1} \circ y \subseteq Q$. In the first case, since $(a^{v-1} \circ a^n) \cap Q \neq \emptyset$, we get $a^{v+n-1} = a^{v-1} \circ a^n \subseteq Q$ which means $a \in \text{rad}(Q)$. In the second case, since $(b^{u-v-1} \circ b^n) \cap Q \neq \emptyset$, we have $b^{u-v+n-1} = b^{u-v-1} \circ b^n \subseteq Q$ which implies $b \in \text{rad}(Q)$. Thus $\text{rad}(Q)$ is a prime hyperideal of A .

(ii) Suppose that Q is a (u, v) -absorbing prime strong $\mathcal{C}$ -hyperideal of A that is not prime. Then we conclude that A is local by Corollary 3.8. Let $u = v + 1$ and M be the only maximal hyperideal of A . By Proposition 3.10, we conclude that $M^v \subseteq Q$ and so $\text{rad}(Q) = M$. $\square$

The following lemma is needed in the proof of our next result.

Lemma 3.13. *Let every hyperideal of A be a prime $\mathcal{C}$ -hyperideal. Then A is a hyperfield.*

Proof. Let $0 \neq x \in A$. Since $\langle x^2 \rangle$ is a prime hyperideal of A and $x^2 \subseteq \langle x^2 \rangle$, we get $x \in \langle x^2 \rangle$ which means $x \in x^2 \circ a$ for some $a \in A$. It follows that $0 \in x^2 \circ a - x$. Since $0 \in x^2 \circ a - x \cap \langle 0 \rangle$, we get the result that

$$(x \circ a - 1) \circ x \subseteq x^2 \circ a - x \subseteq \langle 0 \rangle.$$

Let $t \in x \circ a$. Since $\langle 0 \rangle$ is a prime hyperideal, $(t - 1) \circ x \subseteq \langle 0 \rangle$ and $x \neq 0$, we obtain $t - 1 = 0$ and so $1 \in x \circ a$ which means x is unit. Thus we conclude that A is a hyperfield. $\square$

Theorem 3.14. *Let every hyperideal of A be a strong $\mathcal{C}$ -hyperideal, $J(A) = M$ and $v \in \mathbb{N}$. Then every proper hyperideal in A is a $(v + 1, v)$ -absorbing prime hyperideal if and only if A is local and $M^v = 0$.*

Proof. ($\implies$) Suppose on contrary that A is not local. By Corollary 3.8, we conclude that every proper hyperideal in A is prime. Therefore, by Lemma 3.13, we get the result that A is a hyperfield which is impossible. Then A is local. Let M be the only maximal hyperideal of A . Since $\langle 0 \rangle$ is a $(v+1, v)$ -absorbing prime hyperideal and A is local, we have $M^v = 0$ by the hypothesis and Proposition 3.10.

($\impliedby$) Let Q be an arbitrary hyperideal and $x_1 \circ \cdots \circ x_{v+1} \subseteq Q$ for $x_1, \dots, x_{v+1} \in A \setminus U(A)$. Since A is local, M is the only maximal hyperideal of A and $x_1, \dots, x_v \in A \setminus U(A)$, we conclude that $x_1 \circ \cdots \circ x_v \subseteq M^v = 0$ which means $x_1 \circ \cdots \circ x_v \subseteq Q$, as required. $\square$

Theorem 3.15. *Let Q be a proper hyperideal of A and $u, v \in \mathbb{N}$ with $u > v$. Then Q is a (u, v) -absorbing prime hyperideal of A if and only if for proper hyperideals $Q_1, \dots, Q_u$ of A , $Q_1 \circ \cdots \circ Q_u \subseteq Q$ implies that $Q_1 \circ \cdots \circ Q_v \subseteq Q$ or $Q_{v+1} \circ \cdots \circ Q_u \subseteq Q$.*

Proof. ($\implies$) Let Q be a (u, v) -absorbing prime hyperideal of A . Assume that $Q_1 \circ \cdots \circ Q_u \subseteq Q$ where $Q_1, \dots, Q_u$ are proper hyperideals of A such that $Q_{v+1} \circ \cdots \circ Q_u \not\subseteq Q$. Then there exists $x_i \in Q_i$ for each $i \in \{v+1, \dots, u\}$ such that $x_{v+1} \circ \cdots \circ x_u \not\subseteq Q$. Take any $y_i \in Q_i$ for each $i \in \{1, \dots, v\}$. Therefore we have $y_1 \circ \cdots \circ y_v \circ x_{v+1} \circ \cdots \circ x_u \subseteq Q$. This implies that $y_1 \circ \cdots \circ y_v \subseteq Q$ as Q is a (u, v) -absorbing prime hyperideal of A and $x_{v+1} \circ \cdots \circ x_u \not\subseteq Q$. Hence $Q_1 \circ \cdots \circ Q_v \subseteq Q$, as needed.

($\impliedby$) Let $x_1 \circ \cdots \circ x_u \subseteq Q$ for some $x_1, \dots, x_u \in A \setminus U(A)$ such that $x_1 \circ \cdots \circ x_v \not\subseteq Q$. Put $Q_i = \langle x_i \rangle$ for each $i \in \{1, \dots, u\}$. Hence we have $Q_1 \circ \cdots \circ Q_u \subseteq Q$ and $Q_1 \circ \cdots \circ Q_v \not\subseteq Q$. By the hypothesis, we conclude that $Q_{v+1} \circ \cdots \circ Q_u \subseteq Q$ which implies $x_{v+1} \circ \cdots \circ x_u \subseteq Q$. This means that Q is a (u, v) -absorbing prime hyperideal of A . $\square$

Proposition 3.16. *Let $u, v \in \mathbb{N}$ with $u > v$, Q a (u, v) -absorbing prime hyperideal of A and $x \in A \setminus (Q \cup U(A))$. Then $(Q : x)$ is a $(u-1, v-1)$ -absorbing prime hyperideal of A .*

Proof. Let $x_1 \circ \cdots \circ x_{u-1} \subseteq (Q : x)$ for $x_1, \dots, x_{u-1} \in A \setminus U(A)$ such that $x_1 \circ \cdots \circ x_{v-1} \not\subseteq (Q : x)$. So we have $x \circ x_1 \circ \cdots \circ x_{u-1} \subseteq Q$. Since Q is a (u, v) -absorbing prime hyperideal of A and $x \circ x_1 \circ \cdots \circ x_{v-1} \not\subseteq Q$, we get the result that $x_v \circ \cdots \circ x_{u-1} \subseteq Q \subseteq (Q : x)$. This shows that $(Q : x)$ is a $(u-1, v-1)$ -absorbing prime hyperideal of A . $\square$

Assume that A is a multiplicative hyperring. Then the set of all hypermatrices of A is denoted by $M_m(A)$. Let $I = (I_{ij})_{m \times m}$, $J = (J_{ij})_{m \times m} \in P^*(M_m(A))$. Then $I \subseteq J$ if and only if $I_{ij} \subseteq J_{ij}$ [5].

Theorem 3.17. *Let $u, v \in \mathbb{N}$ with $u > v$ and Q be a hyperideal of A . If $M_m(Q)$ is a (u, v) -absorbing prime hyperideal of $M_m(A)$, then Q is a (u, v) -absorbing prime hyperideal of A .*

Proof. Let $x_1 \circ \cdots \circ x_u \subseteq Q$ for $x_1, \dots, x_u \in A \setminus U(A)$. Therefore we have

$$\begin{pmatrix} x_1 \circ \cdots \circ x_u & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \subseteq M_m(Q).$$

Since $M_m(Q)$ is a (u, v) -absorbing prime hyperideal of $M_m(A)$ and

$$\begin{pmatrix} x_1 \circ \cdots \circ x_u & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} = \begin{pmatrix} x_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \circ \cdots \circ \begin{pmatrix} x_u & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

we conclude that

$$\begin{aligned} & \begin{pmatrix} x_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \circ \cdots \circ \begin{pmatrix} x_v & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \\ &= \begin{pmatrix} x_1 \circ \cdots \circ x_v & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \subseteq M_m(Q) \end{aligned}$$

or

$$\begin{aligned} & \begin{pmatrix} x_{v+1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \circ \cdots \circ \begin{pmatrix} x_u & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \\ &= \begin{pmatrix} x_{v+1} \circ \cdots \circ x_u & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \subseteq M_m(Q). \end{aligned}$$

Thus we get the result that $x_1 \circ \cdots \circ x_v \subseteq Q$ or $x_{v+1} \circ \cdots \circ x_u \subseteq Q$. Hence Q is a (u, v) -absorbing prime hyperideal of A . $\square$

Suppose that $(A, +, \circ)$ is a commutative multiplicative hyperring and x is an indeterminate. Then $(A[x], +, \diamond)$ is a polynomial multiplicative hyperring where $tx^n \diamond sx^m = (t \circ s)x^{n+m}$ [10].

Theorem 3.18. *Let $u, v \in \mathbb{N}$ with $u > v$. If Q is a (u, v) -absorbing prime hyperideal of $(A, +, \circ)$, then $Q[x]$ is a (u, v) -absorbing prime hyperideal of $(A[x], +, \diamond)$.*

Proof. Assume that $t_1(x) \diamond \cdots \diamond t_u(x) \subseteq Q[x]$. Without loss of generality, we may assume that $t_i(x) = a_i x^{n_i}$ for each $i \in \{1, \dots, u\}$ and $a_i \in A$. Therefore $a_1 \circ \cdots \circ a_u x^{n_1 + \cdots + n_u} \subseteq Q[x]$. This implies that $a_1 \circ \cdots \circ a_u \subseteq Q$. Since Q is a (u, v) -absorbing prime hyperideal of $(A, +, \circ)$, we obtain $a_1 \circ \cdots \circ a_v \subseteq Q$ or $a_{v+1} \circ \cdots \circ a_u \subseteq Q$ which implies

$$t_1(x) \diamond \cdots \diamond t_v(x) = a_1 x^{n_1} \diamond \cdots \diamond a_v x^{n_v} = a_1 \circ \cdots \circ a_v x^{n_1 + \cdots + n_v} \subseteq Q[x]$$

or

$$t_{v+1}(x) \diamond \cdots \diamond t_u(x) = a_{v+1} x^{n_{v+1}} \diamond \cdots \diamond a_u x^{n_u} = a_{v+1} \circ \cdots \circ a_u x^{n_{v+1} + \cdots + n_u} \subseteq Q[x].$$

Thus $Q[x]$ is a (u, v) -absorbing prime hyperideal of $(A[x], +, \diamond)$. $\square$

In view of Theorem 3.18, we get the following result.

Corollary 3.19. *Suppose that Q is a (u, v) -absorbing prime hyperideal of A . Then $Q[x]$ is a (u, v) -absorbing prime hyperideal of $A[x]$.*

Recall from [17] that a mapping θ from the multiplicative hyperring $(A_1, +_1, \circ_1)$ into the multiplicative hyperring $(A_2, +_2, \circ_2)$ is a hyperring good homomorphism if $\theta(x +_1 y) = \theta(x) +_2 \theta(y)$ and $\theta(x \circ_1 y) = \theta(x) \circ_2 \theta(y)$ for all $x, y \in A_1$.

Theorem 3.20. *Assume that A_1 and A_2 are two multiplicative hyperrings such that $\theta : A_1 \rightarrow A_2$ is a hyperring good homomorphism and $\theta(x) \notin U(A_2)$ for every $x \in A_1 \setminus U(A_1)$. Let $u, v \in \mathbb{N}$ with $u > v$. Then the followings are satisfied:*

- (i) *If Q_2 is a (u, v) -absorbing prime hyperideal of A_2 , then $\theta^{-1}(Q_2)$ is a (u, v) -absorbing prime hyperideal of A_1 .*
- (ii) *If θ is surjective and Q_1 is a (u, v) -absorbing prime $\mathcal{C}$ -hyperideal of A_1 with $\text{Ker}(\theta) \subseteq Q_1$, then $\theta(Q_1)$ is a (u, v) -absorbing prime hyperideal of A_2 .*

Proof. (i) Let $x_1 \circ_1 \cdots \circ_1 x_u \subseteq \theta^{-1}(Q_2)$ for some $x_1, \dots, x_u \in A_1$. Then we get $\theta(x_1 \circ_1 \cdots \circ_1 x_u) = \theta(x_1) \circ_2 \cdots \circ_2 \theta(x_u) \subseteq Q_2$ because θ is a good homomorphism. Since Q_2 is a (u, v) -absorbing prime hyperideal of A_2 , we

obtain $\theta(x_1 \circ_1 \cdots \circ_1 x_v) = \theta(x_1) \circ_2 \cdots \circ_2 \theta(x_v) \subseteq Q_2$ which means $x_1 \circ_1 \cdots \circ_1 x_v \subseteq \theta^{-1}(Q_2)$ or $\theta(x_{v+1} \circ_1 \cdots \circ_1 x_u) = \theta(x_{v+1}) \circ_2 \cdots \circ_2 \theta(x_u) \subseteq Q_2$ which implies $x_{v+1} \circ_1 \cdots \circ_1 x_u \subseteq \theta^{-1}(Q_2)$. Thus $\theta^{-1}(Q_2)$ is a (u, v) -absorbing prime hyperideal of A_1 .

(ii) Let $y_1 \circ_2 \cdots \circ_2 y_u \subseteq \theta(Q_1)$ for $y_1, \dots, y_u \in A_2$. Then there exists $x_i \in A_1$ such that $\theta(x_i) = y_i$ for each $i \in \{1, \dots, u\}$ because θ is surjective. Therefore $\theta(x_1 \circ_1 \cdots \circ_1 x_u) = \theta(x_1) \circ_2 \cdots \circ_2 \theta(x_u) \subseteq \theta(Q_1)$. Now, pick any $t \in x_1 \circ_1 \cdots \circ_1 x_u$. Then $\theta(t) \in \theta(x_1 \circ_1 \cdots \circ_1 x_u) \subseteq \theta(Q_1)$ and so there exists $s \in Q_1$ such that $\theta(t) = \theta(s)$. Then we get $\theta(t - s) = 0$ which means $t - s \in \text{Ker}(\theta) \subseteq Q_1$ and so $t \in Q_1$. Therefore $x_1 \circ_1 \cdots \circ_1 x_u \subseteq Q_1$ as Q_1 is a $\mathcal{C}$ -hyperideal. Since Q_1 is a (u, v) -absorbing prime hyperideal of A_1 , we get the result that $x_1 \circ_1 \cdots \circ_1 x_v \subseteq Q_1$ or $x_{v+1} \circ_1 \cdots \circ_1 x_u \subseteq Q_1$. This means

$$y_1 \circ_2 \cdots \circ_2 y_v = \theta(x_1 \circ_1 \cdots \circ_1 x_v) \subseteq \theta(Q_1)$$

or $y_{v+1} \circ_2 \cdots \circ_2 y_u = \theta(x_{v+1} \circ_1 \cdots \circ_1 x_u) \subseteq \theta(Q_1)$. Consequently, $\theta(Q_1)$ is a (u, v) -absorbing prime hyperideal of A_2 . $\square$

Now, we have the following result.

Corollary 3.21. *Let the hyperideal Q_1 of A be subset the $\mathcal{C}$ -hyperideal Q_2 of A , $x + Q_1 \notin U(A/Q_1)$ for all $x \in A \setminus U(A)$ and $u, v \in \mathbb{N}$ with $u > v$. Then Q_2 is a (u, v) -absorbing prime hyperideal of A if and only if Q_2/Q_1 is a (u, v) -absorbing prime hyperideal of A/Q_1 .*

Proof. Consider the homomorphism $\pi : A \longrightarrow A/Q_1$ defined by $\pi(a) = a + Q_1$. Since π is a good epimorphism, the claim follows from Theorem 3.20 $\square$

Let S be a non-empty subset of A . If S is closed under the hypermultiplication and S contains 1, then S refers to a multiplicative closed subset (briefly, MCS) [5]. Consider the set $(A \times S / \sim)$ of equivalence classes denoted by $S^{-1}A$ such that $(x_1, r_1) \sim (x_2, r_2)$ if and only if there exists $r \in S$ with $r \circ r_1 \circ x_2 = r \circ r_2 \circ x_1$. The equivalence class of $(x, r) \in A \times S$ is denoted by $\frac{x}{r}$. The triple $(S^{-1}A, \oplus, \odot)$ is a multiplicative hyperring where

$$\begin{aligned} \frac{x_1}{r_1} \oplus \frac{x_2}{r_2} &= \frac{r_1 \circ x_2 + r_2 \circ x_1}{r_1 \circ r_2} = \left\{ \frac{a+b}{c} \mid a \in r_1 \circ x_2, b \in r_2 \circ x_1, c \in r_1 \circ r_2 \right\} \\ \frac{x_1}{r_1} \odot \frac{x_2}{r_2} &= \frac{x_1 \circ a_2}{r_1 \circ r_2} = \left\{ \frac{a}{b} \mid a \in x_1 \circ x_2, b \in r_1 \circ r_2 \right\} \end{aligned}$$

The localization map $\pi : A \longrightarrow S^{-1}A$, defined by $a \mapsto \frac{a}{1}$, is a homomorphism of hyperrings. Moreover, if I is a hyperideal of A , then $S^{-1}I$ is a hyperideal of $S^{-1}A$ [24].

Theorem 3.22. *Assume that Q is a $\mathcal{C}$ -hyperideal of A , S a MCS such that $Q \cap S = \emptyset$ and $u, v \in \mathbb{N}$ with $u > v$. If Q is a (u, v) -absorbing prime*

hyperideal of A , then $S^{-1}Q$ is a $(u-1, v-1)$ -absorbing prime hyperideal of $S^{-1}A$.

Proof. Let $\frac{x_1}{r_1} \odot \cdots \odot \frac{x_{u-1}}{r_{u-1}} = \frac{x_1 \odot \cdots \odot x_{u-1}}{r_1 \odot \cdots \odot r_{u-1}} \subseteq S^{-1}Q$ for $x_1, \dots, x_{u-1} \in A \setminus U(A)$ and $r_1, \dots, r_{u-1} \in S$ such that $\frac{x_1}{r_1} \odot \cdots \odot \frac{x_{v-1}}{r_{v-1}} \not\subseteq S^{-1}Q$. Take any $a \in x_1 \odot \cdots \odot x_{u-1}$ and $r \in r_1 \odot \cdots \odot r_{u-1}$. Hence $\frac{a}{r} \in \frac{x_1 \odot \cdots \odot x_{u-1}}{r_1 \odot \cdots \odot r_{u-1}}$ and so $\frac{a}{r} = \frac{a'}{r'}$ for some $a' \in Q$ and $r' \in S$. Then there exists $t \in S$ such that $t \odot a \odot r' = t \odot a' \odot r$. This means $t \odot a \odot r' \subseteq Q$. Since $a \in x_1 \odot \cdots \odot x_{u-1}$, we conclude that

$$t \odot a \odot r' \subseteq t \odot x_1 \odot \cdots \odot x_{u-1} \odot r'.$$

Therefore we have $t \odot x_1 \odot \cdots \odot x_{u-1} \odot r' \subseteq Q$ as Q is a $\mathcal{C}$ -hyperideal of A and $t \odot x_1 \odot \cdots \odot x_{u-1} \odot r' \cap Q \neq \emptyset$. Take any $s \in t \odot r'$. If $s \odot x_1 \odot \cdots \odot x_{v-1} \subseteq Q$, then we get $\frac{x_1}{r_1} \odot \cdots \odot \frac{x_{v-1}}{r_{v-1}} = \frac{x_1 \odot \cdots \odot x_{v-1}}{r_1 \odot \cdots \odot r_{v-1}} = \frac{s \odot x_1 \odot \cdots \odot x_{v-1}}{s \odot r_1 \odot \cdots \odot r_{v-1}} \subseteq S^{-1}Q$ which is impossible. Since Q is a (u, v) -absorbing prime hyperideal of A , $s \odot x_1 \odot \cdots \odot x_{u-1} \subseteq Q$ and $s \odot x_1 \odot \cdots \odot x_{v-1} \not\subseteq Q$, we have $x_v \odot \cdots \odot x_{u-1} \subseteq Q$ which implies $\frac{x_v}{r_v} \odot \cdots \odot \frac{x_{u-1}}{r_{u-1}} = \frac{x_v \odot \cdots \odot x_{u-1}}{r_v \odot \cdots \odot r_{u-1}} \subseteq S^{-1}Q$. This shows that $S^{-1}Q$ is a $(u-1, v-1)$ -absorbing prime hyperideal of $S^{-1}A$. $\square$

Questions Dear readers can think about the following subjects:

- (i) weakly (u, v) -absorbing (prime) hyperideals.
- (ii) ϕ -(u, v)-absorbing (prime) hyperideals.
- (ii) (u, v) -absorbing primary hyperideals.

4. CONCLUSION

In this paper, we introduced and studied the concept of (u, v) -absorbing hyperideals as a generalization of v -absorbing hyperideals in multiplicative hyperrings where $u, v \in \mathbb{Z}$ with $u > v$. We presented several specific results explaining this structure. We indicated that the radical of a (u, v) -absorbing $\mathcal{C}$ -hyperideal Q in a multiplicative hyperring A is a (k, v) -absorbing hyperideal where $k = \max\{\lceil \frac{u}{v} \rceil, v+1\}$. We concluded that the cartesian product of two (u, v) -absorbing hyperideals may not be (u, v) -absorbing by an example. We got the result that a multiplicative hyperring A admits a (u, v) -absorbing hyperideal which is minimal in the set of all (u, v) -absorbing hyperideals of A containing an arbitrary hyperideal. Then we generalized this notion to (u, v) -absorbing prime hyperideals. We gave an example of a (u, v) -absorbing prime hyperideal that is not a (u, v) -absorbing hyperideal. We discussed the fundamental characteristics of (u, v) -prime hyperideals and examined their behavior in several classes of commutative multiplicative hyperrings.

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(u, v) -ABSORBING (PRIME) HYPERIDEALS OVER COMMUTATIVE
MULTIPLICATIVE HYPERRINGS

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ابرایدهال‌های (u, v) -جاذب در ابرحلقه‌های ضربی جابه‌جایی

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در این مقاله، با در نظر گرفتن کلاس ابرحلقه‌های ضربی به معرفی یک تعمیم از ابرایدهال‌های v -جاذب تحت عنوان ابرایدهال‌های (u, v) -جاذب می‌پردازیم. ابرایدهال سره A از یک ابرحلقه ضربی Q یک ابرایدهال (u, v) -جاذب نامیده می‌شود اگر برای عناصر نایکال $x_1, \dots, x_n \in A$ داشته باشیم $x_1 \circ \dots \circ x_u \subseteq Q$ ، آن‌گاه حاصل ضرب v تا از x_i ها زیرمجموعه Q باشد که $u, v \in \mathbb{N}$ و $u > v$. علاوه بر این، به بررسی مفهوم ابرایدهال‌های اول (u, v) -جاذب به عنوان تعمیمی از ابرایدهال‌های (u, v) -جاذب خواهیم پرداخت. مثالی ارائه می‌شود که نشان می‌دهد یک ابرایدهال اول (u, v) -جاذب لزوماً یک ابرایدهال (u, v) -جاذب نیست. برخی از خصوصیات این مفاهیم جدید ارائه می‌شوند.

کلمات کلیدی: ابرایدهال v -جاذب، ابرایدهال (u, v) -جاذب، ابرایدهال اول (u, v) -جاذب.